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NUMERICAL SENSITIVITY ANALYSIS AND PARTIAL SIMILARITY OF POROUS MEDIA FLOWS*

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Abstract: A numerical optimization approach to identify dominant dimensionless variables in porous media flows by sensitivity analysis is proposed. We have validated the approach at first by examining a simple oil reservoir theoretically and numerically as well. A more complex water-flooding reservoir is examined based on sensitivity analysis of oil recovery to the similarity parameters, thus demonstrating the feasibility of the proposed approach to identify dominant similarity parameters for water-oil two-phase flows.
Keywords: similarity, optimisation approach, sensitivity analysis, flow in porous media, model, prototype

1. INTRODUCTION

Physical modeling is an alternative fundamental approach in oil exploitation studies. Generally speaking, a model is fully similar to the prototype if each of their corresponding dimensionless variables is kept identical [1]. However, complex flows tend to involve many parameters associated with stratum characteristics, the properties of the fluid including injected surfactants, foam and polymer etc. Furthermore, one should consider all kinds of phenomena in different developing processes, such as convection and diffusion of all components, mass transport between phases, interfacial processes including adsorption, temperature variation, etc.[2,3] Therefore, dimensionless variables are often more than ten in general. Consequently, it is impossible to conform similarity criteria strictly, in particular, inconsistencies between those dimensionless constraints may occur in some circumstances.

To tackle this kind of problems, the efficient and practical way out is to identify the major dimensionless parameters and to neglect the influence of minor ones as previous scientists did. G. I. Taylor set up an excellent example in dealing with the problem of an intense explosion [4]. Ship motion is an additional example exhibiting contradiction to arrive at full similarity by keeping the same *Re* and *Fr* numbers [1].

In the area of porous media flows, many researchers have investigated the principle of scaling law theoretically, experimentally and numerically as well [5~10] when modeling such processes as water or steam flooding, in-situ combustion, etc. Unfortunately, few studies, especially regarding polymer flooding reservoirs, have been reported in literatures on the relationship between dimensionless parameters or dominance sorting among them. Therefore, the problem is still open to us. For more practical purpose, one should find an efficient way to determine the similarity conditions and identify major similarity parameters.

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The authors of the present paper have suggested a numerical optimization approach by sensitivity analysis of a target function to each dimensionless variable. In the following sections, we have demonstrated the feasibility of this method by two examples, a single-phase oil flow and a water-flooding one. In the first case, similarity analysis has validated the approach. We have then defined a target function and analyzed its sensitivity to the dimensionless parameters, which is in accordance with the theory. We have further studied the similarity of the more complex water flooding flow by numerical method and quantified the sensitivity parameters of oil recovery to the dimensionless parameters. In this way, we can make sure which of the dimensionless parameters are essential.

2. CASE I: ANALYSIS OF SIMILARITY OF SINGLE-PHASE OIL FLOW

Assume a simple two-dimensional single-phase oil reservoir with two production wells in it. The pressure distribution $p(x,y,t)$ in the reservoir may be described by

$$\left.\begin{aligned}
&\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}+\sum_{i=1}^{2}\frac{\mu}{K}q_i\delta(x-x_i,y-y_i)=\frac{1}{\chi}\frac{\partial p}{\partial t}\\
&p\big|_{x=0}=p_0,\ p\big|_{x=l_x}=p_l\\
&\left.\frac{\partial p}{\partial y}\right|_{y=0}=0,\ \left.\frac{\partial p}{\partial y}\right|_{y=l_y}=0\\
&p(x,y,0)=p_i
\end{aligned}\right\} \tag{1}$$

where $\chi=\dfrac{K}{\phi\mu c_t}$, in which K is the permeability, ϕ is the porosity of media, μ is the viscosity of oil and c_t is the total compressibility; (x_i, y_i) is the coordinate of the wells; q_i is the discharge of the wells, which is positive for injection and negative for production; δ is the Dirac Delta function; p_0, p_l and p_i are constant.

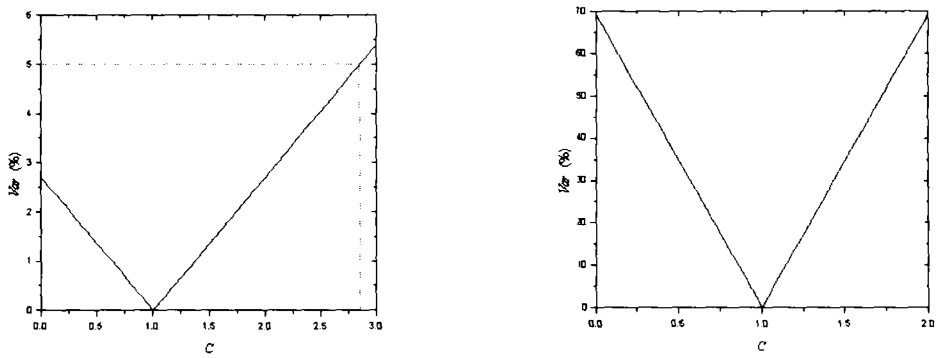
Normalizing Eq. (1) by choosing the following dimensionless parameters

$$p_D=\frac{p-p_0}{p_l-p_0},\quad x_D=\frac{x}{L},\quad y_D=\frac{y}{L},\quad t_D=\frac{t}{T}=\frac{\chi}{L^2}t$$

where L and T are respectively the space and time scale, we may obtain two dimensionless variables $\pi_1=\dfrac{\mu q_1}{K(p_l-p_0)}$ and $\pi_2=\dfrac{\mu q_2}{K(p_l-p_0)}$. Obviously, if $\pi_1>>\pi_2$ in the prototype, then π_1 exerts more considerable influence on pressure distribution than π_2. Now, let us prove this conclusion numerically.

Applying finite difference method to Eq. (1) with forward-time central-space scheme. We may explicitly obtain numerical solution of Eq. (1). Now design a small model reservoir strictly similar to the prototype and define

$$Var=\frac{1}{n_x n_y}\sum_i\sum_j\left|\frac{p_{ij}^m-p_{ij}^p}{p_{ij}^p}\right|,\qquad p_{ij}^m=p_0^p+p_{ijD}^m(p_l^p-p_0^p)$$



(a) $\pi_1^m = \pi_1^p$ and $\pi_2^m = C\pi_2^p$ (b) $\pi_1^m = C\pi_1^p$ and $\pi_2^m = \pi_2^p$

Fig. 1 Variation of *Var* with the sensitivity coefficient C for the cases

as a target function, in which n_x, n_y are grid numbers in x- and y-direction, and the superscripts *m* and *p* (hereafter) indicate variables for the model and the prototype respectively. Apparently, if both $\pi_1^m = \pi_1^p$ and $\pi_2^m = \pi_2^p$ hold, then we have $Var = 0$.

Now, keep one of the two dimensionless variables π_1 and π_2 identical between the model and the prototype and allow the other one slightly deviates from the prototype. In this case, the pressure field of the model may be different from the prototype and the target function from zero. Fig. 1 shows the variation of the target function *Var* with a sensitivity coefficient C that is defined as in the caption below the figure. Clearly, the target function is far more sensitive to the first variable π_1 than to the second π_2, implying the dominant effect of the well with much larger oil production on the pressure distribution. In addition, it can be seen from Fig.1 (a) that π_2 can be as large as 2.8 times of its prototype's value even if 5% error of the target function is acceptable. In this way, not only can we single out the more important similarity parameter but also we can display the acceptable degree of the less influential parameters' deviation.

3. CASE II: WATER-FLOODING OIL FLOW

3.1 Similarity analysis

Consider a five-spot well pattern in a water-flooding reservoir. By normalizing the governing equations, initial and boundary conditions, constitutive relationships, we can derive a set of scaling groups as follows

$$\frac{K_{cwo}}{K_{row}},\quad \frac{K_o}{K_{cwo}},\quad \frac{K_w}{K_{row}},\quad \frac{y_R}{x_R},\quad \frac{x_R y_R}{z_R^2},\quad \frac{x_B}{x_R},\quad \frac{y_B}{y_R},\quad \frac{r_{eo}}{x_R},\quad \frac{r_o}{x_R},\quad \frac{s_{cw}}{\Delta s},\quad \frac{s_{ro}}{\Delta s},$$

$$\frac{s_{wi}-s_{cw}}{\Delta s},\quad \frac{\sigma\sqrt{\frac{\phi_0}{K}}\cos\theta K_{row}h}{q_t\mu_w},\quad \frac{\mu_o}{\mu_w},\ \frac{\rho_{o0}}{\rho_{w0}},\quad \frac{K_{row}h}{q_t\mu_w}\rho_{w0}gz_R,\quad \frac{C_o q_t\mu_w}{K_{row}h},\quad \frac{C_w q_t\mu_w}{K_{row}h},\quad \frac{C_\phi q_w\mu_w}{K_{row}h},$$

$$\frac{p_{w0}K_{row}h}{q_t\mu_w},\quad \frac{p_{o0}K_{row}h}{q_t\mu_w},\quad \frac{p_{wf}K_{row}h}{q_t\mu_w},\quad \frac{p_{oi}K_{row}h}{q_t\mu_w},\quad J(\bar{s}_w)$$

which are identified respectively by $\pi_1, \pi_2, \cdots, \pi_{24}$ in the following text.

It is difficult or sometimes even impossible to ensure full similarity, namely, to keep all these dimensionless parameters identical between a model and its prototype. For example, geometry and gravity similarity contradict with each other if we don't change the gravitational acceleration in laboratory. In this case, we should make clear which of these parameters are vital for model designing. The following section deals this task via numerical approach.

3.2 Sensitivity analysis

A 3D numerical model has been established for water flooding oil flow, which is validated by comparing the results with that of Buckley-Leverett equation as shown in Fig.2. The agreement of waterfront evolution is satisfactory.

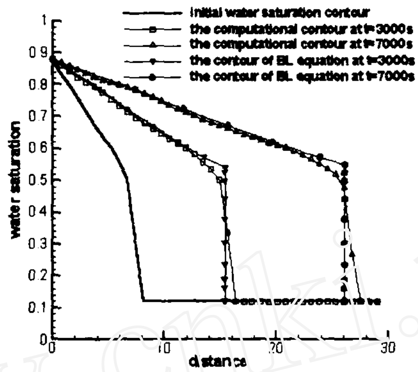


Fig.2 Displacement of iso-water saturation contour at different moment

Now, we define sensitivity coefficients as follows

$$S_i = \frac{\partial f}{\partial \pi_i}, \qquad i=1,2,\ldots 24$$

in which f is a target function defined as $f = \int_0^{T_D} \eta(\pi_1, \pi_2, \ldots, \pi_{24}, t_D) \mathrm{d}t_D$, where T_D is the dimensionless time, and η is the oil recovery curve. Numerically, the sensitivity coefficients read

$$S_i = \frac{\int_0^{T_D} \left|\eta_m - \eta_p\right| \mathrm{d}t_D / f_p}{\left|\pi_{im} - \pi_{ip}\right| / \pi_{ip}}$$

where the subscripts m and p indicate model and prototype respectively. In Table 1 listed are the numerical results of sensitivity coefficients, which range from 10^{-4} to 10^{0}. Apparently, the larger the sensitivity coefficient is, the more important the corresponding dimensionless parameter. Hence, we may easily make a choice of dominant scaling criteria from the numerical results. If we just reserve those parameters ranging from 10^{-1} to 10^{0}, the scaling law looks like

$$\frac{K_{cwo}}{K_{row}}, \quad \frac{K_o}{K_{cwo}}, \quad \frac{K_w}{K_{row}}, \quad \frac{s_{wi} - s_{cw}}{\Delta s}, \quad \frac{\mu_o}{\mu_w}, \quad \frac{\rho_{o0}}{\rho_{w0}}$$

implying that the relation between saturation and permeability, the density and viscosity ratio between water and oil, the initial water saturation are the most important parameters in water flooding modeling.

Table 1 Sensitivity coefficients of scaling group

i	1	2	3	4	5	9	10
S_i	1.8×10^{-1}	1.8×10^{-1}	1.8×10^{-1}	6.4×10^{-2}	2.5×10^{-3}	9.0×10^{-4}	7.3×10^{-3}
i	11	12	13	14	15	16	17
S_i	8.0×10^{-4}	1.2×10^{0}	4.7×10^{-4}	1.8×10^{-1}	1.1×10^{-1}	5.8×10^{-3}	1.4×10^{-3}
i	18	19	20	21	22	23	24
S_i	1.9×10^{-3}	2.1×10^{-3}	1.8×10^{-3}	1.3×10^{-2}	3.2×10^{-2}	2.5×10^{-2}	4.7×10^{-4}

4. CONCLUSIONS

In the present paper, we have proposed a numerical optimization approach to derive oil reservoir similarity. The feasibility of this approach is demonstrated by case study of porous media flows in a single-oil-phase reservoir and a water-flooding two-phase reservoir. The numerical results conform to the theoretical analyses very well. In addition, the foregoing analyses show that the numerical approach can not only single out the dominant parameters to reach approximate or partial similarity but also provide us with to what extent the error between models and prototypes can be induced by neglecting some trivial variables as long as the approximate similarity is concerned. For complex reservoir, the numerical optimization approach seems to be a very efficient method.

NOMENCLATURE

p_o	Pressure of oil phase	ρ_{o0}	Density of oil phase at a given pressure p_{o0}
p_w	Pressure of water phase	ρ_{w0}	Density of water phase at a given pressure p_{w0}
p_c	Capillary pressure for phases water and oil	C_o	Coefficient of compressibility of oil phase
p_{oi}	Initial pressure of oil phase in formation	C_w	Coefficient of compressibility of water phase
p_{o0}	Pressure of oil phase at a given condition	C_f	Coefficient of compressibility of formation
p_{w0}	Pressure of water phase at a given condition	ϕ	Porosity
s_o	Oil saturation	ϕ_0	Porosity at a given condition
s_w	Water saturation	g	Acceleration of gravity
s_{ro}	Residual oil saturation	x_R, y_R, z_R	Characteristic lengths of three directions
s_{cw}	Irreducible water saturation	x_B, y_B	Coordinates of production well
μ_o	Viscosity of oil phase	L	Length of formation
μ_w	Viscosity of water phase	w	width of formation
K	Absolutely permeability	h	Thickness of formation
K_o	Effective permeability to oil phase	σ	Interfacial tension between water and oil
K_w	Effective permeability to water phase	θ	Contact angle between water and oil
K_{cwo}	Effective permeability to oil phase under the condition of irreducible water saturation	r_o	Radius of producing well
K_{row}	Effective permeability to water phase under the condition of residual oil saturation	r_{eo}	Radius of supply
ρ_o	Density of oil phase	q_I	Water volume per unit time of injection well
ρ_w	Density of water phase		

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