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DYNAMIC STRESS ANALYSIS OF THE INTERFACE IN A PARTICLE-REINFORCED COMPOSITE *

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Abstract: The analysis of the dynamic stress on the particle-matrix interface in particle-reinforced composite for the reason that this stress may lead to the microvoids' nucleation due to the interfacial debonding were studied. For simplification, a sphere containing a concentric rigid spherical particle was taken as the representative volume element (RVE). The Laplace transformation was used to derive the basic equations, and the analytical solutions were obtained by means of Hankel transformation. Moreover, the influences of the inertia and viscosity on the debonding damage were also discussed.

Key words: rheological material; dynamic stress; interface debonding; microvoids' nucleation

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Introduction

The interfacial debonding may cause the microvoids' nucleation in a particle-reinforced composite. The interfacial debonding is usually governed by the tension stress criterion[1,2] or energy criterion[3,4]. In the recent research works, the present authors pointed out that the analysis of interfacial stress is the key procedure in the nucleation analysis, no matter which criterion is used as the critical condition of nucleation[5]. If the applied load is static, the interfacial stress can be obtained by means of Eshelby's equivalent inclusion method and Mori-Tanaka's theory. But it is extremely difficult to obtain the exact solution in the case of dynamic loading due to the propagation of the stress wave in the composite. As a preliminary discussion, the average method is used and a sphere containing a concentric rigid spherical particle is taken as

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the Representative Volume Element (RVE). The governing equation of a linear viscoelastic matrix is written in terms of a linear elastic form by taking Laplace transformation. The solution of the problem is divided into two parts: one is a quasi-static solution which satisfies the inhomogeneous mixed boundary conditions, and it can be easily obtained by using the method in elasticity; the other is a dynamic solution which satisfies homogeneous mixed boundary conditions, and can be obtained by using Hankel transformation. The final solution is the superposition of the above two parts. Finally, a numerical example is given to show the fluctuating effect of the interfacial stress, from which influences of the inertia and viscosity on the interfacial debonding are discussed.

1 Basic Equations and Their Laplace Transformation

A spherical cell model is used as a representative volume element (RVE) in the analysis of dynamic nucleation of microvoids. The RVE with outside radius b contains a concentric spherical particle with radius a. The particle volume concentration is $f = a^3/b^3$. A uniform force $\Sigma(t)$ is applied on the outer boundary of the RVE. For a composite material filled with hard particles, the particles may be approximately considered as rigid ones because their stiffness is much higher than that of the matrix. For example in PP/$CaCO_3$ composite systems, the elastic modulus of $CaCO_3$ particles is about 50 times the initial relaxation modulus of PP matrix. Suppose that the Poisson's ratio of the matrix is a constant, and the constitutive relation of the matrix material can be expressed in a linear viscoelastic form as follows[6]:

$$\boldsymbol{\sigma}(\mathrm{x},t) = \int_{-\infty}^{t}\left\{E(t-\tau)\left[\frac{\nu}{(1+\nu)(1-2\nu)}\dot{\varepsilon}_{kk}(\mathrm{x},\tau)\,\mathrm{I} + \frac{1}{1+\nu}\dot{\boldsymbol{\varepsilon}}(\mathrm{x},\tau)\right]\right\}\mathrm{d}\tau, \quad (1)$$

where $E(t)$ is relaxation moduli. The bulk and shear moduli of the matrix and their Laplace transformation are

$$K_m(t) = \frac{E(t)}{3(1-2\nu)}, \quad G_m(t) = \frac{E(t)}{2(1+\nu)}, \quad (2a)$$

$$\bar{K}_m(s) = \frac{\bar{E}(s)}{3(1-2\nu)}, \quad \bar{G}_m(s) = \frac{\bar{E}(s)}{2(1+\nu)}. \quad (2b)$$

Because the problem is spherical symmetry, the equation of motion in Laplace space can be written as

$$\frac{\partial\bar{\sigma}_r}{\partial r} + \frac{2}{r}(\bar{\sigma}_r - \bar{\sigma}_\theta) = \rho s^2\bar{u}. \quad (3)$$

In Laplace space, the constitutive relation of the matrix can be expressed in a stress-displacement form as follows:

$$\bar{\sigma}_r = s\left(\bar{K}_m + \frac{4}{3}\bar{G}_m\right)\frac{\partial\bar{u}}{\partial r} + s\left(2\bar{K}_m - \frac{4}{3}\bar{G}_m\right)\frac{\bar{u}}{r}, \quad (4a)$$

$$\bar{\sigma}_\theta = \bar{\sigma}_\phi = s\left(\bar{K}_m - \frac{2}{3}\bar{G}_m\right)\frac{\partial\bar{u}}{\partial r} + s\left(2\bar{K}_m + \frac{2}{3}\bar{G}_m\right)\frac{\bar{u}}{r}. \quad (4b)$$

Substituting Eq. (4) into Eq. (3), we obtain

$$D(\bar{u}) = \frac{\rho s}{\bar{L}}\bar{u}, \quad (5a)$$

where

$$D(\ ) = \frac{\partial^2(\ )}{\partial r^2} + \frac{2}{r}\frac{\partial(\ )}{\partial r} - \frac{2}{r^2}(\ ), \tag{5b}$$

$$\bar{L} = \bar{K}_m + \frac{4}{3}\bar{G}_m. \tag{5c}$$

The boundary conditions are

$$r = a, \quad \bar{u} = 0, \tag{6a}$$

$$r = b, \quad \bar{\sigma}_r = s\left(\bar{K}_m + \frac{4}{3}\bar{G}_m\right)\frac{\partial \bar{u}}{\partial r} + s\left(2\bar{K}_m - \frac{4}{3}\bar{G}_m\right)\frac{\bar{u}}{r} = \Sigma. \tag{6b}$$

The solution of the above dynamic problem may be obtained by solving Eqs. (5), (6) and by taking the inverse transformation of $\bar{u}$.

2 Solution of the Problem

Supposing that $\bar{u}$ can be divided into two parts

$$\bar{u}(r,s) = \bar{u}_1(r,s) + \sum_n \bar{d}_n(s)U_n(r), \tag{7}$$

where $\bar{u}_1$ satisfies quasi-static equation and inhomogeneous boundary condition (6), i.e.,

$$D(\bar{u}_1) = 0, \tag{8a}$$

$$r = a, \quad \bar{u}_1 = 0, \tag{8b}$$

$$r = b, \quad s\left(\bar{K}_m + \frac{4}{3}\bar{G}_m\right)\frac{\partial \bar{u}_1}{\partial r} + s\left(2\bar{K}_m - \frac{4}{3}\bar{G}_m\right)\frac{\bar{u}_1}{r} = \Sigma. \tag{8c}$$

By solving Eq. (8), we obtain

$$\bar{u}_1(r,s) = \bar{c}_1(s)r + \frac{1}{r^2}\bar{c}_2(s), \tag{9a}$$

$$\bar{c}_1 = \frac{\Sigma}{s(3\bar{K}_m + 4f\bar{G}_m)}, \quad \bar{c}_2 = -\frac{a^3\Sigma}{s(3\bar{K}_m + 4f\bar{G}_m)}. \tag{9b}$$

According to Christensen's method[7], let $U_n(r)$ be the solution of the following eigenvalue problem

$$D(U_n) + k^2U_n = 0, \tag{10a}$$

$$r = a, \quad U_n = 0, \tag{10b}$$

$$r = b, \quad \frac{\partial U_n}{\partial r} + h\frac{U_n}{r} = 0, \tag{10c}$$

where k is to be determined, and

$$h = \frac{2K - 4G/3}{K + 4G/3}, \quad K = \frac{E(0)}{3(1-2\nu)}, \quad G = \frac{E(0)}{2(1+\nu)}. \tag{10d}$$

Taking

$$U_n(r) = r^{-1/2}A_n(r), \tag{11}$$

and substituting Eq. (11) into Eq. (10a), we have

$$\frac{d^2A_n}{dr^2} + \frac{1}{r}\frac{dA_n}{dr} + \left[k^2 - \left(\frac{3/2}{r}\right)^2\right]A_n = 0, \tag{12a}$$

$$A_n(a) = 0, \tag{12b}$$

$$\left(\frac{\partial A_n}{\partial r}\right)_{r=b} + h_1 A_n(b) = 0, \quad h_1 = \left(h - \frac{1}{2}\right)\Big/ b. \tag{12c}$$

Eq. (12a) is Bessel's equation, its solution is

$$A_n(r) = A_1 J_{3/2}(kr) + A_2 Y_{3/2}(kr), \tag{13}$$

where $J_{3/2}(kr)$, $Y_{3/2}(kr)$ are 3/2-th Bessel's functions of the first and second kind, respectively. From the boundary conditions of Eq. (12b) we obtain

$$A_n(r) = A_1'[Y_{3/2}(ka)J_{3/2}(kr) - J_{3/2}(ka)Y_{3/2}(kr)], \tag{14}$$

where A_1' is a constant. Because A_1' can be included in $D_n(t)$, we take $A_1' = 1$. k may be determined by using the boundary conditions of Eq. (12c), i.e.,

$$h_1[Y_{3/2}(ka)J_{3/2}(kb) - J_{3/2}(ka)Y_{3/2}(kb)] + k[Y_{3/2}(ka)J_{3/2}'(kb) - J_{3/2}(ka)Y_{3/2}'(kb)] = 0, \tag{15}$$

$k = k_n$ $(n = 1, 2, \cdots)$. Hence, we obtain

$$U_n(r) = r^{-1/2}[Y_{3/2}(k_n a)J_{3/2}(k_n r) - J_{3/2}(k_n a)Y_{3/2}(k_n r)]. \tag{16}$$

It can be seen that the solution $\bar{u} = \bar{u}_1 + \sum_n \bar{d}_n U_n$ satisfies all boundary conditions. Now we try to determine $\bar{d}_n$, and let it satisfy Eq. (5a). Substituting $\bar{u}$ into Eq. (5) yields

$$D(\bar{u}_1) + \sum_n \bar{d}_n D(U_n) = \frac{\rho s}{L}\left(\bar{u}_1 + \sum_n \bar{d}_n U_n\right). \tag{17}$$

Making use of the results of Eqs. (8a), (10a) and (17), and taking

$$W(r) = r^{3/2} - a^3 r^{-3/2}, \tag{18}$$

we obtain

$$\sum_n (\rho s + k_n^2 \bar{L})\bar{d}_n A_n = -\rho s \bar{c}_1 W. \tag{19}$$

In order to determine $\bar{d}_n(s)$, the following definition of the finite Hankel's transformation is introduced

$$W^*(k_n) = H[W(r)] = \int_a^b r W(r) A_n(r)\,\mathrm{d}r. \tag{20a}$$

Its inverse transformation is[8]

$$W(r) = H^{-1}[W^*(k_n)] = \sum_n \frac{W^*(k_n)}{F(k_n)} A_n(r), \tag{20b}$$

where

$$F(k_n) = \frac{2}{\pi}\,\frac{J_{3/2}(k_n a)\left[h_1^2 + k_n^2\left(1 - \left(\frac{3}{2k_n b}\right)^2\right)\right] - [k_n J_{3/2}'(k_n b) + h_1 J_{3/2}(k_n b)]^2}{k_n^2[k_n J_{3/2}'(k_n b) + h_1 J_{3/2}(k_n b)]^2}. \tag{20c}$$

Substituting Eq. (20) into Eq. (19) yields

$$\bar{d}_n(s) = \bar{c}_{3n}(s)\frac{W^*(k_n)}{F(k_n)}, \quad \bar{c}_{3n}(s) = -\frac{\rho s}{\rho s + k_n^2 \bar{L}}\bar{c}_1(s). \tag{21}$$

If the inverse transformation of $\bar{c}_1(s)$, $\bar{d}_n(s)$ are $c_1(t)$, $d_n(t)$, respectively, the displacement of the matrix is

$$u(r,t) = c_1(t)\left(r - \frac{a^3}{r^2}\right) + \sum_n d_n(t)\, r^{-1/2}[\, Y_{3/2}(k_n a) J_{3/2}(k_n r) - J_{3/2}(k_n a) Y_{3/2}(k_n r)\,]. \tag{22}$$

3 The Normal Stress on the Interface

From Eqs. (2) and (4), the interfacial stress in the Laplace space is obtained as

$$\bar{\sigma}_r(a,s) = s\bar{E}\left[\frac{1-\nu}{(1+\nu)(1-2\nu)}\frac{\partial \bar{u}}{\partial r}\right]_{r=a}. \tag{23}$$

The inverse transformation of Eq. (23) may be written by

$$\sigma_r(a,t) = \int_{\infty}^{t} E(t-\tau)\frac{1-\nu}{(1+\nu)(1-2\nu)}\frac{\partial}{\partial \tau}\left(\frac{\partial u}{\partial r}\right)_{r=a}\mathrm{d}\tau. \tag{24}$$

In the following, the matrix is taken as a standard viscoelastic solid ($E(t) = E_2 + E_1 \exp(-t/t_m)$), and the outer boundary is subjected to a step uniform stress

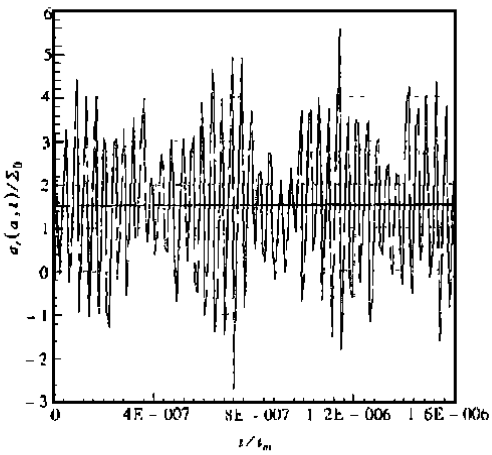
$$\Sigma(t) = \Sigma_o \cdot H(t), \tag{25}$$

where Σ_o is a constant; $H(t)$ is an unit step function.

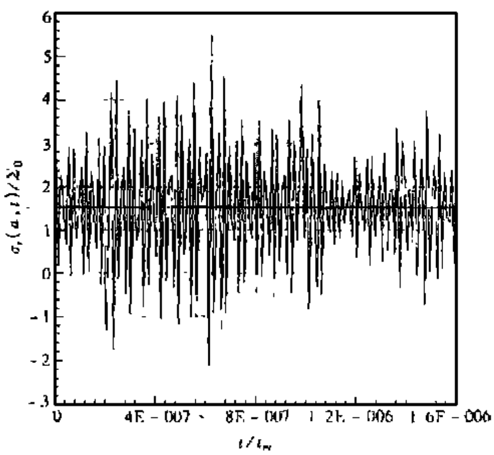
In the calculation, parameters are taken to be $t_m = 5\mathrm{s}$, $\nu = 0.3$, $f = 0.1$, $E_1 = 1.0 \times 10^8\,\mathrm{Pa}$, $E_2 = 5.0 \times 10^8\,\mathrm{Pa}$. The normal stress variations with time are shown in Fig. 1 (a)~(i).

4 Discussion and Conclusions

Fig. 1 shows the interface stress fluttering at the different time with the particle radius $a = 2\mu\mathrm{m}$, $6\mu\mathrm{m}$ and $10\mu\mathrm{m}$, respectively. According to the numerical results, the following conclusions can be drawn:



(a) $a = 2\mu\mathrm{m}$

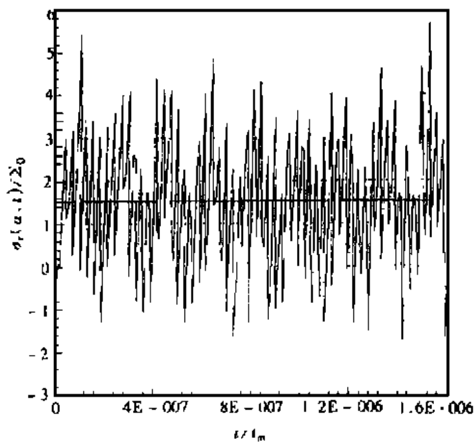


(b) $a = 6\mu\mathrm{m}$

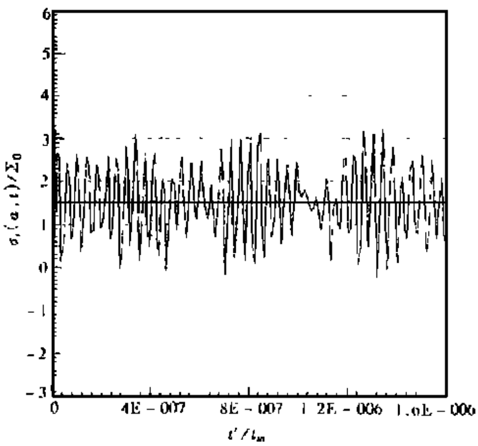
Fig. 1 The relation between interfacial stress and time

1) Under the action of step loading, the interfacial stress shows acute fluctuating effect.

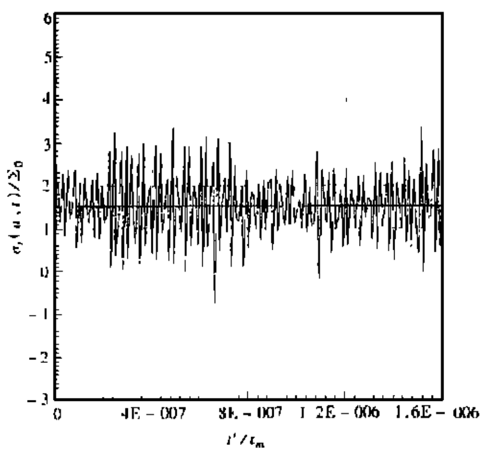
Both the frequency and the peak value of the stress are very high , and the maximun peak value is much higher than that of quasi-static solution. From the computational example shown in Fig. 1 (a) , the quasi-static solution is $\sigma_{rs}(a) = 1.521\Sigma_0$, while the maximun peak value of the solution in dynamic case is $\max\{\sigma_r(a,t)\} = 5.6\Sigma_0$, this means that the value of the dynamic solution considering inertia effect is 3.6 times that of the quasi-static solution in which the inertia effect is neglected. Therefore , if the interface strength is used as a microvoids ' nucleation criterion , the interfacial debonding is prone to occur when the composite is subjected to a dynamic loading. It is obvious that according to the quasi-static solution , the number of the particles debonded will be less , and the elastic modulus of the composite will be overestimated.



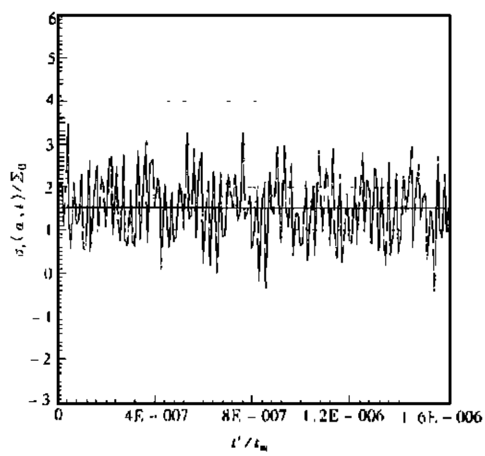
(c) $a = 10\mu m$



(d) $a = 2\mu m$, $t'/t_m = t/t_m - 4$



(e) $a = 6\mu m$, $t'/t_m = t/t_m - 4$

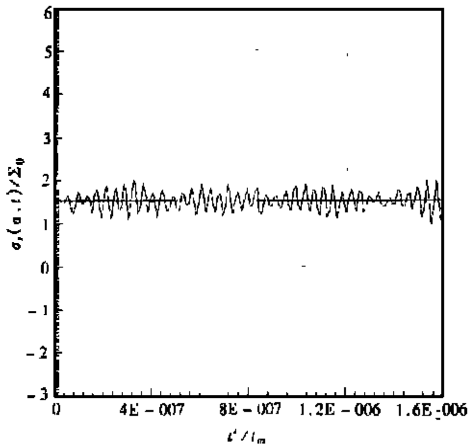


(f) $a = 10\mu m$, $t'/t_m = t/t_m - 4$

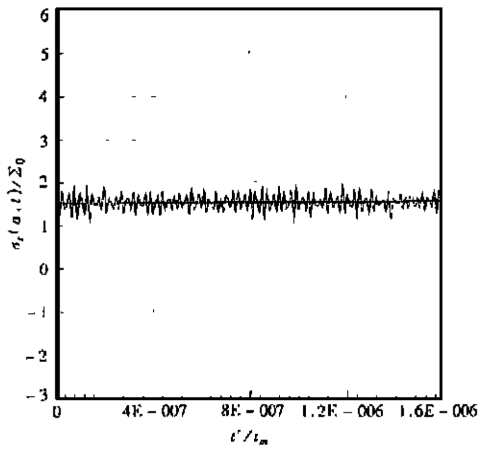
Fig. 1 The relation between interfacial stress and time

2) Because of the viscous effect of the matrix material , the stress in the matrix will attenuate during its propagation course , so the peak value of the interfacial stress will go downhill. Fig. 1

(a) ~ (c) give the attenuation curves of the stress with $a = 2\mu m$. The maximun peak value of the stress is $5.6\Sigma_0$ in Fig. 1 (a) (at the beginning of loading), but in Fig. 1 (b) (at the instant of $t = 4t_m$) and Fig. 1 (c) (at the instant of $t = 12t_m$), it equals to $3.2\Sigma_0$ and $2.0\Sigma_0$, respectively. This is one of characteristics in viscoelastic materials which is quite different from those in elastic ones. It also indicates that the debonding is prone to occur at the beginning of loading. As time goes on, the debonding damage is not easier to occur than it does at the beginning of loading, because the stress relaxation leads to the peak values falling gradually.



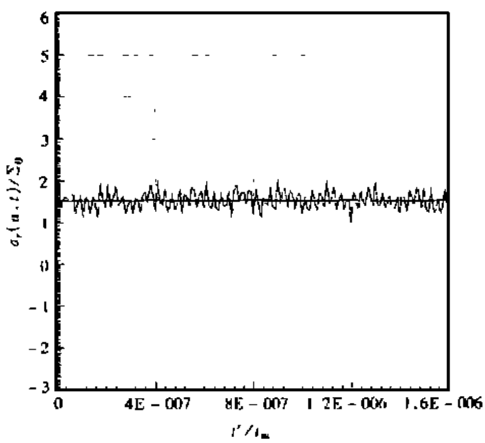
(g) $a = 2\mu m$, $t'/t_m = t/t_m - 12$



(h) $a = 6\mu m$, $t'/t_m = t/t_m - 12$

3) If the particle volume concentration f is given, the influence of particle radius on the interfacial stress is not obvious. Fig. 1 (a), (d) and (g) show the stress fluttering at the beginning of the deformation with $a = 2\mu m$, $6\mu m$ and $10\mu m$. The peak values of them are almost the same (about $5.6\Sigma_0$), while at the instant of $t = 4t_m$ and $t = 12t_m$, the peak values are also almost identical. They are about $3.5\Sigma_0$ and $2.0\Sigma_0$ in Fig. 1 (b), (e), (h), and (c), (f), (i), respectively.

It should be noted that the interfacial stress for different particle radius can not be the same, and it is very difficult to seek the exact solution if there are a great number of particles in the matrix. The above conclusions are obtained by means of the concept of average field, however, they are very helpful for understanding the effects of inertia and viscosity on the interface debonding in the composite under high triaxiality stress.



(i) $a = 10\mu m$, $t'/t_m = t/t_m - 12$

Fig. 1 The relation between interfacial stress and time

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