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# A New Model for Sediment Transport

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**Abstract** The problem of predicting sediment transportation by water waves is treated analytically with the rate of wave energy dissipation or wave damping. With resorting to the theory of shallow water waves and the basis of Yamamoto's Coulomb-damped poroelastic model, the Boussinesq-type equation has been derived over a variation depth bed. For convenience Cnoidal wave is just discussed, The Cnoidal wave with complex wave length and wave velocity, which are as a function of wave frequency, water depth, permeability, Poisson's ratio and complex elastic moduli of bed soil, is applied to analyse the rate of sediment transportation. Considering the sediment transportation depended on the shear stress near-bed or the horizontal velocity, the conclusion of Yamamoto's experiment in clay bed has been extended to general situation. It could be Fig.d out that the model should provide a method to avoid the undistinguishable factors during sediment transport processes and relate mass transport with the sediment peculiarities.

**Key words:** sediment transport, shallow water wave, porous seabed, energy dissipation rate

## INTRODUCTION

To predict the sediment transport rates is generally an important element in morphological studies in coastal marine environments. One of the challenges is to develop improved predictive models of sediment transport rate in wave and current conditions. As we known, models for steady uniform flows were developed a few decades ago and the many empirical or semi-empirical formulas are used for rive applications. But, in the marine coastal environment, the process of sediment transport becomes increasingly complex due to the presence of different types of unsteady flow, as caused by tidal influence and wind waves. Different lines of sediment transport research and model development are followed to cope with coastal flows, leading to various model concepts in the coastal zone (Ribberink, 1998; Davies, et al., 2002; Camenen, et al., 2003). All in all, the practical sediment transport models still has a strong empirical character and relies heavily on physical insights and quantitative data as obtained in laboratory and field studies. Also more refined mathematical models, such as turbulence closures, two–phase flow models, discrete vortex models and, e.g., were made in order to describe the near bed transport processes (Bakker, 1974; Smith, 1977; Fredsøe et al., 1985; Davies, 1990; Asano, 1990; Hansen et al., 1994). Despite these efforts, the present state of knowledge of the dynamics of sediment transport still does not allow a full dynamic description. Thus to construct the dynamic response of sediment to water waves is very important.

Generally speaking, only to take into account the situation of water wave loading, there are two different causes of sediment motion: one due to viscosity and the other to nonlinear waves. In this paper we concern the later. As known, the time-dependent sediment transport through the nonlinear wave cycle is related to the shear stresses directly near seabed and further development of models is generally hampered by the limited the knowledge of vortex distribution on the interface. In this paper we may steer clear of the complex near-bed transport processes and study the problem from the view of wave damping. This is because the wave-induced soil motion dissipates wave energy and causes the water waves to attenuate, in return, the energy dissipated in the marine floor mobilizes the sediment and induces the forward mass transport. This problem stems from the wave-sediment interaction, though many theoretical models have been proposed by adopting various linear mechanical models for the dynamic behavior of soil (Dalrymple & Liu, 1978; Madsen, 1978; Mei & Foda, 1981; Yamamoto, 1978, 1983), but no theory has been developed to model the wave-induced soil mass transport phenomena. Being differ from general nonlinear wave theory, in this paper, we consider the influence of marine sediment on properties of nonlinear wave. Based on Coulomb-damped poro-elastic models, a set of Boussinesq equations over a porous elastic bed with variational depth are derived for the condition of weakly nonlinear wave. In the case of even porous bed, the Cnoidal wave has been obtained and in comparison with some experiments the qualitative results are discussed. Finally, sediment transport rates related with the rate of wave energy dissipation or wave damping are given. The concept is enlightened by Yamamoto's experiments in water-soil flume.

## GOVERNING EQUATIONS AND BOUNDARY CONDITIONS

The problem in question consists of water waves propagating over a poroelastic bed (see fig. 1). The free surface is displaced by $\eta(x,z,t)$ from still water. Free water has a thickness of *h(x)* and the porous layer of thickness d is underlain by an impermeable bottom.

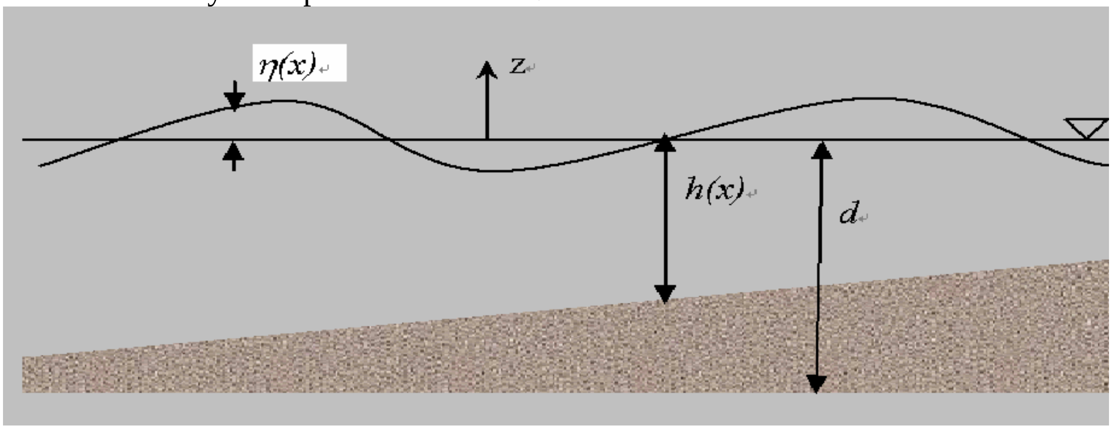


*Fig.1. Definition Sketch for the Wave Propagation Problem*

For the fluid above the seabed we consider incompressible and inviscid and irrotational motion in two dimensions. The velocity potentials $\varphi$ satisfy the Laplace′s equation:

$$\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial z^2}=0 \tag{1}$$

In realistically, the marine soil is a material of three phases (solid, liquid and gas) and the Coulomb friction between gains is proved to play an important role. For this reason the seabed is taken as a slightly inelastic porous bed. Assuming that both soil skeletal frame and pore fluid is compressible and the Darcy´s law is valid, the linear motion equations of marine soil skeleton and fluid for the porous bed based on the Biot's theory may be written as (Referring to Lin, 2001):

$$\begin{cases}\dfrac{\partial^2}{\partial t^2}(\rho\vec{u}+\rho_f\vec{w})=\tilde{\mu}\nabla^2\vec{u}+(\tilde{H}-\tilde{\mu})\nabla e-\tilde{C}\nabla\zeta \\ \dfrac{\partial^2}{\partial t^2}\left(\rho_f\vec{u}+m\vec{w}\right)+\dfrac{\eta_f}{k_s}\dfrac{\partial\vec{w}}{\partial t}=\nabla(\tilde{C}e-\tilde{M}\zeta)\end{cases} \tag{2}$$

where $k_s$, the hydraulic coefficient of permeability of soil; $\eta_f$, the viscosity of the pore fluid; $\rho=(1-\beta)\rho_r+\beta\rho_f$, the bulk density of soil, and $\rho_r$ and $\rho_f$ are densities of grain and fluid, respectively, $\beta$ is the porosity of the soil; $m=\frac{\rho_f}{\beta}(1+\alpha)$, the virtual mass of soil skeletal frame in the accelerated flow field, and $\alpha$ is the added mass coefficient of the skeletal frame; $\zeta=-\nabla\cdot\vec{w}=-\nabla\cdot\beta(\vec{U}-\vec{u})$, in which $\vec{u}(u_x,u_y)$ and $\vec{U}(U_x,U_y)$ are the displacements of soil skeleton frame and pore fluid; $e=\partial u_x/\partial x+\partial u_z/\partial z$, the bulk strain of the skeletal frame of soil; $\tilde{H}$, $\tilde{\mu}$, $\tilde{C}$, $\tilde{M}$, Biot´s complex elastic moduli of porous media, are derived by physical parameters of soil. On account of skeletal frame rigidity and compressibility and pore fluid compressibility, there are three kinds of elastic waves, defined as fast compressible wave and slow compressible wave and shear wave, propagating in the seabed. Noticed that linear Equation (2), each time-independent term of displacement vectors of solid and fluid can be represented by the sum of two longitudinal waves and the transverse wave, that is:

$$\begin{cases} \vec{u} = \nabla \phi_f + \nabla \phi_s + \nabla \times \phi_T \hat{e}_y \\ \vec{w} = \nabla \psi_f + \nabla \psi_s + \nabla \times \psi_T \hat{e}_y \end{cases} \tag{3}$$

and rewrite the solid/fluid-coupled Equation (2) into decoupled scalar equations as:

$$\Delta \phi_{f,s,T} + \tilde{k}^2_{f,s,T} \phi_{f,s,T} = 0 \tag{4}$$

where the subscript $f, s, T$ represent fast wave, slow wave and shear wave respectively and the elastic wave numbers $\tilde{k}_{f,s,T}$ are given in Lin (2001). With these, the Equation (1) and (4) can be applied for water waves and marine soil.

On the free surface $z = \eta(x,t)$ the kinematic and dynamic conditions are:

$$\begin{cases} -\dfrac{\partial \eta}{\partial x}\dfrac{\partial \varphi}{\partial x} + \dfrac{\partial \varphi}{\partial z} = \dfrac{\partial \eta}{\partial t} \\ \dfrac{\partial \varphi}{\partial t} + \dfrac{1}{2}\left[\left(\dfrac{\partial \varphi}{\partial x}\right)^2 + \left(\dfrac{\partial \varphi}{\partial z}\right)^2\right] + g\eta = 0 \end{cases} \tag{5}$$

On the seabed interface, $z = -h(x)$, the mass of the fluid must be conserved, the fluid pressure is transmitted continuously from the sea to the pores medium and the effective stress of solid is continuity also:

$$\begin{cases} \vec{n} \cdot (\vec{u}_t + \vec{w}_t) = \vec{n} \cdot \nabla \varphi \\ p = -\left[\dfrac{\partial \varphi}{\partial t} + \dfrac{1}{2}\left[\left(\dfrac{\partial \varphi}{\partial x}\right)^2 + \left(\dfrac{\partial \varphi}{\partial z}\right)^2\right]\right]\rho_f \\ \vec{n} \cdot \hat{\tau} = 0 \end{cases} \tag{6}$$

where $\vec{n}$ is the unit normal vector. At the impermeable bottom, $z = -d$, the boundary condition between the porous medium and the horizontal bed is:

$$\begin{cases} \vec{u} = 0 \\ w_z = 0 \\ p_z = 0 \end{cases} \tag{7}$$

The problem is now governed by two linear equations constrained by eight boundary conditions. The free-surface conditions and interface continuity of pressure are nonlinear in the new variables $\varphi, \phi_{f,s,T}$ and $\eta$.

## DERIVATION OF BOUSSINESQ EQUATIONS FOR POROUS BEDS

Wave motion is always characterized by three lengths: a water depth $h$, a wave length $l$ and a surface displacement amplitude $a$. To discern the relative importance of terms in the equations, we define length scales for the wave propagation problem as: $\mu \equiv kh_0$, $\varepsilon \equiv a/h_0$, ($k$ is the wave number of incident waves, $h_0$ is the maximum water depth). Dimensionless variables using the relevant characteristic length as follows:

$$x' = kx,\ z' = \frac{z}{h_0},\ h' = \frac{h}{h_0},\ \eta' = \frac{\eta}{a},\ d' = \frac{d}{h_0},\ \vec{u}' = \left(ku_x, \frac{u_z}{h_0}\right),\ \vec{w}' = \left(kw_x, \frac{w_z}{h_0}\right),\ t' = tkc_0$$

$$\tilde{k}'_{f,s,T} = \frac{\tilde{k}_{f,s,T}}{k^2},\ (\tilde{M}', \tilde{C}', \tilde{H}', \tilde{\mu}') = \frac{(\tilde{M}, \tilde{C}, \tilde{H}, \tilde{\mu})}{\rho c_0^2},\ p' = \frac{p}{\rho c_0^2},\ \varphi' = \varphi \frac{kh_0}{ac_0},\ \phi'_{f,s,T} = k^2 \phi_{f,s,T}$$

Omitting the primes for clarity, the above equations and boundary conditions reduced to the following normalized equations:

$$\begin{cases} \mu^2 \varphi_{xx} + \varphi_{zz} = 0 & -h < z < \varepsilon\eta \\ \mu^2 \phi_{(f,s,T)xx} + \tilde{k}^2_{(f,s,T)} \phi_{(f,s,T)zz} = 0 & -h < z < -d \end{cases} \tag{8}$$

And at the free surface, $z = \varepsilon\eta$, the normalized boundary conditions are:

$$\begin{cases} \mu^2 \left(\eta_t + \varepsilon \varphi_x \eta_x\right) = \varphi_z \\ \mu^2 \left(\varphi_t + \eta\right) + \frac{1}{2} \varepsilon \left(\mu^2 \varphi_x^2 + \varphi_z^2\right) = 0 \end{cases} \tag{9}$$

The normalized conditions on the boundary, $z = -h$, between the fluid and the porous medium lead to:

$$\begin{cases} \varepsilon\varphi_z + \varepsilon\mu^2 h_x \varphi_x = \mu^2 \frac{\partial}{\partial t} h_x \left[\left(\tilde{c}_f + 1\right)\phi_{fx} + \left(\tilde{c}_s + 1\right)\phi_{sx} - \frac{1}{\mu}\left(\tilde{c}_T + 1\right)\phi_{Tz}\right] + \frac{\partial}{\partial t}\left[\left(\tilde{c}_f + 1\right)\phi_{fz} + \left(\tilde{c}_s + 1\right)\phi_{sz} + \mu\left(\tilde{c}_T + 1\right)\phi_{Tx}\right] \\ \left(\tilde{M}\tilde{c}_f - \tilde{C}\right)\tilde{k}_f^2 \phi_f + \left(\tilde{M}\tilde{c}_s - \tilde{C}\right)\tilde{k}_s^2 \phi_s = \varepsilon\varphi_t + \frac{1}{2}\varepsilon^2 \varphi_x^2 + \frac{1}{2\mu^2}\varepsilon^2 \varphi_z^2 \\ \left[(\tilde{H} - \tilde{C}) + (\tilde{C} - \tilde{M})\tilde{c}_f\right]\tilde{k}_f^2 \phi_f + \left[(\tilde{H} - \tilde{C}) + (\tilde{C} - \tilde{M})\tilde{c}_s\right]\tilde{k}_s^2 \phi_s = 2\tilde{\mu}(\phi_{fxx} + \phi_{sxx} - \phi_{Txz} / \mu) \end{cases}$$

(10) At the impermeable horizontal bottom, $z = -d$, the normalized boundary conditions are as follows:

$$\begin{cases} \mu\phi_{fx} + \mu\phi_{fx} - \phi_{Tz} = 0 \\ \phi_{fz} + \phi_{sz} + \mu\phi_{Tx} = 0 \\ \tilde{c}_f \phi_{fz} + \tilde{c}_s \phi_{sz} + \mu\tilde{c}_T \phi_{Tx} = 0 \\ \left(\tilde{M}\tilde{c}_f - \tilde{C}\right)\tilde{k}_f^2 \phi_{fz} + \left(\tilde{M}\tilde{c}_s - \tilde{C}\right)\tilde{k}_s^2 \phi_{sz} = 0 \end{cases} \tag{11}$$

In these equations, we can see a distinction in the appearance of horizontal and vertical derivatives of the potential. This suggests that it is possible to decouple the horizontal and vertical dependencies by assuming a certain distribution in one plane and as arbitrary distribution in the other plane. Here the velocity potential $\varphi$ and displacement potential function $\phi_{f,s,T}$ are assumed to admit arbitrary distribution in the horizontal direction and a power series expansion in the vertical direction as follows:

$$\varphi(x,z,t) = \sum_{n=0}^{\infty} \left(z + h(x)\right)^n \varphi^{(n)}(x,t) \tag{12}$$

$$\phi_{f,s,T}(x,z,t) = \sum_{n=0}^{\infty} \left(z + d\right)^n \phi^{(n)}_{f,s,T}(x,t) \tag{13}$$

The above two power series are substituted in the Eqs.(8). Since z is arbitrary, the coefficients of each power of $(z + h(x))$ and $(z + d)$, must vanish, the recurrence relations for $\phi^{(n+2)}, \varphi^{(n+2)}_{f,s,T}$ are,

$$\varphi^{(n+2)} = -\frac{\mu^2 \phi^{(n)}_{xx} + \mu^2 2h_x \left(n+1\right)\varphi^{(n+1)}_x + \mu^2 h_{xx}\left(n+1\right)\varphi^{(n+1)}}{\left(n+1\right)\left(n+2\right)\left(1 + \mu^2 h_x^2\right)} \qquad n = 0,1,2,...... \tag{14}$$

$$\phi_{f,s,T}^{(n+2)} = -\frac{\mu^2(\phi_{(f,s,T)xx}^{(n)} + \tilde{k}_{f,s,T}^2\phi_{f,s,T}^{(n)})}{(n+1)(n+2)} \qquad n = 0,1,2,....... \tag{15}$$

Substituting Eqs.(15) into boundary conditions at impermeable bottom (11), since the terms after the first are all zeroes, that gives:

$$\begin{cases} \phi_{fx}^{(0)} + \phi_{sx}^{(0)} - \dfrac{1}{\mu}\phi_T^{(1)} = 0 \\ \phi_f^{(1)} + \phi_s^{(1)} + \phi_{Tx}^{(0)} = 0 \\ \tilde{c}_f\phi_f^{(1)} + \tilde{c}_s\phi_s^{(1)} + \tilde{c}_T\phi_{Tx}^{(0)} = 0 \\ \left(\tilde{M}\tilde{c}_f - \tilde{C}\right)\tilde{k}_f^2\phi_f^{(1)} + \left(\tilde{M}\tilde{c}_s - \tilde{C}\right)\tilde{k}_s^2\phi_s^{(1)} = 0 \end{cases} \tag{16}$$

which implies from Eqs. (16) that all $\varphi_{f,s}^{(n)}$'s with odd n vanish. For the even $\varphi_{f,s}^{(n)}$'s, we have:

$$\phi_{f,s}^{(2n)} = (-1)^n\frac{\mu^{2n}(\phi_{(f,s)x^{2n}}^{(0)} + \tilde{k}_{f,s}^2\phi_{f,s}^{(0)})}{2n!} \qquad n = 1,2,3,.... \tag{17}$$

The power series for the potential equations are then substituted Eqs. (14) and boundary conditions (9), (10) and retain terms to a given order in $\varepsilon$ and $\mu$, we obtain the following equations for the velocity potential and the surface elevation $\eta$:

$$\begin{cases} \mu^2\eta_t + \varepsilon\mu^2\varphi_x^{(0)}\eta_x + \mu^2(h+\varepsilon\eta)\varphi_{xx}^{(0)} - \dfrac{1}{6}\mu^4h^3\varphi_{xxxx}^{(0)} = A\varphi_x^{(0)} + B\varphi_{xx}^{(0)} + C\varphi_{xxx}^{(0)} + \mu^2 d\left[\tilde{E}\varphi_{xxtt}^{(0)} - \tilde{F}\varphi_{tt}^{(0)}\right] + O\left(\varepsilon\mu^4,\mu^5\right) \\ \mu^2\varphi_t^{(0)} + \mu^2\dfrac{1}{2}\varepsilon\left(\varphi_x^{(0)}\right)^2 + \mu^2\eta - \dfrac{1}{3}\mu^4h^2\varphi_{xxt}^{(0)} = \mu^4\left[\left(hh_x\right)_x\varphi_{xt}^{(0)} + 2hh_x\varphi_{xxt}^{(0)} - dh\tilde{F}\varphi_{ttt}^{(0)}\right] + O\left(\varepsilon\mu^4,\mu^5\right) \end{cases} \tag{18}$$

where

$$A = -\mu^2h_x + \mu^4\left[h_x^3 + 3hh_xh_{xx} + \frac{1}{2}h^2h_{xxx}\right]$$

$$B = 3\mu^4\left[hh_x^2 + h^2h_{xx}\right]$$

$$C = \frac{3}{2}\mu^4h^2h_x$$

$$\tilde{A}_{1,2} = \tilde{M}\tilde{c}_{f,s} + \tilde{C}$$

$$\tilde{A}_{3,4} = \left(\tilde{H} - \tilde{C}\right) + \left(\tilde{C} - \tilde{M}\right)\tilde{c}_{f,s}$$

$$\tilde{D} = \tilde{A}_1\tilde{A}_4\tilde{k}_f^2\tilde{k}_s^2 - \tilde{A}_2\tilde{A}_3\tilde{k}_f^2\tilde{k}_s^2$$

$$\tilde{E} = \left[\left(\tilde{c}_T + \tilde{c}_f\right)\tilde{A}_4\tilde{k}_s^2 - \left(\tilde{c}_T + \tilde{c}_s\right)\tilde{A}_3\tilde{k}_f^2\right]\Big/\tilde{D}$$

$$\tilde{F} = \left[\left(\tilde{c}_f + 1\right)\tilde{A}_4 - \left(\tilde{c}_s + 1\right)\tilde{A}_3\right]\tilde{k}_f^2\tilde{k}_s^2\Big/\tilde{D}$$

Eqs. (18) is the namely Boussinesq-type equation for variation depth, which comprise the first-order approximation for the flow in the porous medium. For practical applications, such as the propagation of a wave on a permeable beach, the wave transformation around artificial reefs and submerged porous structures, and sediment transport, the above equation should be valid. Since the model connects basic characteristics of wave field and marine sediment together, it will be useful to understand the complex physical processes that occur in these environments.

If $h_x = O(\varepsilon)$, for weakly nonlinear wave, the above equation are approximated to include terms of order $O(\varepsilon, \mu^2)$ only, obtaining:

$$\begin{cases} \eta_t + \varepsilon\varphi_x^{(0)}\eta_x + (h+\varepsilon\eta)\varphi_{xx}^{(0)} - \dfrac{1}{6}\mu^2 h^3 \varphi_{xxxx}^{(0)} = d\left[\tilde{E}\varphi_{xxtt}^{(0)} - \tilde{F}\varphi_{tt}^{(0)}\right] + O\left(\varepsilon\mu, \mu^3\right) \\ \varphi_t^{(0)} + \dfrac{1}{2}\varepsilon\left(\varphi_x^{(0)}\right)^2 + \eta - \dfrac{1}{2}\mu^2 h^2 \varphi_{xxt}^{(0)} = \mu^2 dh\left[-\tilde{F}\varphi_{ttt}^{(0)}\right] + O\left(\varepsilon\mu, \mu^3\right) \end{cases} \tag{19}$$

It is obvious that only $\tilde{E}, \tilde{F}$ relate to sediment properties, if $\tilde{E}, \tilde{F}$ =0, Eqs. (19) reduces to Boussinesq equation derived by Peregrine (1967).

## SEDIMENT TRANSPORT DISCUSSION

Firstly, in this section, we will limit the above shallow water equation to describe Cnoidal waves. Assuming that the bottom is even, we derived a single equation for $\varphi$:

$$[1+d\tilde{F}]\varphi_{tt} - \varphi_{xx} = [\frac{1}{2}\mu^2 + d\tilde{E}]\varphi_{xxxx} - \mu^2 d\tilde{F}\varphi_{xxxx} - \frac{1}{6}\mu^2\varphi_{xxxx} + \varepsilon\left(\varphi_x^2 + \frac{3+d\tilde{F}}{2}\varphi_t^2\right)_t \tag{20}$$

where terms smaller than $O(\varepsilon, \mu^2)$ have been ignored. Supposing that the motion is harmonics:

$\varphi = \varphi(\xi) = \varphi\left(x - ct\right)$

and

$$\frac{\partial}{\partial x} = \frac{d}{d\xi} \equiv (\ )', \qquad \frac{\partial}{\partial t} = -c\frac{d}{d\xi} \equiv -c(\ )'$$

Substituted above relations into Eqs. (20), after cumbersome deducing, Eqs. (20) can be written with respect to $\xi$:

$$\frac{\nu^2}{\varepsilon}\left(\frac{1}{3} + d\tilde{E} - d\tilde{F}\right)\xi = -\xi^3 + \frac{\left[1+d\tilde{F}\right]c^2 - 1}{\varepsilon(3+d\tilde{F})}\xi^2 + C_1\xi + C_2$$

where $C_1$ and $C_2$ are integration constants. Finally the surface profile is:

$$\begin{cases} \eta = HCn^2\left[\dfrac{2K(m)}{\lambda}(x-ct)\right] \\ \lambda = 4K(m)h\left[\dfrac{mh}{3H(1-d\rho g\tilde{F}/6)}\left(1+3d\rho g\tilde{F} - 3d\rho g\tilde{E}/h^2\right)\right]^{1/2} \\ c^2 = \dfrac{gh}{1+d\rho g\tilde{F}}\left[1 + H(1-d\rho g\tilde{F}/6)\dfrac{1}{h}\dfrac{1}{m}\left(2-m-3E(m)/2K(m)\right)\right] \end{cases} \tag{21}$$

where $Cn$ is the Jacobian elliptic function with modulus $m$; $K(m)$ is the complete elliptic integral of the first kind; $E(m)$ is the complete elliptic integral of the second kind. It is noticed interestingly that the wave length and the wave velocity are the complex number as function of many parameters of loading waves and soil bed. As $\tilde{E}, \tilde{F}$ is zero, the above equation reduced to horizontal rigid bottom situation (Mei, 1983). In order to confirm our derived results, we compare the wavenumber and wave profile with Cheng's experiments (See Table 1 and Fig.2, the solid line correspond to calculation results and dots to experiments measurements). Virtually no difference is found between the theory and the experiments for this case.

Because the soil motion can be directly coupled to the instantaneous bed shear stress or near-bed horizontal orbital velocity, we concern the properties of the loading wave and horizontal velocity near bed. The surface profile and maximum horizontal velocity near bed with modulus and two kinds of seabed (sand bed and rigid bed) are plotted in Fig. 3 and Fig. 4. It is particularly interesting to note that the wave length over sand bed becomes longer than that over rigid bed, but the maximum horizontal velocity smaller, that is to say, the soil motion is decreased. The results are agreement with some experiments qualitatively (Dibajnia, et al, 1992).

*Table 1 Values of Wavenumber*

| | Wave Hight | Wave Period | Wavenumber (experiments） | Wavenumber (calculation) |
|---|---|---|---|---|
| Case 1 | 3.05(cm) | 1.025(s) | $4.2488+i2.7357\times10^{-3}$ ($m^{-1}$) | $4.1464+i2.4321\times10^{-3}$($m^{-1}$) |
| Case 2 | 2.52(cm) | 1.26(s) | $3.1593+i3.1299\times10^{-3}$($m^{-1}$) | $3.0387+i2.8937\times10^{-3}$($m^{-1}$) |
| Case 3 | 3.57(cm) | 1.55(s) | $2.4262+i3.0259\times10^{-3}$($m^{-1}$) | $2.5356+i2.9462\times10^{-3}$($m^{-1}$) |

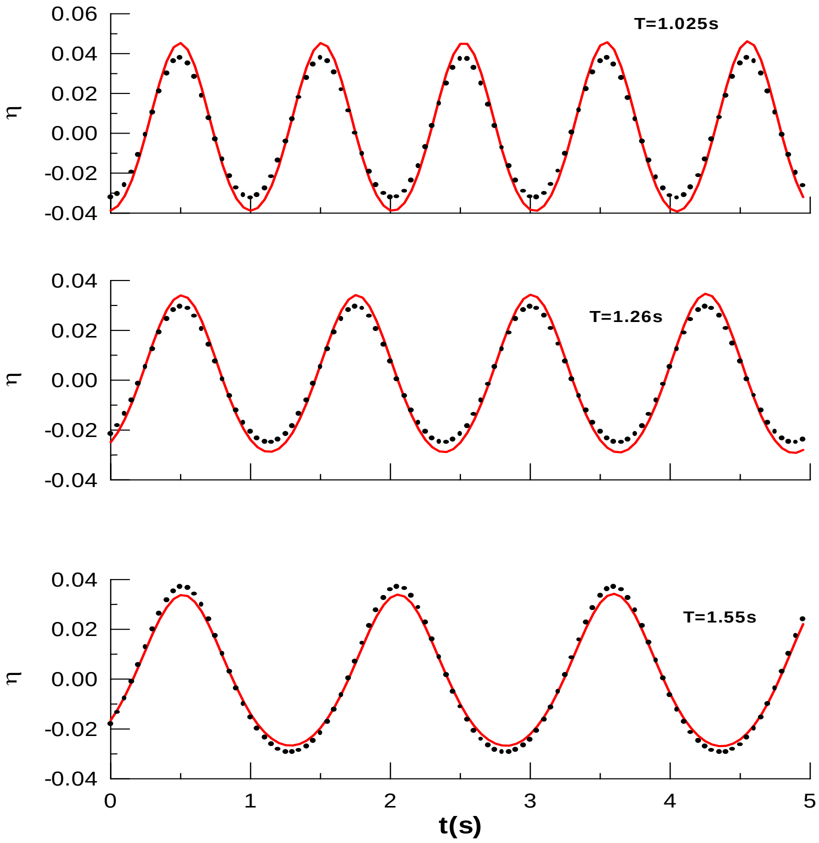


*Fig.2 Comparison Results of Theory and Eperiment*

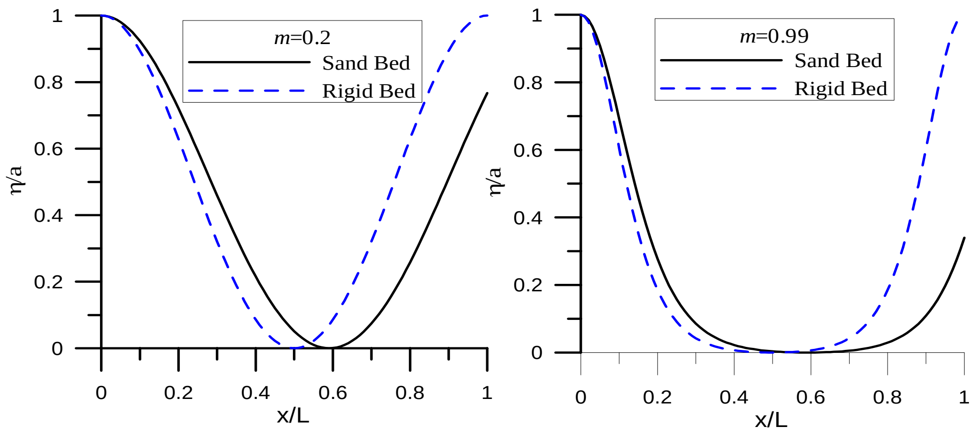


*Fig.3 Surface Profile with Different Seabed*

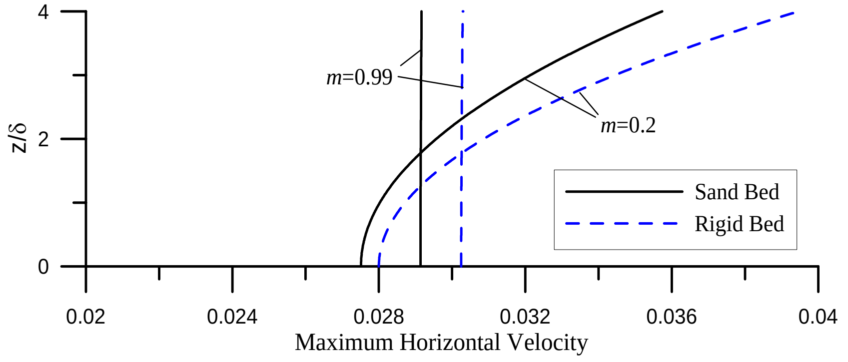


*Fig.4 Distribution of Mximum Horizental Velocity*

Secondly, we will discuss the problem of sediment transport. As water waves propagate over the porous medium, if Shields number is greater than the critical Shields number, the grains, which continuously contact with the bed, begin to roll, slide and jump. The transportation process is rather complicated. For the sake of keeping away from the cumbrance, in this section, we will discuss the problem of sediment transport as viewed from the wave energy dissipation.

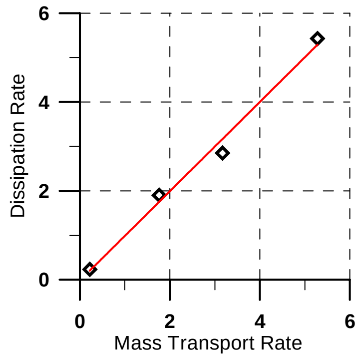


*Fig.5 Measured Mass Transport and Wave Energy Dissipation Rates*

It is demonstrated in the Yamamoto's experiments on wave-soil interaction, that the wave damping is uniquely governed by the sediment motion near interface and the rate of wave-induced mass transport is found to be linearly related to the rate of wave energy dissipation (as shown in Fig.5). Now we extend the results to general situation. Thus the rate of mass transport can be determined by the value of wave energy dissipation rate J:

$$J = 2DUE$$

in which J is the rate of energy dissipation per unit area of interface, $U$ is the wave energy transport velocity, and $E(=\rho g H^2/8)$ is total wave energy density per unit surface area. In the above expression $D$, the damping coefficient, is defined by:

$$k = k_0 + \Delta k = k' + iD$$

where $k_0$ is the rigid bed wavenumber, $\Delta k$ is a complex deviation from $k_0$ and $k'$ is the change in wavenumber. Therefore, to search damping coefficient D is the key problem. From Eqs.(21), we can obtain the relation for the case of the Cnoidal wave.

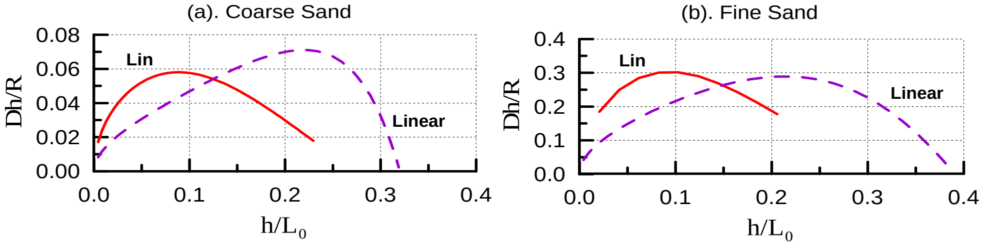


*Fig.6. Wave Damping vs. Water Depth*

Fig.6 is plotted the normalized wave damping rate with various normalized water depth, in which $R \equiv \omega k_s / \nu$ is the nondimensional permeability and $h / L_0 \equiv \omega^2 h / 2\pi g$. Because the original Boussinesq equations for impermeable beds are valid up to $h/L_0$=0.22 and this bound becomes smaller in the presence of the porous layer, we propose the valid range of the damping rate is $h/L_0$<0.2. As $h/L_0 \geq 0.2$ the damping rate may fit the results of linear theory. It is noticed that the normalized damping rates reaches the maximum around $h/L_0$=0.08. This implies that high transportation will occur here. Referring to the Yamamoto's experiment value, $h/L_0$=0.064 ($h$=0.4m, $T$=2s), we calculate the rates in the clay bed and coarse sand bed. This is in good agreement with the measurements in the clay bed (see Fig. 7). For the coarse sand bed, the results is based on the assumption that rate of mass transport may proportionate to the value of wave energy dissipation rate. We expect the assumption will be confirmed by experiments in the near future.

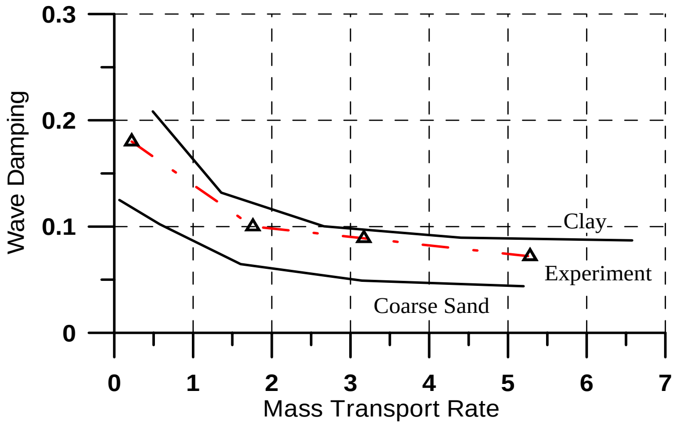


*Fig.7. Mass Transport vs. Wave Damping*

## CONCLUSIONS

This paper proposes a model to study sediment transportation by water waves. Being that the nonlinear characteristic of water waves is the dominant cause of the sediment net transportation, the nonlinear dispersive shallow water equations developed for the description of water waves propagating over a porous seabed with depth variation has been derived. The equations, taken into account the interaction of water wave and porous seabed, include the dynamic response near interface. In order to investigate sediment transportation, the Cnoidal wave surface profile, with complex wave length and complex wave velocity, have been obtained by simplifying the equations to even bed. And then, the mass transport rates have been obtained by using the complex wave length relation. It should be stressed that the model is based on the idea of substituting the rate of wave energy dissipation for the rate of mass transport. Although the conclusion of Yamamoto's experiment is for clay bed, we consider it should be extended to common situation according to the knowledge of sediment motion mechanics.

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