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NUMERICAL SOLUTION FOR POLYMER TRANSIENT FLOWS IN A CIRCLE BOUNDED COMPOSITE FORMATION*

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Abstract: A new numerical model for transient flows of polymer solution in a circular bounded composite formation is presented in this paper. Typical curves of the wellbore transient pressure are yielded by FEM. The effects of non-Newtonian power-law index, mobility and boundary distance have been considered. It is found that for the mobility ratio larger than 1, which is favorable for the polymer flooding, the pressure derivative curve in log-log form rises up without any hollow. On the other hand, if the pressure derivative curve has a hollow and then is raised up, we say that the polymer flooding fails. Finally, the new model has been extended to more complicated boundary cases.
Keywords: polymer flow, numerical solution, transient flow, circular boundary, composite formation

1. INTRODUCTION

The tertiary recovery technology is widely applied to enhance recovery efficiency. Major tertiary recovery operations include surfactant flooding, surfactant-polymer flooding, polymer flooding, alkaline flooding and alkaline-surfactant-polymer flooding etc. Among them, only the polymer injection technology is widely used in China. There is more than 10 million tons of oil produced annually by using this kind of technology. The recovery efficiency of the polymer tertiary recovery operation can be estimated according to the mobility ratio of polymer solution and oil flows in the reservoir. In the polymer tertiary recovery operation, it is advantageous that the velocity of polymer solution at any point is less than that of oil phase if the mobility ratio is less than 1, then more oil may be recovered. Therefore, it is very likely to significantly increase oil recovery, if one could change the relative magnitudes of the mobility ratio.

Dilute polymer solutions, however, do not possess a constant viscosity and exhibit a non-Newtonian rheological behavior. Most of them closely approximate the Ostwald de Waele power-law fluid [1~3]. Mobility characteristics of fluid banks in a formation can be obtained by pressure transient testing. The methods to find the fluid mobility are well established for Newtonian fluids, but thesemethods are not directly applicable to pressure transient tests of formations for non-Newtonian fluid banks. Methods to analyze this kind of system for power law fluids were developed by Ikoku and Ramey [4] and Odeh and Yang [5]. Other authors have developed some new models based on Ikoku and Odeh's work [6~13].

Transient flow of power law fluids in porous media is relatively a new subject to well testing. The first paper in this field was written by Van Poollen and Jargon [2] in 1969.The authors investigated both steady

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and unsteady flows of non-Newtonian fluids using a numerical simulator. An analytical study of the transient flow of power-law fluids was performed by Odeh and Yang [5]. Using the modified Blake-Kozeny model and the Ostwald de Waele power-law relationship, Ikoku and Ramey [4] derived new partial differential equations for radial flow of power-law fluids through porous media.

This paper presents a new numerical model in the analysis of transient flow of non-Newtonian power-law fluid in a circular bounded reservoir. In the present work, flows of oil and water are simultaneously considered. The results show that for the successful polymer flooding with the mobility ratio larger than 1, the pressure derivative curve in log-log form is rising up without any hollow. The straight line slope in the pressure derivative curve is determined by the power-law index.

2. MODEL DESCRIPTION

2.1 Description of the physical model

We assumed that the reservoir is homogeneous, horizontal and of uniform thickness throughout. The polymer solution is assumed to obey the Ostwald de Waele power-law. Furthermore, the compressibility of the polymer solution remain constant in the range of the temperature and pressure variation encountered in the formation. The density of fluid in the formation obeys an exponential type law. $\rho = \rho_0 e^{-C_t(p_i - p)}$, where ρ is the density at some pressure p, ρ_0 is the density at some standard pressure (conveniently taken as the original pressure p_i and C_t is the compressibility (assumed constant). The polymer solution distribute in the formation in circular band form, see Fig.1. For every band, the power-law index and the equivalent viscosity are different. The well is operated with constant production or injected with constant rate. The outer boundary is the circular closed one.

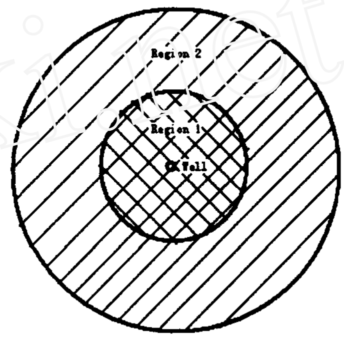

Fig.1 Distribution of the fluid in the formation

2.2 Mathematical formulation

Therefore, the governing equations for the polymer transient flow in the circular bounded reservoir are as follows

$$\frac{\partial^2 p_{1D}}{\partial R_D^2} + \frac{n_1}{R_D}\frac{\partial p_{1D}}{\partial R_D} = \frac{R_D^{1-n_1}}{C_D e^{2S}}\frac{\partial p_{1D}}{\partial T_D} \qquad 1 < R_D < R_{aD} \qquad \text{(inner region)} \tag{1}$$

$$\frac{\partial^2 p_{2D}}{\partial R_D^2} + \frac{n_2}{R_D}\frac{\partial p_{1D}}{\partial R_D} = \frac{R_D^{1-n_2}}{C_D e^{2S}}\frac{\partial p_{2D}}{\partial T_D} \qquad R_{aD} < R_D < R_{eD} \qquad \text{(outer region)} \tag{2}$$

with initial condition

$$p_{1D}(R_D,0) = p_{2D}(R_D,0) = 0 \tag{3}$$

with inner boundary condition

$$\frac{\partial p_{wD}}{\partial T_D} - \frac{\partial p_{1D}}{\partial R_D}\bigg|_{R_D=1} = 1 \tag{4}$$

with outer closed boundary condition

$$\frac{\partial p_{2D}}{\partial R_D}\bigg|_{R_D=R_{eD}} = 0 \tag{5}$$

Interface pressure boundary conditions

$$p_{1D}(R_{aD},T_D)=p_{2D}(R_{aD},T_D) \tag{6}$$

Flow rate continue interface condition

$$R_D^{1-n_1}\frac{\partial p_{1D}}{\partial R_D}=\lambda_m R_D^{1-n_2}\frac{\partial p_{2D}}{\partial R_D} \tag{7}$$

Where C_D is the dimensionless wellbore storage coefficient, $C_D=\dfrac{1.592C}{\phi hC_t r_w}$. p_D is the dimensionless pressure, $p_D=\dfrac{kh(p_i-p)}{1.1842\times10^{-3}q\mu B}$. R_D is the dimensionless distance, $R_D=\dfrac{r}{r_{we}}$. R_{aD} is the dimensionless distance, $R_{aD}=\dfrac{R_a}{r_{we}}$. r_{we} is the effective wellbore radius, $r_{we}=r_w\cdot e^{-S}$. t_D is the dimensionless time, $t_D=\dfrac{3.6kt}{\phi\mu C_t r_w^2}$. T_D is the dimensionless time, $T_D=\dfrac{t_D}{C_D}$. B is the volume factor; C is the wellbore storage factor. C_t is the total compress coefficient for the test formation. h is the net pay of the test formation. k is the permeability of the reservoir; n, n_2 is the power-law index in region 1 and 2. p_i is the initial formation pressure. p is the pressure in the formation. q is the production rate. r_w is the wellbore radius. R_a is the distance of the interface. R_e is the distance of the outer boundary. S is the skin factor. ϕ is the porosity of the reservoir. μ_1,μ_2 is the viscosity of the fluid in region 1 and 2. λ_m is the mobility ratio, $\lambda_m=\dfrac{k_2/\mu_2}{k_1/\mu_1}$.

3. NUMERICAL CACULATION

It's not easy to get analytical solution from equation (1) and (2), so the FEM is selected to solve the polymer transient flow problem. In order to use the FEM, we needed to divide the studied region into a large number of elements. According to the mesh automatic generation method given in the literatures[14~15], the meshes of the reservoir showed in Fig. 1 are generated in Fig. 2. Then, we can derive the finite element equations for every unit in the calculated region, which are shown as followings

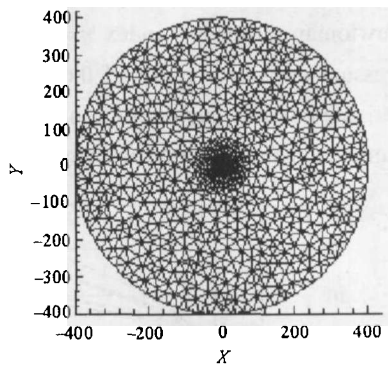

Fig.2 The triangle meshes of the reservoir

$$\iint_A \varphi_i^e\left(\frac{\partial^2 p_{1D}^e}{\partial R_D^2}+\frac{n_1}{R_D}\frac{\partial p_{1D}^e}{\partial R_D}-\frac{R_D^{1-n_1}}{C_De^{2S}}\frac{\partial p_{1D}^e}{\partial T_D}\right)\mathrm{d}A=0,\qquad 1<R_D<R_{aD}\qquad i=1,2,3 \tag{8}$$

$$\iint_A \varphi_i^e\left(\frac{\partial^2 p_{D2}^e}{\partial R_D^2}+\frac{n_2}{R_D}\frac{\partial p_{2D}^e}{\partial R_D}-\frac{R_D^{1-n_2}}{C_De^{2S}}\frac{\partial p_{2D}^e}{\partial T_D}\right)\mathrm{d}A=0,\qquad R_{aD}<R_D<R_{eD}\qquad i=1,2,3 \tag{9}$$

Where φ_i^e is the interpolating function; p_D^e is the pressure at every node of the calculating element.

By FEM, we have solved the transient flow problem for the well located in the centre of the closed circle reservoir. Typical curves of the wellbore pressure vs. time for different C_De^{2S} are calculated as shown in Fig. 3 and Fig.4.

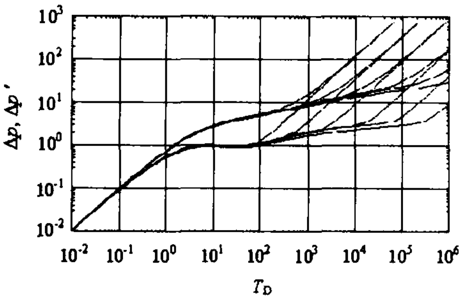

Fig. 3 Numerical solution for $m = 0.5$
(with $R_{eD} = 50, 100, 200, 500, 1\,000, 2\,000$)

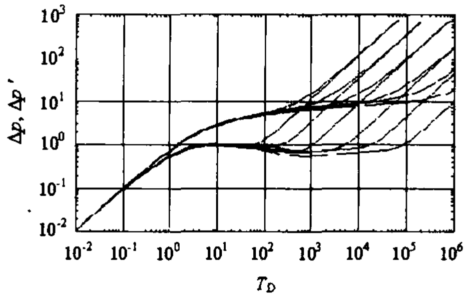

Fig. 4 Numerical solution for $m = 2$
(with $R_{eD} = 50, 100, 200, 500, 1\,000, 2\,000$)

For the polymer injection well, if the mobility ratio is less than 1, the pressure derivative curve rises up. In this case the polymer solution has a higher velocity than the oil in the formation. On the other hand, if the mobility ratio is larger than 1, the pressure derivative cure falls down with a hollow. In this case we say that the polymer flooding is failed.

4. DISCUSSION

The new model can be simplified to solve the transient flow problem in a single connected region. If the outer boundary is large enough, we can get the transient wellbore pressure typical curve in log~log form just like Ikoku and Ramey's analytical solution shown in Fig.5. It is shown in Fig.5 that if the non-Newtonian power-low index $n \neq 1$, we have a parallel straight lines in the last part of the pressure and pressure derivative curve. If the value of the power-low index is equal to 1, the value of the pressure derivate curve is equal to 0.5, namely the Newtonian transient flow case. The comparison of the special numerical solution with the analytical solution for the circular closed boundary is just shown in Fig. 6.

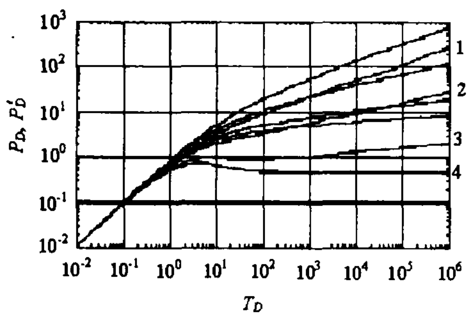

Fig.5 Typical curves for one region in infinite reservoir
(With n=0.3,0.5,0.7,1.0)

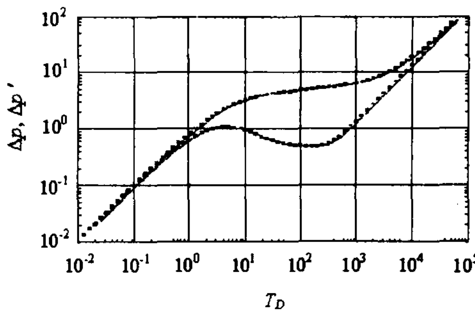

Fig.6 Comparison with the Newtonian problem solution
(With n=1.0)

This paper presents a new numerical model for transient flow of polymer solutions in a circular bounded composite formation. The wellbore transient pressure typical curves are got by FEM. The effects of non-Newtonian power-law index, mobility and the boundary distance have been considered. By analyzing typical curve, we have found that for successful polymer flooding with the mobility ratio less than 1, the pressure derivative curve in log-log form for the injection well is rising up without any hollow. The new numerical model can be expended to the other boundary conditions such as infinite boundary and circular constant pressure boundary. Otherwise, if the power-law index equals 1, the numerical solution is reduced to the Newtonian fluid flow problem.

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