

文章编号:1000-0887(2002)07-0667-06

激光辐照下圆薄板的动态屈曲研究^{*}

黄晨光, 段祝平

(中国科学院 力学研究所,北京 100080)

(我刊编委段祝平来稿)

摘要: 对强激光辐照下薄板(铜片)的动态热失稳过程进行了分析,得出了简支圆薄板在热冲击下发生的屈曲及后屈曲过程,并给出了临界激光功率密度与薄板厚径比的关系曲线,方法计入了温度分布、惯性项和缺陷大小对于失稳过程的影响.这一工作有利于人们对强激光引起的硬目标破坏机理的认识.

关键词: 激光; 屈曲; 热冲击

中图分类号: O343.9 **文献标识码:** A

引 言

在强激光诱导的结构破坏中,结构的动态屈曲是一种重要的现象.它能迅速引起靶目标承载能力的丧失,并能诱发其它的破坏和失效形式.在我们首次发现的激光破坏机理的新现象“反冲击塞效应”中,薄板的屈曲也是重要的一环^[1].

关于弹性结构的动态屈曲问题,近四十年来引起了广泛的关注^[2~5].最早的工作可以追溯到 Knoing 和 Taub 等人的含缺陷圆柱体在固定周期外载作用下的稳定性分析.到了 60 年代,力学工作者在这一领域达成了一些共识,认为在最普遍意义下的弹性系统的稳定性问题,应该从动力学的角度加以统一研究,李雅普诺夫的思想和方法被广泛采用.至于圆薄板的屈曲问题,在动态方面有代表性的工作是 Edstrom, Zizicas, Birkgam 等人完成的.

本文中,作者将静态和动态、含缺陷与不含缺陷结构的屈曲问题以统一的形式来研究,在求解了板中的温度场和得出热弹性薄板的控制方程后,利用小扰动方法和数值积分方法研究激光辐照下薄板的稳定性问题,得出的结论与相应的有限元计算结果基本吻合.

1 温度场的解

在“反冲击塞效应”的研究中,我们通过实验观察和数值模拟发现,圆薄板的屈曲是形成“塞片”的重要步骤,屈曲直接诱发了不可逆变形在光斑边缘处的集中.在其它由激光诱导的结构破坏中,也经常能观察到屈曲现象的发生.

图 1 给出了本文分析激光诱导屈曲的简单模型.薄板的直径为 a ,厚度为 d .我们将激

* 收稿日期: 2000-11-20; 修订日期: 2002-01-28

基金项目: 国家自然科学基金资助(10002021); 国家高技术计划激光技术领域的资助(863-802-4)

作者简介: 黄晨光(1969—),男,湖北汉川人,副研究员,博士(E-mail: huangcg@imech.ac.cn).

光的辐照理解成第二类热边界条件,板的底边和周边为绝热边界条件.另外,板的四周为简支边界条件.

对于图 1 所示的问题,我们先求解温度场,其控制方程和初始、边界条件如下所示.

$$-\frac{\partial^2 T}{\partial z^2} = \frac{\partial T}{\partial t}, \quad (1)$$

$$t = 0, T = 0, \quad (2)$$

$$\left\{ \begin{array}{l} \frac{\partial T}{\partial z} \Big|_{z=d/2} = q, \\ \frac{\partial T}{\partial z} \Big|_{z=-d/2} = q. \end{array} \right. \quad (3)$$

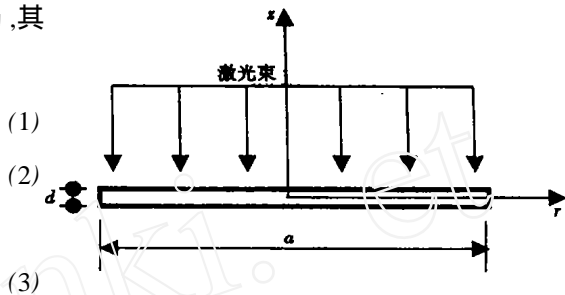


图 1 激光诱导薄板热屈曲的示意图

利用 Laplace 变换,可以获得板内温度场的解,

$$T = \frac{2q(d/2)}{c} \left[\frac{t}{2} + \frac{3z^2 + 6hz - h^2}{24h^2} - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} e^{-n^2 t} \cos \frac{n}{2} \left(\frac{z}{h} + 1 \right) \right], \quad (4)$$

$$= \frac{2}{c 4h^2}, \quad (5)$$

如果激光的功率密度在板的上表面分布并不均匀,为高斯分布或双高斯分布,那么就是一个二维的热传导问题,解析解的形式就更加复杂,一般为 Bessel 函数 $J_0(x)$ 的级数形式.

2 薄板的控制方程

设板中面的位移为 u, v, w , 其中 u, v 面内之径向、周向位移,而 w 则表示板的挠度,设板内各点的位移与板中面位移场的关系为:

$$\left\{ \begin{array}{l} u_r = u - z(r, t), \\ u = 0, \\ u_z = w(r, t). \end{array} \right. \quad (6)$$

不计板 z 向的应变,可以获得应变场.

$$\epsilon_{11} = \frac{\partial u}{\partial r} + \frac{1}{2} \left(\frac{\partial w}{\partial r} \right)^2 - z \frac{\partial^2 w}{\partial r^2}, \quad (7)$$

$$\epsilon_{22} = \frac{u}{r} - \frac{z}{r} \frac{\partial w}{\partial r}, \quad (8)$$

$$\epsilon_{12} = 0. \quad (9)$$

如果板预先存在一个挠度 w_0 , 它往往被视为缺陷,此时的应变场则为,

$$\epsilon_{11} = \frac{\partial u}{\partial r} + \frac{1}{2} \left(\frac{\partial w}{\partial r} \right)^2 - \frac{1}{2} \left(\frac{\partial w_0}{\partial r} \right)^2 - z \frac{\partial^2 w}{\partial r^2} + z \frac{\partial^2 w_0}{\partial r^2}, \quad (10)$$

$$\epsilon_{22} = \frac{u}{r} - \frac{z}{r} \frac{\partial w}{\partial r} + \frac{z}{r} \frac{\partial w_0}{\partial r}, \quad (11)$$

$$\epsilon_{12} = 0, \quad (12)$$

其中,记

$$\tilde{\epsilon}_{11} = \frac{\partial u}{\partial r} + \frac{1}{2} \left(\frac{\partial w}{\partial r} \right)^2 - \frac{1}{2} \left(\frac{\partial w_0}{\partial r} \right)^2, \quad (13)$$

$$\tilde{\epsilon}_{22} = \frac{u}{r}. \quad (14)$$

由广义 Hooke 定律,有,

$$\epsilon_{11} = \frac{E(\epsilon_{11} + \nu_{22})}{1 - \nu^2} - \frac{E}{1 - \nu^2}, \tag{15}$$

$$\epsilon_{22} = \frac{E(\epsilon_{22} + \nu_{11})}{1 - \nu^2} - \frac{E}{1 - \nu^2}, \quad \epsilon_{12} = 0. \tag{16}$$

于是薄板内的弹性能可以写成:

$$= \frac{Ed^3}{12(1 - \nu^2)} \int_0^a \left\{ \frac{12}{d^2} \epsilon_1^2 + [\nabla^2 (w - w_0)]^2 + 2(1 - \nu) \frac{\partial^2 (w - w_0)}{\partial r^2} \frac{\partial (w - w_0)}{\partial r} \right\} r dr - \frac{2E}{1 - \nu} \int_0^a [\epsilon_1 - \nabla^2 (w - w_0)] r dr - 2 \int_0^a (p - d\ddot{w}) wr dr, \tag{17}$$

其中,

$$\bar{\epsilon}_1 = \int_{-d/2}^{d/2} \epsilon_1(r, z) dz, \tag{18}$$

$$\tilde{\epsilon}_1 = \int_{-d/2}^{d/2} \epsilon_1(r, z) z dz, \tag{19}$$

$$\epsilon_1 = \tilde{\epsilon}_1 + \bar{\epsilon}_1. \tag{20}$$

由虚位移原理,可以得出薄板的控制方程,

$$D \nabla^4 (w - w_0) + K^2 \nabla^2 w + \frac{E}{1 - \nu} \nabla^2 \tilde{\epsilon}_1 = p - d\ddot{w}, \tag{21}$$

$$\frac{12D}{d^2} \epsilon_1 - E \bar{\epsilon}_1 / (1 - \nu) = K^2, \tag{22}$$

$$D = \frac{Eh^3}{12(1 - \nu^2)}, \tag{23}$$

D 为板的抗弯刚度. 用挠度表示的简支边界条件可以写成,

$$\begin{cases} u = 0, \\ w = 0, \\ D \left[\frac{\partial^2 (w - w_0)}{\partial r^2} + \frac{1}{r} \frac{\partial (w - w_0)}{\partial r} \right] + \frac{E}{1 - \nu} \bar{\epsilon}_1 = 0. \end{cases} \tag{24}$$

3 挠曲过程与稳定性分析

引入无量纲参数:

$$w = \frac{w}{d}, \quad w_0 = \frac{w_0}{d}, \quad r = \frac{r}{d}, \quad C_0 = \frac{d}{a}, \tag{25}$$

$$\bar{\epsilon}_1 = \frac{\bar{\epsilon}_1}{dT^*}, \quad \tilde{\epsilon}_1 = T^* \tilde{\epsilon}_1, \quad \tilde{\epsilon}_1 = \frac{\tilde{\epsilon}_1}{d^2 T^*}. \tag{26}$$

将板的控制方程重新写为:

$$\frac{1}{12(1 - \nu^2)} C_0^2 \frac{\partial}{\partial r} \left\{ \frac{\partial}{\partial r} \left[\frac{\partial}{\partial r} \left(\frac{\partial (w - w_0)}{\partial r} \right) \right] \right\} - \frac{K^2}{Ed} \frac{\partial}{\partial r} \left(\frac{\partial w}{\partial r} \right) + \frac{1}{1 - \nu} \frac{\partial}{\partial r} \left(\frac{\partial \tilde{\epsilon}_1}{\partial r} \right) + \frac{d}{EC_0^2} d\ddot{w} = 0, \tag{27}$$

$$\frac{1}{1 - \nu^2} \epsilon_1 - \bar{\epsilon}_1 = \frac{K^2}{E}. \tag{28}$$

按照 Poincare 等采用的级数解法^[4~5],设板心处的挠度为 w_0 ,可以得到,

$$\frac{1}{12(1 - \nu^2)} C_0^2 (w - w_0) = \frac{4K^2}{Ed^2} \left(\frac{A_2}{64} r^4 + \frac{A_4}{144} r^6 \right) - \frac{1}{1 - \nu} \sum_j \frac{\tilde{\epsilon}_j}{(j + 2)^2} r^{j+2}.$$

$$\frac{d}{EC_0^2} \left(\frac{1}{64} + \frac{A_2}{576} + \frac{A_4}{2304} \right) + \frac{C_1}{4} (lg - 1) + \frac{C_2}{4} + C_3 lg + C_4, \tag{29}$$

$$\frac{K^2}{Ed} = \left[\left(\frac{-}{d} \right)^2 - \left(\frac{-}{d} \right)^2 \right]^{1 - \frac{2}{1 - i}} \frac{-}{i + 2}, \tag{30}$$

其中,

$$C_1 = C_3 = 0, \quad A_2 = \frac{-2(3 +)}{5 +}, \quad A_4 = \frac{1 +}{5 +}, \tag{31}$$

$$C_2 = \frac{-2}{1 +} \left\{ \frac{4K^2}{Ed^2} \left[\frac{A_2}{16} (3 +) + \frac{A_4}{24} (5 +) \right] \right\} - \frac{2}{1 +} \frac{-}{j} \frac{-}{j + 2} + \frac{2d}{1 +} \frac{-}{EC_0^2} \left[\frac{3 +}{16} + \frac{5 +}{96} A_2 + \frac{7 +}{288} A_4 \right], \tag{32}$$

$$C_4 = - \frac{K^2}{ED^2} \left(\frac{A_2}{16} + \frac{A_4}{36} \right) + \frac{-}{1 -} \frac{-}{j} \frac{-}{(j + 2)^2} + \frac{d}{EC_0^2} \left(\frac{1}{64} + \frac{A_2}{576} + \frac{A_4}{2304} \right) - \frac{C_2}{4}. \tag{33}$$

将薄板上的温度场和温度矩用下式展开,

$$\bar{-} = \bar{-}_i^i \quad (i = 0, 2, 4, \dots), \tag{34}$$

$$\tilde{-} = \tilde{-}_j^j \quad (j = 0, 2, 4, \dots). \tag{35}$$

如果只考虑板心处的挠度,有,

$$f_1 \frac{-}{d} + f_2 \left(\frac{-}{d} \right)^3 + f_3 \frac{-}{d} + f_4 = 0, \tag{36}$$

其中,

$$f_1 = \frac{-d^2}{E} \left(3 - \frac{5}{2(1 +)} \right), \quad f_2 = \frac{1}{2(1 +)} - 2, \tag{37}$$

$$f_3 = f_2 \left(\frac{-}{d} \right)^2 - \frac{1}{12(1 - ^2)} C_0^2 - \frac{f_2}{12} \frac{2}{1 -} \frac{-}{i} \frac{-}{i + 2}, \tag{38}$$

$$f_4 = \frac{[4 + (1 +)j] \tilde{-}_j}{2(1 - ^2)(j + 2)^2} + \frac{1}{12(1 - ^2)} C_0^2 \frac{-}{d}, \tag{39}$$

$$1 = \frac{1}{1 - ^2} C_0^2 \left(A_2^2 + 2A_4^2 + \frac{8}{3} A_2 A_4 \right), \quad 2 = \left(\frac{A_2}{16} + \frac{A_4}{36} \right), \tag{40}$$

$$3 = \frac{1}{64} + \frac{A_2}{576} + \frac{A_4}{2304}, \quad 4 = \frac{A_2}{16} (3 +) + \frac{A_4}{24} (5 +), \tag{41}$$

$$5 = \frac{+3}{16} + \frac{5 +}{96} A_2 + \frac{7 +}{288} A_4. \tag{42}$$

相应的初始条件为,

$$t = 0, \quad \frac{-}{d} = 0, \quad \frac{-}{d} = \frac{-}{d}. \tag{43}$$

如果令,

$$\bar{-} = \bar{-}_0^* + \bar{-}_1^*, \tag{44}$$

其中, $\bar{-}_0^*$, $\bar{-}_1^*$ 分别为稳态解和小扰动解,于是,

$$f_1 \bar{-}_1^* + 3f_2 \bar{-}_0^{*2} + f_3 \bar{-}_1^* = 0. \tag{45}$$

设 $\bar{-}_1^*$ 的解有如下形式,

$$\bar{-}_1^* = \exp(-t). \tag{46}$$

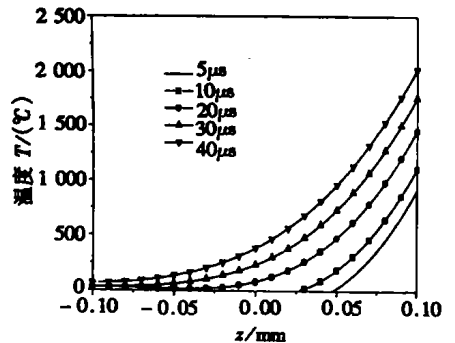


图 2 薄板内温度沿厚度的分布

如果取 $\dot{\delta}^* = 0$, 稳定的临界条件 ($\ddot{\delta} = 0$) 要求,

$$3 \frac{d^0}{d} f_2 + f_3 = 0 \tag{47}$$

4 结果讨论

我们首先给出激光、材料和结构参数,

$$\begin{aligned} q &= 10^7 \text{W/cm}^2, & \rho &= 8900 \text{Kg/m}^3, & E &= 10^{11} \text{Pa}, & \nu &= 0.163, \\ \alpha &= 1.09 \text{W/cm} \cdot \text{K}, & c &= 250 \text{J/Kg} \cdot \text{K}, & \beta &= 2 \times 10^{-5} \text{K}^{-1}, \\ a &= 10 \text{mm}, & d &= 0.2 \text{mm} \end{aligned} \tag{48}$$

根据公式(4), 我们可以得到如图 2 所示的温度分布, 随着激光辐照时间的增长, 板表面的温度上升, 热沿板厚度方向下表面传播.

利用二级二阶半隐式 Runge-Kutta 方法, 对板心处的控制方程(36)进行数值积分, 可得出图 3 所示的挠度-激光辐照时间曲线, 它反映了薄板的屈曲及后屈曲的过程, 展现了屈曲对于初始缺陷等因素的敏感性. 从中我们发现:

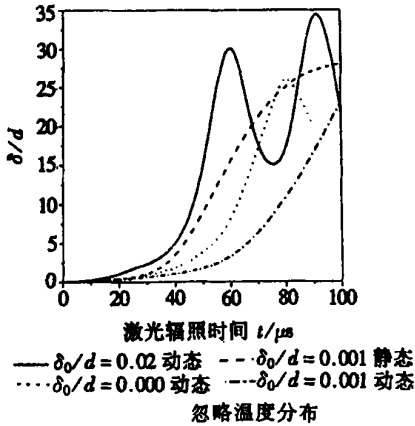


图 3 挠度-激光辐照时间曲线

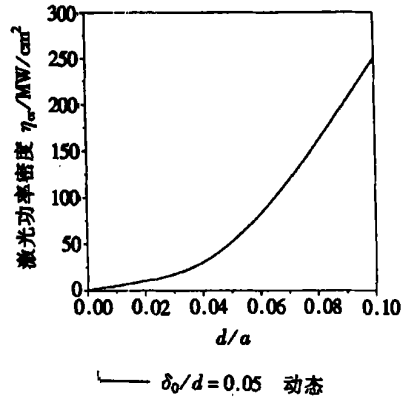


图 4 临界激光功率密度 q_{cr} 与厚径比的关系

- A. 惯性项的引入, 迟滞了屈曲的发生.
- B. 屈曲对于初始缺陷的大小较为敏感, 缺陷增大, 屈曲的发生将提前.
- C. 考虑温度厚度方向的分布将提前屈曲的发生. 如果只是计入平均温度效应, 对屈曲的预测有较大的误差.

另外, 在上述问题中, 我们按照 Budiansky 等人的作法, 认为挠度-激光辐照时间曲线中斜率突然变大即为屈曲点. 这一作法明显带有一定的经验性, 但应用于数值研究中较为方便.

下面, 我们利用生活上扰动方法得出的屈曲判据, 获得了薄铜片屈曲所对应的临界激光功率密度与薄板厚径比 d/a 的关系, 如图 4 所示. 它表明, 激光临界功率密度与厚径比的平方线性相关.

图 3 和图 4 所得的预测结果与实验观察^[1]和数值计算的结果^[6]吻合较好.

[参 考 文 献]

[1] DUAN Zhu-ping, ZHOU Yi-chun, HUANG Chen-guang, et al. On laser induced reverse plugging effect [A]. In: Zheng Z M, Tan Q M Eds. IUTAM Symposium on Impact Dyn [C]. Beijing: Peking

University Press ,1994.

- [2] 朱兆祥. 弹性结构的动态屈曲[A]. 见:朱兆祥 著. 材料和结构的不稳定性[M]. 北京:科学出版社,1993,158—173.
- [3] Hermann G. Dynamic stability of structures[M]. New York: Pergamon Press ,1967.
- [4] Edstrom R E. Dynamic buckling of a rectangular orthotropic plate[J]. AIAA Journal ,1972, **11**(12) : 75—83.
- [5] Pal M C. Large deflection of heated circular plates[J]. Acta Mechanica ,1969, **8**(1) :82—103.
- [6] 黄晨光. 激光辐照下几种金属材料的动态力学性质和结构热-力耦合的变形与破坏[D]. 北京:中国科学院力学研究所,1996.

Studies On the Dynamic Buckling of Circular Plate Irradiated by Laser Beam

HUANG Chen-guang, DUAN Zhu-ping

(Institute of Mechanics , Chinese Academy of Sciences , Beijing 100080 , P R China)

Abstract: The dynamic buckling of thin copper plate induced by laser beam, was analyzed with the numerical integration and disturbance methods of controlling equation. The buckling and post-buckling of thin plate were shown, with the consideration of the temperature distribution, inertia effect and initial deflection. At last, the buckling criterion about the circular plate was obtained and used to investigate the relation between the critical laser intensity and the ratio of thickness and diameter of the plate. The results fit the experimental observation and the FEM simulation very well, and benefit to the understanding of failure phenomenon of structures irradiated by laser beam.

Key words: laser; buckling; thermal shock