Analysis of behaviors of shock focusing in the inner cavities of double wedge and cone

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Abstract. The mechanisms of shock focusing in inner cavities of double wedge and cone are compared with that of traditional curved-surface shock focusing. The results show that there are many high temperature regions just behind shock surface which appear in two place alternately, one is near the surface of wall and the other is near the centerline. Also, changes in temperature, pressure, energy and power of the high temperature regions were analyzed and the results show that energy and power per unit volume increase, but total energy and power in the high temperature regions decrease during the process of shock moving forward the apex of double wedge or cone.

1 Background

Shock focusing can be classified into two types: one is the focusing of curved-surface shock wave, which is traditional one, the other is the shock focusing which takes place while planar shock enters into an inner double wedge or cone. Obviously, these two phenomena mentioned above are different. According to the viewpoint of Milton ([3]), shock focusing is that the maximisation of the pressure and volume of high pressure gas (maximisation of energy) at or near a point is obtain, that is, focusing is the maximisation of the wave energy transferred to a single point (called the focus) in the flow field.

Recently, shock focusing as a new method for igniting combustible mixture has been an interesting research topic, especially for the shock focusing in an inner cavity of double wedge or cone ([2], [1]).

However, theoretical analysis to the behavior of parameters in case of shock focusing in the inner cavities of double wedge and cone has not been studied systematically. So the purpose of this paper is to study the changes in temperature, energy and power in the high temperature region during the process of shock focusing.

2 Behaviors of shock focusing

When incident shock moves into an inner cavity of double wedge or cone, two reflection modes on the wall may take place, regular reflection or Mach reflection.

Either in an inner double wedge or cone, when regular reflection on the wall takes place, because the intensity of incident shock whick moves forward the apex of double wedge or cone does not change, so shock focusing does not occur. Fig. 1 illustrates the interactions of shock waves while regular reflection on the wall takes place. Obviously, temperature and pressure limits exist in case of regular reflection in an inner cavity of either double wedge or cone.

In case of Mach reflection, when incident shock enters into the inner cavity of wedge or cone, shock-shock disturbance loci are generated from the wall, so as Mach stems and triple-points. Then, Mach stems from the wall grow until Mach stems from upper wall and from lower wall collide. At this time, incident shock disappears, and shock front is compose of the generated Mach stems. Next, because the new shock front cannot maintain its shape when travelling forwards, new shock-shock disturbance loci from the centerline will be generated, so as Mach

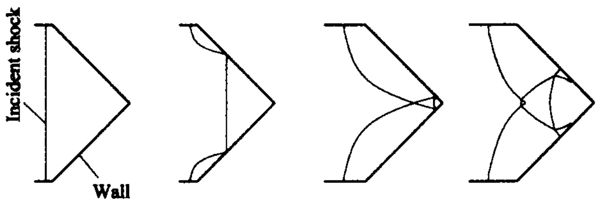

Fig. 1. Regular reflection in a wedge/cone.

stems and triple-points from the centerline. Mach stems from the centerline grow gradually until reach to the wall.

Consequently, this process will be repeated and repeated, that is, new shock-shock disturbance loci, Mach stems and triple-points from the wall and from the centerline are generated alternately, as shown in Fig. 2. At last, the shock is strengthened step by step and shock focusing forms. In other words, shock focusing in case of Mach reflection is realized by the travelling of triple-points between the wall and centerline and shock-shock disturbances propagating on the shock surface continuously. As a result, the Mach number of shock front will be stronger and stronger, so as temperature and pressure just behind the shock front. In theory, the temperature and pressure of shock focusing can reach arbitrarily high value if the space of shock focusing region approaches to infinitesimal.

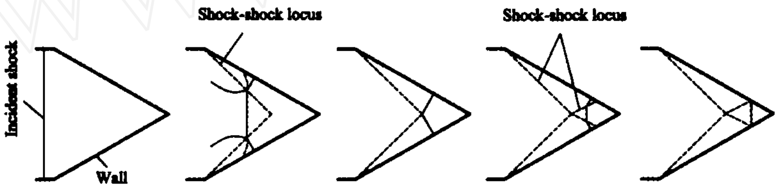

Fig. 2. Shock focusing regime in an inner wedge in case of Mach reflection.

In all cases of shock focusing above, there are some high temperature regions in the flowfield. They are distinctly surrounded by the Mach stem, contact surface and wall or centerline, as illustrated in Fig. 3.

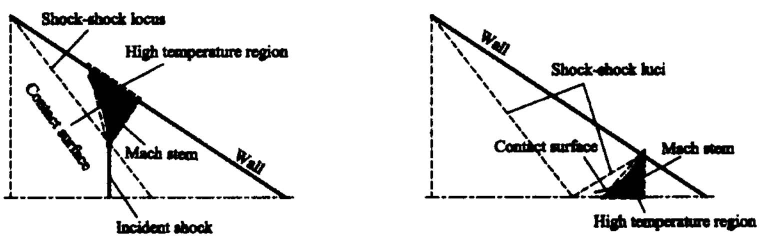

Fig. 3. High temperature regions just behind shock front.

From the results of analysis above, we can also find that while moving shock travels forwards, the more high temperature region appears alternately in two places, one is near the surface of wall, the other is near the centerline. This is different from the traditional curved-surface shock focusing. While the moving shock travels to the apex of wedge or cone, the high temperature region approaches to infinitesimal, and temperature approaches infinity, which is the same as that of traditional curved-surface shock focusing.

3 Methods of shock dynamics for shock focusing

For planer shock focusing which takes place in inner cavities of double wedge or cone, shock dynamics is a simple method to estimate parameters of shock focusing.

As mentioned before, there are mainly two shock-shock regimes for the focusing of planer shock in inner wedge or cone: shock-shock loci from the wall and shock-shock loci from the centerline. These two regimes can be conveniently modelled by methods of shock dynamics, as shown in Fig. 4. The difference of shock-shock loci in an inner wedge and shock-shock loci in an inner cone is that shock-shock loci in an inner cone are curves, but shock-shock loci in an inner double wedge are straight lines. For shock focusing in an inner double wedge, both shock-shock on the wall and shock-shock on the centerline can be described by the following equations:

$$\begin{cases} \dfrac{\cos(\chi+\theta_w)}{M_0} = \dfrac{\cos(\chi)}{M_1} \\ \dfrac{\sin(\chi+\theta_w)}{A_0} = \dfrac{\sin(\chi)}{A_1} \\ \dfrac{A_1}{A_0} = \dfrac{f(M_1)}{f(M_0)} \end{cases} \tag{1}$$

where $f(M)$ can be deduced by CCW relation and $f(M) = e^{-\int \frac{2MdM}{(M^2-1)\kappa(m)}}$

For shock focusing in an inner cone, shock-shock on the wall and shock-shock on the centerline must be handled respectively. Control equations for shock-shock on the wall:

$$\begin{cases} \dfrac{A_1}{A_0} = \dfrac{\sin(\chi_0-\theta)}{\sin\chi_0}\left[\dfrac{r-\frac{(1-r)\tan(\chi_0-\theta)}{2\tan\theta}}{1-\frac{(1-r)\sin\chi_0}{2\sin\theta\cos(\chi_0-\theta)}}\right] \\ \dfrac{d\chi_0}{dr} = \left[\dfrac{M_0}{M_1} - \dfrac{\cos\chi_0}{\cos(\chi_0-\theta)}\right]\left[\dfrac{\cos^2(\chi_0-\theta)}{(1-r)\sin\theta}\right] \\ \dfrac{A_1}{A_0} = \dfrac{f(M_1)}{f(M_0)} \end{cases} \tag{2}$$

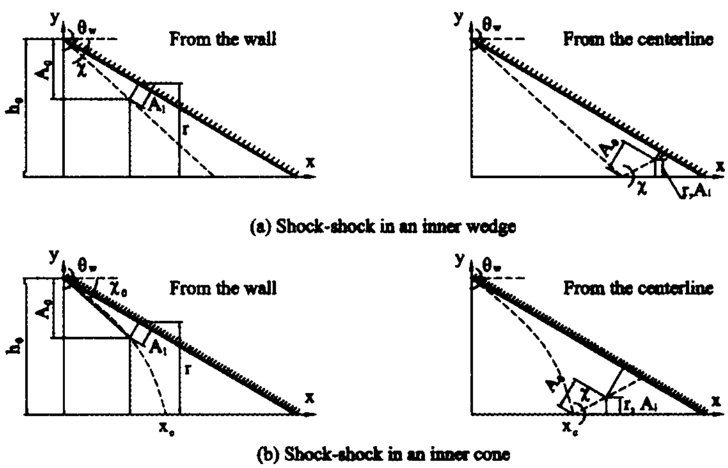

Fig. 4. Shock-shock regimes of shock focusing in inner wedge and cone.

and equations for shock-shock on the centerline:

$$\begin{cases} \frac{A_1}{A_0} = \frac{\sin^2(\chi+\theta)}{\sin^2\chi} \\ \frac{d\chi}{dr} = \frac{\sin\chi}{r}\left[\frac{\frac{M_1}{M_0}\cos(\chi+\theta)-\cos(\chi)}{\frac{M_1}{M_0}\cos\theta-1}\right] \\ \frac{A_1}{A_0} = \frac{f(M_1)}{f(M_0)} \end{cases} \tag{3}$$

where $f(M)$ is same as that of shock-shock in an inner wedge.

It should be noticed that these two equations are differential equations and must be solved by method of integral. In our calculation, we use forth-order Runge-Kutta differential algorithm.

4 Changes in energy and power in the process of shock focusing

When planar shock focuses in an inner double wedge or cone, there are some high temperature regions just behind the shock front, which are surrounded by the Mach stems, contact surfaces and wall or centerline. Energy and power in these regions can be calculated. Integrating the following equation, we can get the energy in the high temperature region:

$$E_i = \int_{\Omega} \rho c_v T dv \tag{4}$$

where Ω is the region surrounded by Mach stem, contact surface and wall or centerline, and ρ, c_v, T are parameters in this region. Then, selecting t_c, the time of shock front propagating from initial disturbance point to the current location of shock front, as the characteristic time of shock propagating, we can get the power in the high temperature region:

$$P_i = E_i/t_c \tag{5}$$

When incident shock moves to the point at which the shock-shock locus intersects with centerline, the length of Mach stem reaches to maximum value, as shown in Fig. 3. Similarly, in the successive reflection of shocks, we have a lot of maximum length of Mach stems. The maximum length of Mach stem approaches to infinitesimal when the moving shock reaches to the apex.

Also, energy and power per unit volume are also important parameters for shock focusing. They can be calculated by the following expressions:

$$E_v = E/V \tag{6}$$

$$P_v = P/V \tag{7}$$

where V is the volume of high temperature region Ω.

5 Results and discussion

Fig. 5 illustrates an example of shock focusing in an inner double wedge. Calculation conditions are designed as followings: $p_0 = 0.02MPa$, $T_0 = 300K$, $M_s = 2.5$, half height of entrance $h_0 = 0.047m$, wall angle $\theta_w = 20°$. From the results of calculation, we can find that: during the process of shock travelling forwards, shock-shock loci are generated from wall and centerline and shock Mach number, temperature and pressure in the high temperature regions are increased step by step.

When Mach stem grows from initial disturbance point to its maximum length, energy and power per unit volume are kept unchanged for the wedge, but due to enlargement of high temperature region, total energy and power increase until next Mach stem appears. During the whole process of shock moving forwards to the apex of wedge, because the maximum Mach stem length decreases sharply, total energy and power in the high temperature region decrease, but energy and power per unit volume increase greatly.

Fig. 6 illustrates an example of shock focusing in an inner cone. While the shock moves forwards to apex of the cone, shock Mach number, temperature and pressure in high temperature regions increase, same as that of shock focusing in an inner wedge. We can notice that different from shock focusing in an inner double wedge, the shock-shock loci are curves, which means that when Mach stem grows, shock becomes stronger and stronger. Besides, the energy and power per unit volume in high temperature regions increase and total energy and power decrease in the whole process of shock focusing.

Compare the results of shock focusing in an inner double wedge with that of shock focusing in an inner cone, we can also conclude that shock focusing in an inner cone is more effective than that in an inner wedge, if other conditions given same.

6 Concluding remarks

It follows from the results that only in case of Mach reflection on the wall, does shock focusing exist. From the results of analysis and calculation, we find that while the moving shock travels forwards, the more high temperature region appears alternately in two places, one is near the surface of wall, the other is near the centerline. This is different from the traditional curved-shock focusing. While the moving shock travels to the apex of wedge or cone, the high temperature region approaches to infinitesimal, and the temperature approaches infinity, which is the same as that of traditional curved-surface shock focusing.

During the whole process of shock focusing, energy and power per unit volume in the high temperature region increase, but total energy and power decrease.

Comparison of the changes in temperature and pressure of shock focusing between inner wedge and inner cone shows that inner cone for shock focusing is more effective.

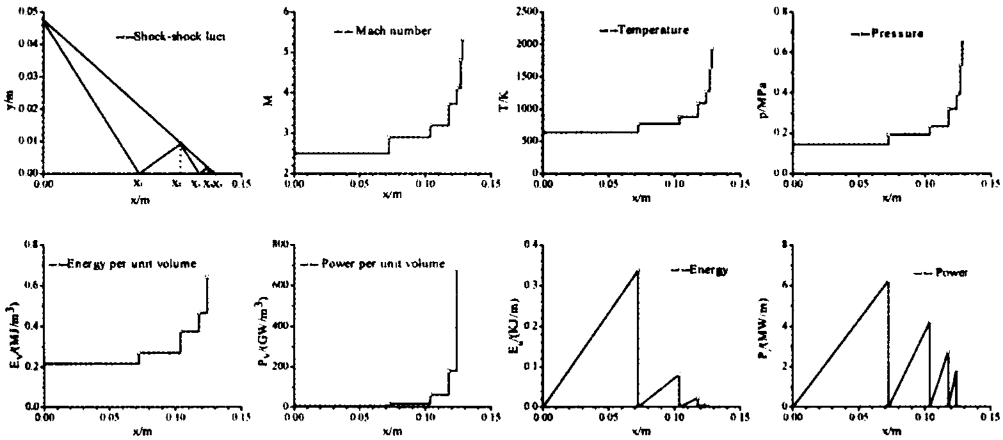

Fig. 5. Parameters of high temperature regions for shock focusing in an inner double wedge ($M_0 = 2.5$, $\theta_w = 20°$, $H_0 = 0.047m$).

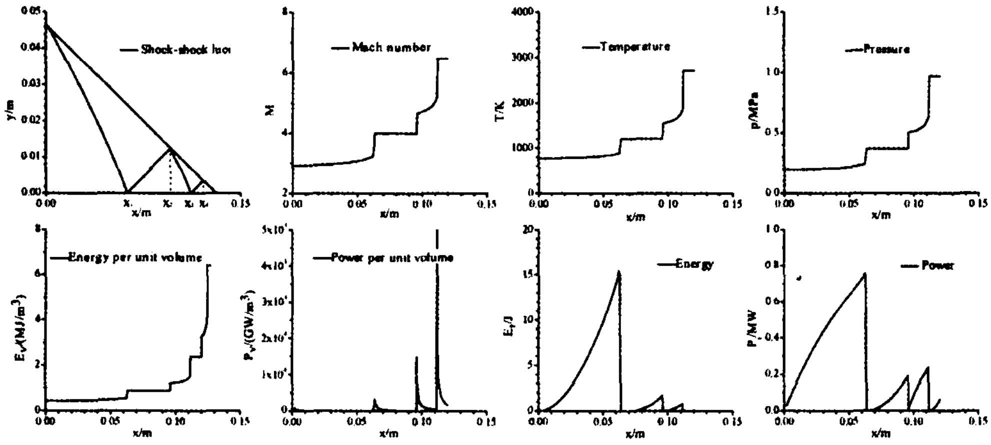

Fig. 6. Parameters of high temperature regions for shock focusing in an inner double cone ($M_0 = 2.5$, $\theta_w = 20°$, $H_0 = 0.047m$).

Acknowledgements

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