Thin layer splitting along the elastic-plastic solid surface

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Abstract. Thin layer splitting along the elastic-plastic solid surface is studied based on the elastic-plastic fracture mechanics method. In the splitting process, since the split arm does not undergo the reversed plastic bending, comparing with the conventional peel test method, the split test has remarkable advantages in measuring the material fracture behavior and is recommended as a new test method. Moreover, besides the driving force parameter, the split test method provides an additional measurable parameter, a residual curvature (or curvature radius) of the split arm. Comparing with the peeling force, the split force also has the connection with the total energy release rate, which is related with the crack tip separation energy (or material fracture toughness), separation strength, and the plastic dissipation work. Through measuring the driving force and the residual curvature, the fracture toughness and separation strength can be obtained. The primary objective of the present research is to develop a series of relations of the split force, the residual curvature, as well as the crack tip slope angle, respectively with the split layer thickness and material parameters, when crack tip advances steadily. Frictionless (or smooth) contact between splitter head and split arm surface is assumed. Another objective of the present research is to explore a connection between the split test solutions and the peel test solutions. Finally, the split test analysis is applied to a wedge-loaded double-cantilever beam experiment for Al-alloy material, a considerably similar test method with the split test, conducted by Thouless and his collaborators, and the fracture parameters from both test systems are correlated.

Key words: Elastic-plastic solids, fracture toughness, peel test, residual curvature, split test, steady-state condition.

1. Introduction

The study of the thin layer splitting on the solid surface can be taken as a continue work of the peel test researches (Kim et al., 1989; Kim and Aravas, 1988; Kim and Kim, 1988; Kinloch et al., 1994; Moidu et al., 1995; Wei and Hutchinson, 1998; Yang et al., 1999). Although the split test method is different from the conventional peel test method, however, through analysis, some connections between both test methods can be established.

The peel test have subjected to the extensive researches, cited in the papers (Kim et al., 1989; Kim and Aravas, 1988; Kim and Kim, 1988; Kinloch et al., 1994; Moidu et al., 1995; Wei and Hutchinson, 1998; Yang et al., 1999). In the peel test, usually, the thin film and the substrate are treated as the elastic-plastic material and elastic material, respectively. The previous researches were primarily focused attention to developing the relation of the peel force with the geometrical parameter (the thin film thickness) and some physical parameters, including the interface adhesion strength and material parameters, etc (Kim et al., 1989; Kim and Aravas, 1988; Kim and Kim, 1988; Kinloch et al., 1994; Moidu et al., 1995; Wei and Hutchinson, 1998; Yang et al., 1999). In the earlier researches on the peel test, the peeled layer (thin film) was treated as the bend of elastic-plastic beam or plate (Kim et al., 1989; Kim and Aravas, 1988; Kim and Kim, 1988; Kinloch et al., 1994; Moidu et al., 1995), and a critical slope angle of the peeled arm at crack tip was taken as the peeling advance condition (Kim

Figure 1. The thin layer splitting along the elastic-plastic solid surface. The split test geometry and the analytical models are shown: the crack tip separation mechanism is characterized by EPZ model, and the deformation of the split thin layer is characterized by plate bending. The active plastic zones (dark shading zones except splitter) and wake zone (light shading zones) are shown.

et al., 1989; Kim and Aravas, 1988; Kim and Kim, 1988; Kinloch et al., 1994; Moidu et al., 1995). More recently, Wei and Hutchinson (1998) presented a new analysis method for the peel test analyses by dividing the total peeling geometry into two parts. One part, an elastic-plastic bend of the peeled film under large deformation, was considered. For this case an exact solution was obtained. Another part, an elastic-plastic steadily delaminating for the thin film along the substrate under plane strain and small deformation conditions, was analyzed. Both interface separation strength and fracture toughness were taken as the dominated parameters in the analyses. For the solution of the second part, the elastic-plastic finite element method was used. In the analyses, they first applied the embedded process zone (EPZ) model to the peel test analysis, wherein a traction-separation relation was employed to model the interface as a condition linking continuum descriptions of film and substrate. With regard to the EPZ model, referencing a sketch in Figure 1, the traction-separation relation is characterized by the work of adhesion (separation energy per unit area) Γ_0 and a maximum separation strength $\hat{\sigma}$, which can be regarded as the interface strength under normal stressing. Under steady-state conditions, the model provides the peel force per unit width of the film (the total work of fracture per unit area) in terms of Γ_0 , $\hat{\sigma}$, and the material parameters, as well as the peel angle. Shortly after Wei and Hutchinson, Yang et al. (1999) adopted the EPZ model to simulate their wedge test and peel test problem, and to compare the simulation result with their experimental results successfully.

In the analyses of the peel test, the effect of the reversed plastic bending of the peeled arm on the interface fracture behavior was concerned (Kim et al., 1989; Kim and Aravas, 1988; Kim and Kim, 1988; Kinloch et al., 1994; Moidu et al., 1995; Wei and Hutchinson, 1998). Such reversed phenomena lead to the fracture behavior to be understood very complicatedly. System total energy release rate comprises the interface fracture toughness and the plastic dissipation work. However, the plastic dissipation work strongly depends on the reverse bend-

ing behavior. Due to the reverse bending effect, the application of peel test to assessing the interface behavior has been limited considerably.

Comparing with the peel test, split test recommended here and a wedge-loaded double-cantilever beam experimental method used by Thouless and his collaborators (Yang et al., 1999; Thouless et al., 1997, 1998) are not puzzled by the reverse bending. During splitting, the split arm undergoes a deformation process from elastic bending, elastic-plastic bending, and unloading, until to a free state (zero-bending moment) with a constant residual curvature left. The residual curvature can be taken as a measurable parameter easily measured in the split test. Therefore, the split test has remarkable advantages for practical applications. The thin layer split test easily connects to the assessment of the product quality in micro machining techniques in MEMS area.

Recently, Thouless and his collaborators carried out a series of experimental and numerical researches for interface adhesion properties by using a blunt wedge-loaded double-cantilever beam specimen (Yang et al., 1999; Thouless et al., 1997, 1998). During testing, two arms of the double-cantilever beam were separated along the bonded interface by moving the wedge and exerting a pair of forces respectively on the each arm. Unlike the peel test, the split arms in the split test didn't undergo the reverse banding and were deformed into an arc of circle. Simply, through measuring the residual curvature radius for split arm after test, they obtained the adhesive toughness, weakly depended on the wedge head curvature radius. Unlike the wedge test adopted by Thouless and his collaborators (Yang et al., 1999; Thouless et al., 1997, 1998), in the present research, the considered splitter is a sharp wedge, easily connected with an indenter in the scratch test, and the splitter angle is taken as an additional geometrical parameter in the split test. Here it is also expected that the splitter angle weakly influences the measured material fracture behavior.

In the present study of the thin layer splitting along the solid surface, for convenience' sake, consider a homogeneous, elastic-plastic material. The total problem is decomposed into two sub-problems as treatment for peel test problem by Wei and Hutchinson (1998). Similarly, one problem is about the large bending of split arm under plastic unloading, and another problem is about a thin layer undergoing an elastic-plastic steadily splitting on the material surface under plane strain and small deformation conditions. In the present analyses, the EPZ model will be also adopted to characterize the crack tip separation behavior. The model provides two important parameters. One is the separation energy rate (or material fracture toughness), and another is the separation strength. One of the primary objectives in the present research is to develop a series of relations of the split force, residual curvature, as well as the crack tip slope angle respectively with the thin layer thickness, material parameters and model parameters, when crack tip steadily advances. The split force has an important physical meaning, that is, it is equal to the total energy release rate (per unit area), and it has a connection with the crack tip separation energy rate (or material fracture toughness) and separation strength, as well as plastic dissipation. Another objective in this research is to explore a connection between the split test and the peel test. Finally, the split test analysis is applied to a wedge-loaded double-cantilever beam experiment for Al-alloy material, studied by Thouless and his collaborators, and two test methods are correlated parametrically.

Figure 2. During a thin layer splitting, the split arm is deformed into arc of circle in downstream region. The residual curvature depends on the interfacial and material properties.

2. Characterization of the split test

The split test geometry and related analytical model are sketched in Figure 1. A thin layer splitting along the surface of elastic-plastic solid is considered. The split depth is t . Splitter head (usually a diamond material) is treated as a rigid material. Suppose that the splitter head contacts smoothly with the split arm and substrate surface. The splitter head contacts with the split arm at the lower edge of the Section C. The interfacial fracture processing near crack tip is characterized by the EPZ model. The material is elastic-plastic one. As crack tip steadily advances, the active plastic zones depicted by the dark shading in Figure 1 around the fracture process zone and the opposite top surface move with the crack tip. During the process, unloading zones depicted by the light shading area in Figure 1 are formed, swept by the moving active plastic zones.

2.1. EMBEDDED PROCESS ZONE (EPZ) MODEL

In this subsection, EPZ model is outlined concisely. A detail description of the EPZ model can be found in Tvergaard and Hutchinson (1992, 1993), Wei and Hutchinson (1997a) and Needleman (1987). The traction separation relation $\sigma(\delta)$ within the fracture process zone is sketched in Figure 1.

Several model parameters ($\hat{\sigma}$, Γ_0 , δ_c , λ_1 , λ_2), are included, and there exists a relation among them:

$$\Gamma_0 = (1/2)\hat{\sigma}\delta_c(1 - \lambda_1 + \lambda_2). \quad (1)$$

So there are four-independent parameters. According to previous researches (Wei and Hutchinson, 1997a, 1998; Tvergaard and Hutchinson, 1992, 1993), the model parameters, λ_1 and λ_2 , are second important to the problems, so in the present research, take $\lambda_1 = 0.15$ and $\lambda_2 = 0.5$. Thus, there are only two important and independent model parameters ($\hat{\sigma}$, Γ_0) in the EPZ model, another important parameter δ_c , crack tip opening displacement, not independent, can be related with above two parameters through formula (1).

2.2. SOLUTIONS OF THIN SPLIT ARM

The thin arm undergoes the elastic-plastic bending. The moment-curvature relation is sketched in Figure 1. Note that the reversed bending and reversed plastic yielding behaviors appeared in the peel test would not take place during the thin layer splitting. In downstream region of the split arm, a constant residual curvature is left with zero moment, referencing Figure 2. It is readily to obtain the solutions for the region behind the Section C:

$$M = 0, \quad \kappa = \kappa_0, \quad (\theta > \theta_c = \beta), \quad (2)$$

where θ_c is the slope angle of C-section inclined with x_1 direction, κ_0 is the residual curvature due to the plastic loading and unloading process.

For the solution within the region $\theta_B < \theta \leq \theta_c$, from fundamental differential equation of plate bending

$$K \, dM = -(Q/\sin \beta) \cos(\beta - \theta) d\theta \quad (3)$$

and from the moment-curvature relation sketched in Figure 1, we have

$$M = Bk_0 \{ \sqrt{1 + (2Q/BK_0^2 \sin \beta) \sin(\beta - \theta)} - 1 \}, \quad \kappa = \kappa_0 + M/B, \quad (\theta_B < \theta \leq \theta_c), \quad (4)$$

where $B = Et^3/12(1 - \nu^2)$ is the bending modulus of the split arm, E and ν are Young's modulus and Poisson's ratio, respectively. The definitions of the split force Q and the splitter angle β are shown in Figure 1. A formula for calculating the residual curvature κ_0 through the plastic strains based on the second part solution can be easily dictated as (Wei and Hutchinson, 1998)

$$\kappa_0 = -(12/t^3) \int_0^t (x_2 - \frac{1}{2}t) \varepsilon_{11}^p(x_2) dx_2. \quad (5)$$

2.3. FURTHER DESCRIPTIONS OF THE SPLIT TEST

In order to describe and analyze the split test problem effectively, it is a comprehensive method to decompose the total split test problem into two sub-problems. One is a 1D bending of the split arm subjected to a large elastic-plastic deformation and rotation in the downstream region until to the section L , a cut section, as shown in Figure 1. Another problem corresponds to a 2D problem, an elastic-plastic crack growth under plane strain and small deformation conditions, which covers the all other material region, from section L to upstream. The chose of the point L located in the unloading zone near the active plastic zone is only according to that at this point the section rotation is small. Otherwise, the location of this matching point will be checked to have essentially no effect on the solution as long as the slope there θ_L is small. The continuous conditions at the section L can be confirmed by exerting the moment and traction obtained from the solution of the 1D bending problem on that section according to a static equivalence. The applied moment and traction on the section L will be the functions of the slope angle at that section and the residual curvature. Through the full 2D continuum analysis of the thin film delaminating matching with the 1D bending solution at the cut section, the slope angle of the section L and the residual curvature can be obtained. That is to say, the two parts of problem are analyzed separately and coupled by requiring continuity of force, moment, displacement and rotation at L section. The solution of the plate bending has been

obtained and given in Equation (4). So the primary aim in the following is to analyze the 2D elastic-plastic crack growth under steady-state condition.

3. The 2D elastic-plastic crack growth under steady-state condition

3.1. STEADY-STATE FORMULATIONS

Consider a thin layer with the thickness t to be split on the material surface subject to the force and moment at section L. The moment is calculated from (4) in terms of the slope ($\theta = \theta_L$) and the residual curvature κ_0 . The force ($Q/\sin \beta$) is also applying on there along the direction of $\pi/2 + \beta$, if consider the smooth contact case. The actions of the moment and force can be replaced by a linearly distributed traction according to static equivalence. κ_0 and θ_L will be computed through solving the 2D crack growth problem iteratively and through using (5).

The present aim is to solve the 2D elastic-plastic crack growth problem under steady state, plane strain and small deformation conditions. The primary relations and solving procedures are summarized in the following.

The material tensile stress-strain curve is taken to be

$$\varepsilon = \sigma/E, \quad \text{for } \sigma < \sigma_y; \quad \varepsilon = (\sigma_y/E)(\sigma/\sigma_y)^{1/N}, \quad \text{for } \sigma \geq \sigma_y. \quad (6)$$

This relation is generalized to multi-axial stress states by the J_2 flow theory for small strain incremental plasticity (von Mises theory). For plastic loading, the incremental relation of the stress and strain is read as

$$\dot{\sigma}_{ij} = D_{ijkl} \dot{\varepsilon}_{kl} \quad (7)$$

where

$$D_{ijkl} = \frac{E}{1+v} \left\{ \delta_{ik} \delta_{jl} + \frac{v}{1-2v} \delta_{ij} \delta_{kl} - \frac{(3/2)\Omega}{[1 + (2/3)(1+v)H/E]\sigma_e^2} \sigma'_{ij} \sigma'_{kl} \right\}, \quad (8)$$

σ'_{ij} is the stress deviator, $\sigma_e = \sqrt{3\sigma'_{ij}\sigma'_{ij}/2}$ is the effective stress, for plastic loading $\Omega = 1$ otherwise $\Omega = 0$, and H is plastic hardening modulus from (6)

$$H = E \left\{ (1/N) \left(\frac{\sigma_e}{\sigma_y} \right)^{1/N-1} - 1 \right\}^{-1}. \quad (9)$$

The emphasis here is on the steady-state growth wherein the crack has advanced sufficiently far from initiation such that stresses and strains no longer change from the vantagepoint of an observer translating with the crack tip. The crack problem is posed for steady-state crack growth under constant driving force Q . A zone of active plasticity moves with tip and a wake (or unloading) of plastically deformed, but elastically unloaded, material extends in the right of the tip, as depicted in Figure 1. The steady-state condition for any quantity such as a component of incremental stress is

$$\dot{\sigma}_{ij} = V \frac{\partial \sigma_{ij}}{\partial x_1}, \quad (10)$$

where V is the leftward velocity of the crack tip. Substituting the all related steady-state formulas like (10) form into the rate-independent incremental constitutive relation (7), a partial differential equation for stress and strain to x_1 in full quantity form

$$\partial \sigma_{ij} / \partial x_1 = D_{ijkl} \partial \varepsilon_{kl} / \partial x_1 \quad (11)$$

is obtained, which is independent of V .

3.2. FINITE ELEMENT METHOD

A numerical method (Wei and Hutchinson, 1997a, 1998; Dean and Hutchinson, 1980), which employs iteration procedures to solve the special stress-strain differential equation (see (11)) is used to directly obtain the convergent solution. The finite element approach and the solution procedures are outlined below.

Anticipating numerical implementation within a finite element framework, let $\mathbf{E}$ be the strain vector with components comprising the strains, and let Σ be the stress vector containing components of stress. The matrix of incremental moduli for plastic loading is denoted by $\mathbf{D}$ such that $\dot{\Sigma} = \mathbf{D}\dot{\mathbf{E}}$. Let $\mathbf{U}$ be the vector of nodal displacements and let $\mathbf{B}$ be the strain matrix such that $\mathbf{E} = \mathbf{B}\mathbf{U}$. The finite element problem for $\mathbf{U}$ in terms of applied boundary forces $\mathbf{F}$ (distributions by a force and moment are prescribed at L section) and any specified plastic strain $\mathbf{E}^p$ is represented in the standard notation as

$$\mathbf{K}^e \mathbf{U} = \int_S \mathbf{N}^T \mathbf{F} dS + \int_V \mathbf{B}^T \mathbf{D}^e \mathbf{E}^p dV \quad \text{where} \quad \mathbf{K}^e = \int_V \mathbf{B}^T \mathbf{D}^e \mathbf{B} dV, \quad (12)$$

where the superscription ' e ' and ' p ' represent the elastic part and plastic part for corresponding quantities. The iteration steps are as follows

(1) Use the distribution of $\mathbf{E}^p$ from the previous iteration in (12) to determine $\mathbf{U}$. In the first iteration take $\mathbf{E}^p = \mathbf{0}$.

(2) Compute $\mathbf{E}$ from $\mathbf{U}$.

(3) Obtain a new estimate of the distribution of Σ . Use $\Sigma = \mathbf{D}^e \mathbf{E}$ in the region upstream of the current estimate of active plastic zone and use $\Sigma = \mathbf{D}^e (\mathbf{E} - \mathbf{E}^p)$ downstream from the active zone. Where yield is currently met, make use the fact that for steady-state growth, $\dot{\Sigma} = \mathbf{D}\dot{\mathbf{E}}$ can be replaced by $\partial \Sigma / \partial x_1 = \mathbf{D} \partial \mathbf{E} / \partial x_1$ such that for any point (x_1, x_2) within the active plastic zone

$$\Sigma(x_1, x_2) = \Sigma(x_1^*, x_2) - \int_{x_1}^{x_1^*} \mathbf{D} \partial \mathbf{E} / \partial x_1 dx_1, \quad (13)$$

where (x_1^*, x_2) is corresponding point on the leading edge of the active plastic zone (i.e., the left edge of the active zone). The integration in (13) is performed for fixed x_2 and applies to all points within the active plastic zone.

(4) Use $\mathbf{E}^p = \mathbf{E} - \mathbf{D}^{e-1} \Sigma$ to compute the new estimate of $\mathbf{E}^p$ for the next iteration. Revise the active plastic zone using the new estimate of Σ . To the right of the active plastic zone in the downstream unloading region, $\mathbf{E}^p$ is a function only of x_2 , corresponding to its value at the right edge of the active zone.

(5) If satisfactory convergence has not been achieved, repeat steps (1)–(4).

A finite element procedure with the equal-height mesh specially designed for the regions around the crack surface near crack tip to cope with the steady-state wake has been used to carry out the calculations, as discussed above or detailed descriptions in (Wei and Hutchinson, 1997b, 1998; Dean and Hutchinson, 1980). In the present analysis, the adopted finite element mesh for crack steadily growing is shown in Figure 3. In the calculation, the eight nodal isoparametric elements and four Gauss integration points in each element are used. The total element number adopted is 1964.

Figure 3. The form of the finite element mesh specially designed for thin layer delaminating elastic-plastically and steadily under the plane strain case. L section is corresponded to that shown in Figure 1.

3.3. SOLUTION FORM

For the split test problem, the total energy rate is just equal to the split force Q (the force per unit length). It can be decomposed into two parts: the crack tip separation energy rate (or material fracture toughness) and plastic dissipation work rate Γ_p . Thus

$$Q = \Gamma_0 + \Gamma_p. \quad (14)$$

For the elastic splitting: $Q \equiv \Gamma_0$.

Summoning all independent parameters, referencing Figures 1 and 2, the solutions of the split test problem can be expressed as

$$\begin{aligned} \frac{Q}{\Gamma_0} &= F_1 \left(\frac{E}{\sigma_Y}, \frac{\hat{\sigma}}{\sigma_Y}, N, \frac{t}{R_0}, \beta, v \right), \quad \frac{Et\kappa_0}{\sigma_Y(1-v^2)} = F_2 \left(\frac{E}{\sigma_Y}, \frac{\hat{\sigma}}{\sigma_Y}, N, \frac{t}{R_0}, \beta, v \right), \\ \alpha &= F_3 \left(\frac{E}{\sigma_Y}, \frac{\hat{\sigma}}{\sigma_Y}, N, \frac{t}{R_0}, \beta, v \right). \end{aligned} \quad (15)$$

In formulas (15), the first solution function $Q/\Gamma_0 = F_1(\dots)$ shows the normalized split force (or normalized total work rate) changing with the independent parameters. The fracture toughness Γ_0 is taken as the normalized quantity. The second function is about the residual curvature κ_0 changing with the independent parameters. The third relation $\alpha = F_3(\dots)$ describes the variation of the crack tip slope angle (defined in Figure 2) with the independent parameters. In the formulas (15), a length parameter R_0 is introduced and it is defined by

$$R_0 = \frac{E\Gamma_0}{3\pi(1-v^2)\sigma_Y^2}, \quad (16)$$

Figure 4. The variation of the normalized split force with the normalized split arm thickness for several splitter head angles.

R_0 characterizes the plastic zone height in small scale yielding case.

The detailed expressions of the three solution functions in (15) will be implemented by solving the splitting test problem numerically using the finite element method.

4. Results and analyses

In this section, the results of the split force, residual curvature and the crack tip slope will be presented and discussed in detail.

The normalized split forces Q/Γ_0 changing with the normalized thin layer thickness t/R_0 are shown in Figure 4. From Figure 4, split force Q/Γ_0 increases and asymptotes to some stable values as t/R_0 increases. When t/R_0 is larger than 8, the solutions become insensitive to the increase of t/R_0 , which corresponds to the conventional small scale yielding case. Thereby, the plastic zone size tends to a stable value, which is far smaller than the split layer thickness. From Figure 4, when t/R_0 is small and as it decreases, the split force Q/Γ_0 decreases and tends to unity. Although in this case the plastic zone height covers entire split layer thickness and the large scale plastic yielding takes place for the split arm, however, the plastic work contribution to the total system is neglected (Wei and Hutchinson, 1997a, 1998). From Figure 4, Q/Γ_0 increases with increasing β , however the solution is not sensitive too much to the β variation.

Figure 5 shows the normalized residual curvatures changing with the normalized thin layer thickness for several splitter angles. From Figure 5, the residual curvature increases as the split layer thickness decreases. Specifically when t/R_0 is smaller than 3, the residual curvature increases strongly with decrease of t/R_0 . However, when t/R_0 is larger than 3, the residual curvature increases slowly with decrease of the split layer thickness. From Figure 5, the residual curvature changes insensitively with the splitting angles β , specifically when t/R_0 is larger than 3.

Figure 5. The variation of the normalized residual curvature with the normalized split layer thickness for several splitter head angles. The results are not sensitive too much to the splitter angle.

Figure 6. The variation of the crack tip slope angle with the normalized split layer thickness for several splitter angles.

The crack tip slope angle α changing with the normalized split layer thickness is shown in Figure 6. From Figure 6, the crack tip slope angle decreases with increase of t/R_0 , but it is insensitive to the change of t/R_0 when t/R_0 is larger than 6. When t/R_0 is smaller than 3, the slope angle sharply increases with decrease of t/R_0 . When t/R_0 is larger than 9, the α tends to a constant around 3 deg for different splitter angle β . The crack tip slope angle is inversely proportional to the splitter angle β .

Figure 7. The relations of normalized split force versus split arm thickness for different yielding strains (σ_Y/E). The material yielding strain has the strong effects on the results.

Figure 8. The effect of the material yielding strain on the normalized split force for a weak separation strength case.

In Figure 7, the curves of Q/Γ_0 vs. t/R_0 are plotted for different values of E/σ_Y . From Figure 7, the variation of Q/Γ_0 is very sensitive to the material yielding strain ($\varepsilon_Y = \sigma_Y/E$). Obviously, E/σ_Y is an important parameter in the split test as in the peel test (Yang et al., 1999). With increasing E/σ_Y , the split force increases very quickly, especially for $t/R_0 < 4$, thereafter, its value quickly asymptotes to a stable value, which corresponds to the small scale yielding limit. The important influence of parameter E/σ_Y on the total energy rate can be interpreted as that the smaller the material yielding strain, the stronger the plastic deformation, so that the stronger the plastic dissipation.

Figure 9. The effect of the yielding strain on the normalized residual curvature for several split layer thickness cases. The material yielding strain has the high influence on the residual curvature.

As another investigation for the influence of the yielding strain on the total energy rate, Figure 8 shows the results for the case of weakly separated strength, $\hat{\sigma}/\sigma_Y = 1$. From Figure 8, with decreasing the yielding strain (or with increasing the ratio E/σ_Y), because there is a considerable plastic dissipation, even though the separation strength is weak, a big energy rate is still needed for realizing the split process. In this case, the split force seems weakly depended on the split layer thickness.

The influence of parameter E/σ_Y on the residual curvature κ_0 is shown in Figure 9. From Figure 9, there is a significant influence on the residual curvature from E/σ_Y , and the residual curvature strongly depends on the split layer thickness t/R_0 . From Figure 9, clearly, two influence stages of E/σ_Y on the residual curvature exist: the influence is weak when $E/\sigma_Y < 500$, and it is strong when $E/\sigma_Y > 500$.

Figure 10 shows the influence of E/σ_Y on the crack tip slope angle α . Obviously, there exist the double straight linear relations for the solution characteristics: when $E/\sigma_Y < 500$, one straight linear relation with a negative slope is corresponded and α is strong dependent on the value of E/σ_Y , however, when $E/\sigma_Y > 500$, another straight linear relation, insensitive to E/σ_Y , is corresponded. The whole solutions seem weakly related on the split layer thickness.

In Figure 11, the solution dependence on the normalized separation strength $\hat{\sigma}/\sigma_Y$ and the material strain-hardening exponent N are shown. Figure 11a–c correspond to the normalized split force Q/Γ_0 , the normalized residual curvature κ_0 and the crack tip slope α , respectively related with $\hat{\sigma}/\sigma_Y$ and N . From Figure 11a–c, the normalized parameter solutions, $(Q/\Gamma_0, Et\kappa_0/\sigma_Y(1-\nu^2), \alpha)$ vs. $\hat{\sigma}/\sigma_Y$ and N , have the similar trends. When $N = 0.1$, at about $\hat{\sigma}/\sigma_Y = 4$, a turning point is corresponded, thereby three normalized quantities turn to increase sharply. When $N = 0.2$, the turning point is at about $\hat{\sigma}/\sigma_Y = 5$. When $N = 0.3$, it is at about $\hat{\sigma}/\sigma_Y = 6.5$, etc. The solution presents a critical crack tip strength $\hat{\sigma}_c$, thereafter, a strong plastic dissipation leading to a high split force, big residual curvature and big crack tip slope is taken place. The solutions imply that it is very difficult or even impossible to split a layer when the separation strength is beyond $\hat{\sigma}_c$, no matter how high the split force

Figure 10. The effect of the yielding strain on the crack tip slope angle for several split layer thickness case.

is exerted. This contradictory conclusion has been realized to come from the limitation of the conventional elastic-plastic theory (Wei and Hutchinson, 1997a, b, 1998; Tvergaard and Hutchinson, 1992, 1993; Evans et al., 1999).

5. Some connections between split test and peel test

Check the peel test first. Peel force is exerted at the peeled arm terminal along the direction inclined by angle Φ with horizontal direction. Consider that the peel angle Φ is larger than $\pi/2$. Let $\Phi = \pi/2 + \beta$ for conveniently connecting and comparing with the split test. Comparing the test principle sketches in Figures 1 and 12, there may exist the corresponding relations between peel test and split test. The aim in this section is to explore the relations between the peel test solution and the split test solution.

The bending solution of the peeled arm in peel test can be dictated as (Wei and Hutchinson, 1998)

$$M = B\kappa_0 \left\{ \sqrt{1 + [2Q/B\kappa_0^2(1 + \sin \beta)][1 + \sin(\beta - \theta)] - w_0} - 1 \right\} \quad (17)$$

$$\kappa = \kappa_0 + M/B \quad (\theta_B < \theta < \Phi = \pi/2 + \beta),$$

where a relation of effective peel force (or a total energy rate) Q in peel test $Q = P(1 + \sin \beta)$, has been used. The formula for computing κ_0 is same as (5). Note that κ_0 in the peel test problem is difficult to be measured unlike the split test, because the peeled arm undergoes a very complicated deformation due to the reversed plastic bending. Parameter w_0 was first introduced by Wei and Hutchinson (1998), and it characterized the Bauschinger's behavior of plastic material bending, which was defined to be equal to a ratio of the shading area to the area of triangle OCG in moment-curvature sketch, seeing Figure 12. From w_0 definition, one has an effective region of the parameter value: $0 \leq w_0 \leq 1$.

The peel test problem was decomposed into two sub-problems by Wei and Hutchinson (1998): the plate bending and the thin film delaminating. Section L is the interface between

Figure 11. The influences of the normalized separation strength on the normalized split force (a), the normalized residual curvature (b) and the crack tip slope angle (c) for different strain hardening exponent values N . From figures, there is a similar trends: existing the turning point, thereafter the plastic dissipation elevates sharply.

Figure 12. The peel test geometry and analytical models. The shading area characterizes the reversed bending behavior. Case 1 is no reversed bending. From case 1 to case 4, the reversed bending strength w_0 increases. The peel angle $\Phi = 90^\circ + \beta$ is larger than 90° .

Figure 13. The variation of the normalized energy rate with the normalized peeled layer thickness for several values of the peel test problem. The dashed line corresponds to the split test result.

two problems, see Figure 12. The choice of the section L and the connection of both problem solutions at L for peel test are same as that described for split test above. The plate bending solution has been obtained, as given in formula (17). For solving the thin film delamination problem, the plate bending solution (17) is applied on the L section. Obviously, the difference between the peel test problem and the split test problem comes from the difference of the plate bending solutions (17) and (4), respectively for peel test and for split test. Note that at the section L , $\theta = \theta_L$, and the slope of the section L is small, $\theta_L \ll \beta$. When $w_0 = 0$, the difference of the solutions (17) and (4) at L comes from the two terms $(1 + \sin(\beta - \theta_L))/(1 + \sin \beta)$ and $\sin(\beta - \theta_L)/\sin \beta$. Because $\theta_L \ll \beta$, so the difference of both solutions (17) and (4) is very little. With increasing w_0 , the difference between both solutions increases.

The variations of the normalized total energy rate Q/Γ_0 (or normalized peel force) with increasing the peeled arm thickness t/R_0 for several w_0 values are plotted in Figure 13. From Figure 13, obviously, the parameter w_0 has the considerable influence on the total energy rate when peeling force angle Φ is larger than $\pi/2$, comparing with the case when peeling force angle is smaller than $\pi/2$ as discussed by Wei and Hutchinson (1998). With the increase of w_0 , corresponding to the increase of shading area in Figure 12, the value of Q/Γ_0 increases considerably. In contrast, in Figure 13, a solution of split test is also shown. From Figure 13, the peel test solution is very close to the split test solution for the parameter $w_0 = 0$, corresponding to the shading area in Figure 12 tending to zero.

6. Application to a wedge-loaded experiment for Al-alloy double-cantilever beam

Recently, Thouless' research group carried out a series of wedge-loaded experiments for the Al-alloy double-cantilever beam specimens (Yang et al., 1999; Thouless et al., 1997, 1998). Having a little difference with the splitter head geometry discussed above, a wedge with tip curvature radius was considered in their experiment. With loading, by moving the wedge towards crack tip direction and through contact points between the wedge head surface with the split arm surface, a pair of driving forces is exerted on the each split arms and makes the double-cantilever beam delaminate along the bonded interface. The contact points move forward with crack tip. They obtained the influence of the curvature radius of wedge head on the residual radius of the split arm. Obviously, the contact point between wedge edge and split arm for a smaller curvature radius wedge is closer to crack tip. Similarly, for a sharp wedge (splitter head) as discussed in the present research, as the splitter angle β decreases, the contact point between splitter face and split arm is close to crack tip. So the splitter angle β for a sharp wedge has a similar function as the curvature radius of a blunt wedge. So in the present section, the split test analysis can be applied to the test geometry of a symmetrical double-cantilever beam as adopted by Thouless and his collaborators. Note that for a double-cantilever beam specimen by wedge-loaded split test, β is referred to as the half wedge tip angle. For comparing, let's consider the same test geometry and Al-alloy material problem as that experimentally researched by Thouless' research group (Yang et al., 1999; Thouless et al., 1997, 1998). Through comparing the present analysis with their experimental results, the connection between the curvature radius of blunt wedge and the splitter angle of the sharp wedge will be developed.

For the material Al alloy considered by Yang et al. (1999), from stress-strain experimental relation shown by Yang et al. (1999), material yielding stress $\sigma_Y \approx 100$ MPa. Young's modulus and Poisson's ratio for a typical Al-alloy are about 70 GPa and 0.3, respectively, from material handbook. So take $(E/\sigma_Y, \nu) = (700, 0.3)$ in the present analysis. For the weakly hardening metal, a typical value $N = 0.2$ is taken for a piece of stress-strain relations, see (6). For weakly bonded double-cantilever beam specimen, corresponding a large fracture process zone size, measured $\Gamma_0 = 1.4 \text{ kJ m}^{-2}$ and $\hat{\sigma}/\sigma_Y = 1$ (Yang et al., 1999). From (16), the plastic zone size in small scale yielding case is computed as $R_0 = 1.15 \text{ mm}$.

Using symmetrical condition of the double-cantilever beam specimen, only one wing of the double-cantilever beam from bonded interface needs to be considered. On the interface (symmetrical surface), the mode I crack conditions are dominated. After the symmetrical condition is used, the problem geometry becomes the same as Figure 1, as long as the substrate is taken as a rigid material in there. Correspondingly, the driving force Q in Figure 1 is only a

Figure 14. Normalized residual curvature changing with splitter shape: sharp wedge with angle β (solid lines) and blunt wedge with tip curvature diameter D (circles; experiment for Al alloy from Yang et al. (1999)).

half total driving force of the double-cantilever beam case, and the critical crack tip opening displacement is also a half one, so that the corresponding interface fracture toughness Γ_0 is also a half total value of the double-cantilever specimen case. So the ratio Q/Γ_0 is unchanged, equal to the total ratio of the double-cantilever beam case.

Figure 14 shows the normalized residual curvature changing with the splitter angle for different arm thickness. From Figure 14, the normalized residual curvature slowly increases with the increase of β . However, with decreasing the split arm thickness, the residual curvature increases considerably. In Figure 14, experimental results in Yang et al. (1998) are also shown under another coordinate system: normalized residual curvature with wedge tip diameter D . Obviously, the analytical results are surprisingly consistent with the experimental results within the typical regions of geometrical parameters, β and D . In Figure 14, $1/\kappa_0$ is the residual curvature radius of the split arm. In this analysis, $E/\sigma_Y(1-\nu^2) = 769$, so from Figure 14 at $\beta = 50^\circ$, the residual radii are about 14 mm, 40 mm and 77 mm, respectively, for split arm thickness $t = 1$ mm, 2 mm and 3 mm. From Figure 14, clearly, there is a strong correspondence between the sharp wedge parameter β and the blunt wedge parameter D . From figure, $D = 1$ mm of blunt wedge case corresponds to $\beta = 32.5^\circ$ of sharp wedge case, and $D = 3.6$ mm corresponds to $\beta = 65^\circ$.

In the application, the split force Q (driving force, per specimen width) is also obtained. For example, at $\beta = 50^\circ$ the split forces (per specimen width) are 4.4 N mm^{-1} , 5.8 N mm^{-1} and 6.9 N mm^{-1} , respectively, for split arm thickness $t = 1$ mm, 2 mm and 3 mm.

7. Concluding remarks

As a new test method, split test has been presented and analyzed in the present research. The split test method has some advantages comparing with peel test method. Firstly, in the split test, no any reversed plastic bending is taken place for the split arm deformation, so the split mechanism is concisely and simply to be understood in concept. The split arm has

Figure 15. The influence of length parameter L in strain gradient plasticity on the ratio of the total work of fracture to the work of the fracture process Γ_{ss}/Γ_0 versus normalized separation strength for mode I, plane strain crack growth in a homogeneous elastic-plastic solid (from Wei and Hutchinson, 1996).

been deformed into a circle, very simple geometry to be measured. Secondly, the split test provides an additional measurable parameter, a residual curvature. Through measuring the split force and the residual curvature (or residual radius R_κ , see Figure 2), it is possible to obtain the material separation energy rate (or fracture toughness) and the separation strength from Figure 11a and 11b.

In the present research, the analyses of the split test have been carried out using elastic-plastic fracture mechanics. Although the analyses have been confined to the homogeneous material case, however, the method can be generalized directly to a thin film/substrate bonded system for the different material property combinations. For the thin film/substrate bonded system, the solution of split arm in formula (4) remains effective. There concerned solutions are still the variations of the total energy release rate, residual curvature and the crack tip slope, respectively with the thin film thickness, interface parameters (maximum separation strength and fracture toughness) and material parameters of the thin film/substrate system.

The embedded process zone model (EPZ model) has been used to analyze the split test, and the two important parameters, i.e., the crack tip separating energy rate (material fracture toughness) and the separation strength, are taken as the governing parameters of system. For a strongly bonded interface, from Figure 11, when the separation strength is high and reaches some critical value, and corresponding interface fracture toughness Γ_0 is at the order of 1 J m^{-2} , plastic zone height in the small scale yielding is about $R_0 \approx 1 \mu\text{m}$ from (16). The split layer thickness is also at the order of several microns. From Figure 11, it is very difficult even impossible to realize a split process at this length scale. The characteristic has been also observed at the peel test result, the thin film delamination result and the interface debonding result (Wei and Hutchinson, 1980, 1997a,b, 1998; Evans et al., 1999). The limitation has been understood recently that it comes from the defect of the conventional plasticity theory. Many researches have displayed that the conventional plasticity theory does not give rise to sufficient stress elevation at the crack tip to produce separation when the separation strength is larger

than about 4 to 6 times the yielding stress (Wei and Hutchinson, 1997a, b, 1998; Tvergaard and Hutchinson, 1992, 1993; Evans et al., 1999). For this reason, several version strain gradient plasticity theories have been developed recently (Fleck and Hutchinson, 1997; Aifantis, 1992; Gao et al., 1999; Acharya and Bassani, 1995) which incorporate the some length parameter L characterizing the scale at which strain gradient becomes important. A quantitative study of the effect of stress elevation on plane strain, Mode I crack growth in homogeneous metals was carried out by Wei and Hutchinson (1997b) using strain gradient plasticity theory. The result displayed that the crack tip stress increased very high and caused crack propagation at much larger normalized separation strength $\hat{\sigma}/\sigma_Y$ than that predicted by the conventional plasticity theory, referencing the Figure 15. Besides the EPZ model, the other fracture models can be adopted to simulate the splitting process (Suo et al., 1993; Wei and Hutchinson, 1999).

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