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DIFFUSIVE WAVES IN A CHANNEL WITH CONCENTRATED LATERAL INFLOW*

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Abstract: In the current paper an analytical solution for diffusive wave equation with the concentrate-distributed lateral inflow is yielded. Finite-difference numerical method is also employed to validate this model. The backwater effects drawn from lateral inflow on the mainstream are examined finally.
Keywords: diffusive wave, lateral inflow, analytical solution, finite-difference method, backwater effect

1. INTRODUCTION

With the advances in computational hydraulics, one-dimensional, gradually varied, unsteady flow is usually dealt with by numerically solving the *Saint-Venant* equations. However, the inertial terms in the momentum equation can be neglected in most practical applications. Therefore the system is reduced to only a single parabolic equation: the diffusive wave equation. By assuming the two parameters in the diffusive wave equation celerity and diffusivity are constant, theoretical solutions may be obtained to facilitate analyzing.

Dooge [1] gained a solution for the diffusion model with respect to discharge without lateral inflow. Furthermore the solution for the diffusion model with respect to water level with lateral inflow was obtained by Tingsanchali [2]. Moussa [3] solved the diffusive wave equation with lateral inflow or outflow uniformly distributed along a channel reach. Moussa [4] pointed out that the choices of each wave types are related to the magnitude of temporal characteristics of flood waves. Perkins[5] used the diffusive wave routing method to study the stream-aquifer coupling problem. Cappelaere [6] developed a HAND method (high-accuracy nonlinear diffusion method) to improve the accuracy of the general non-linear diffusive wave approach, through a modification of the variable-parameter diffusion equation.

Since flood wave propagation in a channel with tributary is a most familiar hydraulic phenomenon in natural water system, flood routing in channel with concentrated lateral inflow is a very practical problem. In this paper, we derived an analytical solution for the diffusive wave in channels with concentrated lateral inflow. A numerical FDM solution by four-point implicit scheme is yielded based on the complete *Saint-Venant* equations to validate this solution. Finally we have made comparison to examine the influence of lateral inflow on the mainstream.

2. ANALYTICAL SOLUTION FOR DIFFUSIVE WAVE WITH CONCENTRATED LATERAL INFLOW

Figure 1 illuminates Moussa's model [3], in which the lateral inflow or outflow is uniformly distributed along the channel reach. In contrast, Fig.2 depicts the channel with concentrated inflow.

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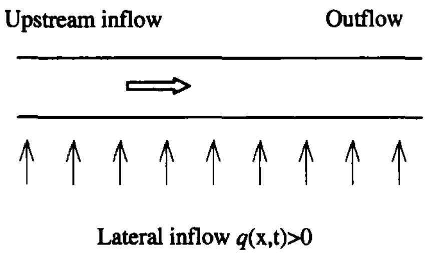

Fig.1 Sketch of river reach with lateral inflow distributed uniformly

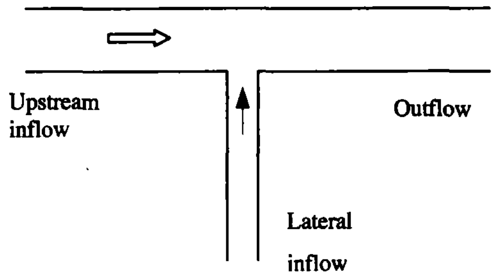

Fig.2 Sketch of river reach with concentrated lateral inflow

The Saint-Venant equations with source terms can be written as

$$\frac{\partial h}{\partial t}+\frac{\partial q}{\partial x}=q_l \tag{1a}$$

$$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial x}+g\frac{\partial h}{\partial x}=g\left(S_0-S_f\right)+q_l\frac{u_l}{h} \tag{1b}$$

where h is the water depth, v the mean cross-sectional velocity, q the discharge per unit width, t the time, x the distance along the flow direction. q_l indicates the concentrated lateral inflow per length. S_0 denotes the bed slope, S_f the friction slope, g the gravitational acceleration, u_l the velocity component of lateral inflow along the mainstream flow direction.

With assumption of Fr^2<<1 and eliminating variable h from Eqs.1(a) and 1(b), a single diffusive wave equation with lateral inflow in terms of discharge can be rendered as

$$\frac{\partial Q}{\partial t}+C\frac{\partial Q}{\partial x}=D\frac{\partial^2 Q}{\partial x^2}+Cq_l-D\frac{\partial q_l}{\partial x} \tag{2a}$$

where $C=\mathrm{d}Q/(B\cdot \mathrm{d}h)$, $D=Q/(2BS_0)$, and B are the diffusion wave celerity, diffusion coefficient, and river width respectively.

The initial and boundary conditions of this model can usually be written as

$$Q(x,0)=Q_0(x)\,,\,(-\infty<x<+\infty),\ Q(x\to+\infty,t)=Q_u(t)\,,\ Q(x\to-\infty,t)=Q_d(t) \tag{2b}$$

If the coefficients C and D are assumed constant in this diffusive wave model, the solution of Eqs.1(a) and 1(b), $\varphi(x,t)$, can be regarded as the summation of two components

$$\varphi(x,t)=\varphi_1+\varphi_2 \tag{3}$$

where φ_1 is the solution of model (A) in the following

$$\frac{\partial \varphi_1}{\partial t}+C\frac{\partial \varphi_1}{\partial x}=D\frac{\partial^2 \varphi_1}{\partial x^2} \tag{4a}$$

with initial and boundary conditions as below

$$\varphi_1(x,0)=Q_0(x)\,,\varphi_1(x\to+\infty,t)=Q_u(t)\,,\varphi_1(x\to-\infty,t)=Q_d(t) \tag{4b}$$

And φ_2 is the solution of model (B) described as

$$\frac{\partial \varphi_2}{\partial t}+C\frac{\partial \varphi_2}{\partial x}=D\frac{\partial^2 \varphi_2}{\partial x^2}+Cq_l-D\frac{\partial q_l}{\partial x} \tag{5a}$$

with initial and boundary conditions

$$\varphi_2(x,0)=0\,,\varphi_2(x\to\pm\infty,t)=0 \tag{5b}$$

In the current paper we have found the solution of model (B), φ_2, which is

$$\varphi_2(x,t)=\frac{1}{4\sqrt{D\pi}}\exp\left(\frac{Cx-Cx_0}{2D}-\frac{C^2t}{4D}\right)\int_0^t\frac{q_l(\tau)[x-x_0+C(t-\tau)]}{\sqrt{(t-\tau)^3}}\exp\left(-\frac{(x-x_0)^2}{4D(t-\tau)}+\frac{C^2\tau}{4D}\right)\mathrm{d}\tau \tag{6}$$

while the solution of model (A), φ_1, was presented by Dooge[1].

To ascertain the validity of the analytical solution obtained previously, numerical simulation is carried out to handle the one-dimensional Saint-Venant equation with Preissmann implicit scheme. The parameters are chosen as: the mainstream's river length is L =100 km; the confluent point is 50km from the upstream end of the mainstream; the widths of mainstream and tributary are 1 000m and 500m respectively. The upstream discharge of mainstream is kept fixed at 5000m^3/s; Manning roughness coefficient is taken as 0.02. A transient flood course of the tributary $Q_l(t)=5\,000\sin(\pi t/432\,000)$ is represented in Fig.3 with period $[0\le t\le 5\mathrm{d}]$.

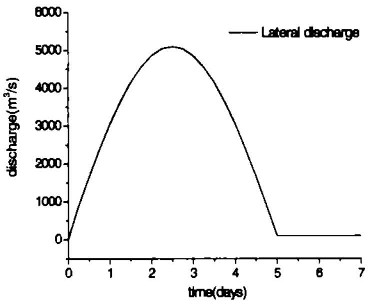

Fig.3 Lateral discharge course

The bed slope of mainstream S_0 is at first taken as 0.0001. In this case, the diffusive wave model is preferable according to the theory of Ponce[7] and Lijize [3]. Consequently the wave celerity C and diffusive coefficient D are calculated to be 2.04m/s and 25 000m^2/s respectively. Figure 4 and Fig.5 show comparisons between analytical solution and numerical results 5km upstream and downstream the confluent area in the mainstream. It can be seen clearly that the two results are in excellent agreement.

3. INFLUENCES OF THE LATERAL INFLOW

Tingsanchali's[2] revealed that the backwater effects of the tributary raise the water level in the mainstream. We further show in this paper that the lateral inflow has marked effects on the discharge in the mainstream. Comparing the discharge course in Fig.4 and Fig.5, we can clearly see that upstream the confluent area in the mainstream, the discharge decreases or increases during the rising or falling phase of the tributary flood wave. It means that upstream the confluent area in the mainstream, the partial derivative of the discharge with respect to time $\partial Q/\partial t$ is related to the minus derivative of the lateral inflow discharge with respect to time $-\mathrm{d}q_l/\mathrm{d}t$. As $\mathrm{d}q_l/\mathrm{d}t$ is equal to zero at the peak of the lateral inflow, $\partial Q/\partial t$ should also vanish.

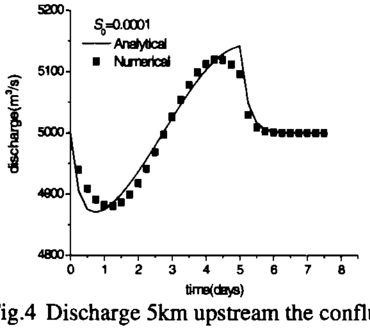

Fig.4 Discharge 5km upstream the confluent area in mainstream bed slope of mainstream S_0 is 0.000 1 .

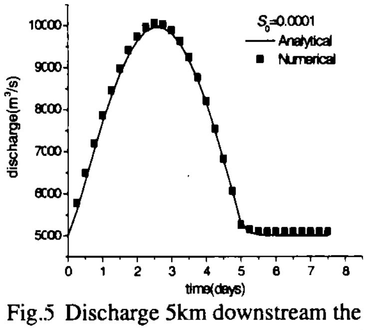

Fig.5 Discharge 5km downstream the confluent area in mainstream bed slope of mainstream S_0 is 0.000 1

Numerical simulations are carried out to study the different lateral effects for the various parameters in the mainstream such as slope S_0, width B and roughness coefficient n. Three groups of the parameters are chosen, that is, case1: B=1000m, n=0.02 and S_0 varying from 0.000 1, 0.000 13 and 0.000 15 to 0.000 5; case2: S_0=0.000 1, n=0.02 and B varying from 500, 1 000 and 1 500 to 2 000m; case3, S_0=0.0001, B=1 000m and n varying from 0.015, 0.02 and 0.025 to 0.03. Discharge courses at the location 20km upstream the confluent area in the mainstream for the different parameters of the mainstream S_0, B and n are compared in Fig.6. It can be clearly seen that the backwater effect diminishes with S_0 and B, or grows with n, as shown in Figs.6(a), 6(b) and 6(c), respectively.

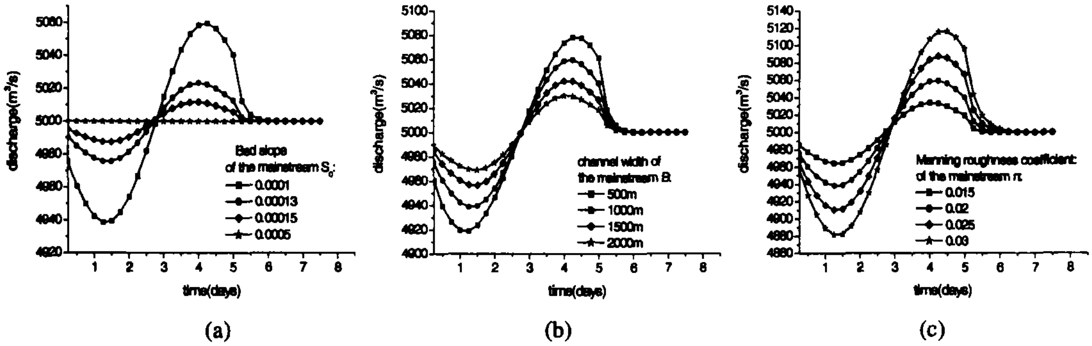

Fig.6 Discharge 20km upstream the confluent area in mainstream under different S_0, B and n

4. CONCLUSIONS

An analytical solution for the diffusive wave equation with the concentrated lateral inflow is presented and finite-difference model is established to validate it. The agreement between numerical and analytical results is satisfactory. Backwater effect is finally examined in detail. Upstream the confluent area, the discharge in the mainstream decreases at first and then increases, due to the lateral inflow. In contrast downstream the confluent area, the discharge of the mainstream increases with the lateral inflow. Different backwater effects for various parameters of mainstream S_0, B and n are also compared, leading to a conclusion that upstream the confluent area, the backwater effect decreases with the bed slope and the width of mainstream, or increases with the roughness coefficient of the mainstream. On the other hand, downstream the confluent area in the mainstream the propagation is slightly influenced by these parameters of mainstream.

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