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Collective Evolution Characteristics and Computer Simulation of Voids Near the Crack Tip of Ductile Metal

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ABSTRACT

In this paper, we consider the collective evolution of voids in front of crack tip, which always means high stress/strain gradient existed there. We use the equilibrium equation of void number density to describe the evolution behavior of voids during the process of crack growth. With the assumption and computation, we are able to solve the equilibrium equation and obtain corresponding results, which present similar tendency with experimental measurements.

1. INTRODUCTION

The nucleation, growth and coalescence of voids are the major characteristics of the failure process of ductile materials. The study on this field has received rich achievements. Since Gurson published his paper on stress field near a single void in a matrix, many scientists have been focusing on the behavior of single void or limited number of voids. In recent years, the damages as a whole were paid more and more attention during the process of failure of material. Bai et al [1] proposed an equilibrium equation of damage number density to describe a mean-field damage. Hong et al[2] applied this method to describe the property of collective evolution of short cracks during fatigue damage and of overall crack number density. From the experiments on two weld materials, Hong and Zheng[3] found isolated voids did not coalesce with crack tip and the coalescence of limited voids might not be the critical stage of crack growth. Also the void area fraction increases with increasing value of COD. As the COD reaches its critical value or beyond, the overall void damages tend to a steady distribution. This phenomenon suggests that we could analyze the collective evolution of voids with equilibrium equation of void number density.

2. EQUILIBRIUM EQUATION OF VOID NUMBER DENSITY

In this paper, we consider the static growth of ductile crack. Here we attempt to use the equilibrium equation of damage density. Based on the evolution process of voids, we may develop an equation to express the collective evolution of voids[4].

$$\int_{\sigma_0}^{\sigma_{\max}} \frac{\partial n}{\partial \xi} \mathrm{d}\sigma + \frac{\partial}{\partial r}\left(\int_{\sigma_0}^{\sigma_{\max}} Rn \, \mathrm{d}\sigma \right) = N_{\mathrm{g}} \int_{\sigma_0}^{\sigma_{\max}} n_{\mathrm{N}} \mathrm{d}\sigma \qquad (1)$$

where n is void number density, σ is local stress, ξ is COD which also means generalized time of crack growth, r represents the radius of voids and R is void growth rate which depends on the dimension of the void and the stress state there. Two characteristic stresses σ_0 and σ_{max} are local stresses near crack tip when it starts to growth and when it grows steadily. n_N is nucleation rate of voids and N_g is dimensionless coefficient.

The equation above implies that the total number of voids of given size depends on the nucleation and the growth of voids before the applied load reaches the critical value of growth.

3. THEORETICAL ANALYSIS AND SIMULATION

Based on the equilibrium equation, we can get more simple expressions according to the following assumptions. Because the voids are near the crack tip, the stress/strain field there should be under careful consideration for the high stress/strain gradient existed. Though the equation itself does not show materials parameters, the high gradient of stress can be included in the local stress σ in the equation.

First we discuss the condition when the growth process is near the steady state, which means the stress distribution does not depend on time. It can be a small time interval during whole crack growth process that the variation of stress with time can be neglected. From Eq. (1) we have

$$\frac{\partial n}{\partial t}+\frac{\partial (Rn)}{\partial r}=N_g n_N \tag{2}$$

The stress is not explicit in the equation. But it is in fact one of independent variables of n. So the stress gradient is included. Then we use HRR field as stress field[5] . It gives

$$\sigma(x)=\sigma_W \ln\left(\frac{x_0}{x}\right)^{\frac{1}{m-1}} \tag{3}$$

where x is the distance to the crack tip, x_0 is the dimension of plastic zone of crack tip, m is strain-hardening exponent of material, and σ_W is related to the stress field. From Eq.3 we notice the singularity of stress exists at the point of x=0.

The nucleation rate of voids depends on both the void size and local stress state. So we choose the nucleation formula as[6]

$$n_N=\dot{N}P\left(\frac{r}{r_{max}}\right)\exp\left(\frac{\sigma-\sigma_0}{\sigma_1}\right) \tag{4}$$

where σ_0, σ_1 and r_{max} are material constants, $\dot{N}$ is average void nucleation rate and $P(\ldots)$ is a probability distribution function expressing the positive relation between nucleation rates and the dimensions of voids.

The void growth rate should also be given. Here we use the rate based on Gurson Model[7]. Such that

$$R=\frac{\dot{\varepsilon}_0}{2}\left(\frac{3n}{2}d^2\right)^{1/m} r(r^2+d^2)^{\ 1/m} \tag{5}$$

where $\dot{\varepsilon}_0$ is a material constant and d is the distance between two voids. From above we can get the analytic solution due to the first assumption.

Then we assume that the stress distribution does depend on time. Similar to Eq. (2) Let

$$\frac{\partial \sigma}{\partial t} = \Sigma \tag{6}$$

We have

$$\frac{\partial n}{\partial t} + \frac{\partial (Rn)}{\partial r} + \frac{\partial (\Sigma n)}{\partial \sigma} = N_{\mathrm{g}} n_{\mathrm{N}} \tag{7}$$

We still chose HRR stress field. In order to eliminate the singularity of stress field at the crack tip we design a kind of stress gradient. Here we use Weibull distribution as stress distribution near the crack tip. The general expression of Weibull distribution is

$$Weibull(s,x) = sx^{s-1}\exp(-x^{s}) \quad \text{for } x>0 \quad \text{and } s>0 \tag{8}$$

Then the stress field can be

$$\begin{cases} \sigma(x,t) = \sigma_{\mathrm{W}}(t)\ln\left(\dfrac{x_0}{x}\right)^{\frac{1}{m-1}} & x > x_1 \\ \dfrac{\partial \sigma(x,t)}{\partial x} = \sigma_{\mathrm{W}}(t)\dfrac{\partial Weibull(s,x)}{\partial x} & x \le x_1 \end{cases} \tag{9}$$

where x_1 is a point near the crack tip. The formula for void nucleation rate is the same as the first assumption. Thus the analytic solution can also be made. Here we assume stress near crack tip is positive related to COD which is described by t according to strip yield model[8]. $\sigma_{\mathrm{W}}(t) = K\sigma_{\mathrm{Y}} t / t_0$, in which t_0 is characteristic time, K is a constant, σ_{Y} is yield stress when t reaches t_0 and COD reaches critical value..

4. RESULTS AND DISCUSSION

The analytical solutions to the first and second assumption are as the following

$$n(r,x,t) = \frac{1}{R}\int_{A}^{r} n_{\mathrm{N}} N_{\mathrm{g}}\, \mathrm{d}r \tag{10}$$

$$n(r,x,t) = \frac{\left\{1-\exp\left[\int_0^t -\left(\dfrac{\partial R}{\partial r}+\dfrac{\partial \Sigma}{\partial \sigma}\right)\mathrm{d}t\right]\right\} N_{\mathrm{g}} n_{\mathrm{N}}}{\dfrac{\partial R}{\partial r}+\dfrac{\partial \Sigma}{\partial \sigma}} \tag{11}$$

And each order moment can be then solved from above. The zero-th order moment:

$$nn(x,t) = \int_0^{r_{max}} n(r,x,t)\,dr \tag{12}$$

The second order moment:

$$f(x,t) = \int_0^{r_{max}} n(r,x,t)r^2\,dr \tag{13}$$

Void area fraction

$$ff(x,t) = \frac{f(x,t)}{\int_0^{r_{max}} n(r,x,t)d^2\,dr} \tag{14}$$

We consider the condition are shown of $\dot{N}$ =10000, K_{IC}=120MPa$\sqrt{m}$ and σ_Y=450MPa. So we can get the numerical results which in Figs. 1-4.

From the results of the first assumption (Figs. 1 and 2) we can see that when the crack growth process is near the steady state, the number of voids and void area fraction are decreasing to steady state away from the crack tip. Also the distribution of void area fraction is converging to a steady distribution. Such distribution does not vary as time increases. It means that the collective evolution of voids reaches a saturation state.

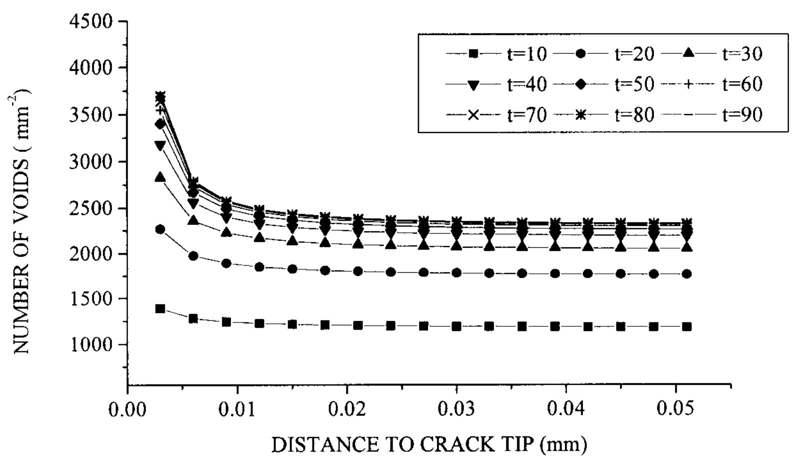

Fig. 1 Relation between number of voids and distance to crack tip.
(t represents time)

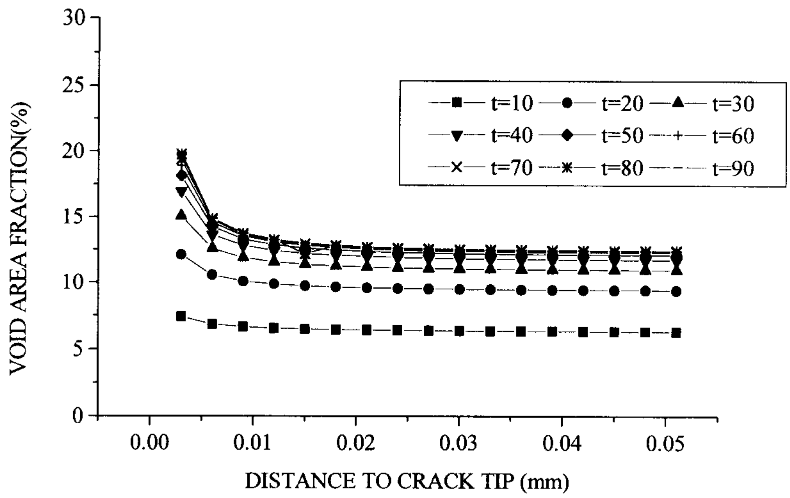

Fig. 2 Relation between void area fraction and distance to crack tip.
(*t* represents time)

The results of the second assumption (Figs. 3 and 4) also show the similar tendency. Though stress field varies with time, saturate tendency of collective evolution of voids still exists. It should be a critical value for further growth of crack tip.

Comparing two groups of results with the experimental measurements of Hong et al[3] on two materials(Fig. 5), we can observe a similar tendency. Near the crack tip the number of voids and void area fraction increase. As time goes by, the void fraction distribution increases and tends to a steady one. The critical stage does exist.

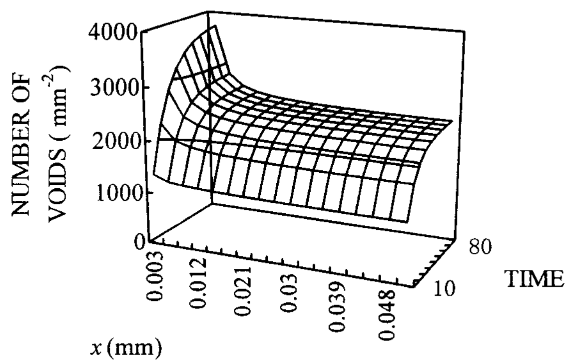

Fig. 3 Relation between number of voids, distance to crack tip x and time.

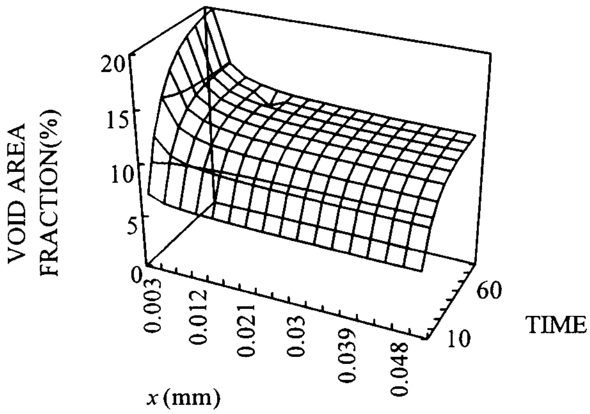

Fig. 4 Relation between void area fraction, distance to crack tip x and time.

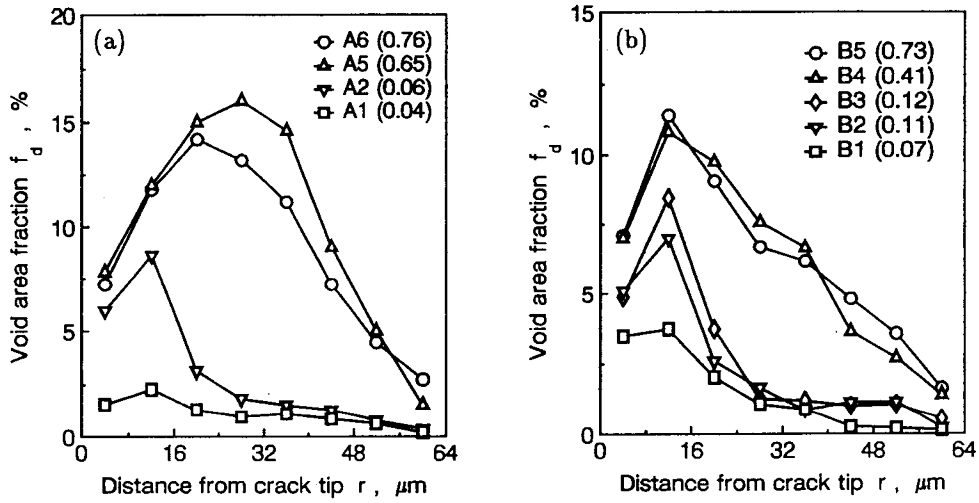

Fig.5. Void area fraction away from crack tip for Material A (a) and B (b). Number in brackets denotes the value of COD, dimensions in mm.

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