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A LARGE EDDY SIMULATION OF THE NEAR WAKE OF A CIRCULAR CYLINDER*

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ABSTRACT: Viscous flow around a circular cylinder at a subcritical Reynolds number is investigated using a large eddy simulation (LES) coupled with the Smagorinsky subgrid-scale (SGS) model. A fractional-step method with a secondorder in time and a combined finite-difference/spectral approximations are used to solve the filtered three-dimensional incompressible Navier-Stokes equations. Calculations have been performed with and without the SGS model. Turbulence statistical behaviors and flow structures in the near wake of the cylinder are studied. Some calculated results, including the lift and drag coefficients, shedding frequency, peak Reynolds stresses, and time-average velocity profile, are in good agreement with the experimental and computational data, which shows that the Smagorinsky model can reasonably predict the global features of the flow and some turbulent statistical behaviors.

KEY WORDS: large eddy simulation (LES), turbulence, subgrid-scale model (SGS), unsteady flow, finite difference method

1 INTRODUCTION

Viscous flow past a bluff body has received a great deal of attention owing mainly to its theoretical and practical significance. The turbulent vortex formation behind a circular cylinder and the subsequent transport of the vortices downstream have many important and practical implications. Recently, a large eddy simulation (LES) method has been widely used to investigate fundamental and practical turbulent flows. Since the LES approach is with a full resolution of the large-scale turbulent motion, the numerical results can provide useful physics regarding the formation of vortices behind the cylinder and their evolution near the downstream.

Even though LES has been used by many investigators, most applications have been limited to flows with simple geometries. In recent years, some researchers^[1~8] have investigated the three-dimensional flows past a circular cylinder using LES method coupled

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with the Smagorinsky^[9] subgrid-scale (SGS) model and/or dynamic SGS model. Su et al.^[6] have simulated 2D and 3D viscous flows past a cylinder by using LES technique with the Smagorinsky SGS model and it is shown that there is a big influence of three-dimensionality on the drag force at higher Reynolds numbers. However, their study did not reveal any information on the transition wave occurring in the separated shear layer, which is the critical problem at this Reynolds number region. Further the LES method was also employed to study 3D oscillating flows past a circular cylinder^[7,8]. But in the LES calculations for viscous flows past a circular cylinder, only the effects of numerical methods and schemes on calculated results were discussed. Numerical and modeling aspects for LES for the flow past a circular cylinder in detail by Breuer^[10], who performed computations with five different numerical schemes and with the dynamic and Smagorinsky SGS models. The findings of the previous numerical studies^[3,4] were confirmed and it was shown that simulations with central difference schemes were in better agreement with the experimental data and the influence of the SGS model was found not strong.

Based on the three-dimensional direct numerical simulation (DNS) of the Navier-Stokes equations, some work has been performed to investigate the transition to turbulence of the flow around a circular cylinder in the subcritical Reynolds number regime^[11~13]. Ducros et al.^[14] employed LES method to deal with the fundamental problem of transition to turbulence in a boundary layer developed spatially over a flat plate. Evidence in this study has shown that LES results are in acceptable agreement with experiments and empirical laws for transitional flows. Now, one may use LES to describe the characteristics of the transition in the cylinder wake flow and to verify how well the Smagorinsky model predicts the turbulent behaviors.

In the present study, the temporal and spatial evolution of the wake and the turbulence statistical behaviors in the near wake of the cylinder at subcritical Reynolds number are studied. The governing equations for LES are the three-dimensional filtered incompressible Navier-Stokes equations to be solved using the fractional-step method proposed by Chorin^[15] and Kim & Moin^[16]. It may be commented here that viscous flow past a circular cylinder is an ideal problem for computation because the entire sequence of states from steady flow to turbulence, as indicated by Henderson^[12], can be studied in an extremely small range of Reynolds number. On the other hand, as suggested by Roshko^[17], the transition to turbulence is essentially complete at $Re \approx 300$. So, what is investigated in this study is the flow behaviors at Re = 500. Here, we use the Smagorinsky SGS model to deal with its capability to predict turbulent flow around a cylinder. Some typical dynamical SGS models will be applied in further work.

2 GOVERNING EQUATIONS

The governing equations are the filtered incompressible Navier-Stokes equations. To make the governing equations nondimensional, we use the radius of the cylinder, R, for the length scale; the free-stream approach velocity, U, for the velocity scale; and U/R for the time scale. The nondimensional governing equations are

$$\nabla \cdot \boldsymbol{V} = 0 \tag{1}$$

$$\frac{\partial V}{\partial t} + (\nabla \times V) \times V = -\nabla \Phi + \frac{2}{Re} \nabla^2 V - \nabla \cdot T$$
(2)

where V is the resolved velocity vector, T is the turbulent stress tensor, which may also be written as τ_{ij} , Φ is the nondimensional pressure head, defined as $\Phi = p + \rho V \cdot V/2$, ρ is the fluid density, t is the nondimensional time, and p is the pressure. A natural choice of coordinate system is cylindrical coordinate system. Thus the governing equations (1) and (2) in the cylindrical system are solved in the present study.

The eddy viscosity model of Smagorinsky^[9] is used to model the turbulent stress. The Smagorinsky eddy viscosity model is given as

$$\tau_{ij} = -2\nu_T S_{ij} = -2C_s^2 \bar{\Delta}^2 \sqrt{2S_{ij}S_{ij}} S_{ij}$$
(3)

where S_{ij} is the strain rate tensor, C_s and $\overline{\Delta}$ are the model constant and filtered scale, respectively. A value of $C_s=0.1$ was used for all calculations and $\overline{\Delta} = (\Delta_r \Delta_\theta \Delta_z)^{1/3}$, where Δ_r , Δ_θ and Δ_z are the filtered widths in the radial, circumferential and axial directions.

Note that, in this paper, non-uniform grid must be used. Commutation of the filtering operation with the temporal and spatial differentiations is only strictly valid for uniform grid systems as discussed by $\text{Ven}^{[18]}$. However, the commutation error can usually be neglected for mildly stretched grids as indicated by Ribault et al.^[19] and Moin^[20], but does exist in general. The most common assumption is, as suggested by Moin^[20], to assume that the effect of the commutation error is lumped into the subgrid model. The modeling error is found by Ribault et al.^[19] to be generally smaller than the discretization error.

In the present calculation, the initial velocity is taken as the 2D potential flow velocity. On the wall of the cylinder, the no-slip and no-penetration boundary conditions for velocity are used. The Neumann boundary condition is used because the far field boundary is sufficiently far from the cylinder. In the circumferential and axial directions, we use periodic boundary conditions.

3 NUMERICAL METHODS

To discretize the governing equations, a second-order fractional step method proposed by Chorin^[15] and Kim and Moin^[16] is used. In this approach, we first obtain an intermediate velocity, \hat{V} , by omitting pressure and using the second-order Adams-Bashforth scheme on the convective terms and the Crank-Nicolson scheme on the viscous terms. This intermediate velocity is corrected by pressure, which is obtained through a Poisson equation, to satisfy the continuity equation. Finally, the boundary conditions on velocity are applied to get the velocity at the next time step. In this section, this procedure in a semi-discrete formulation in vector form is presented.

The intermediate velocity, \hat{V} , is obtained from

$$\frac{\hat{\boldsymbol{V}} - \boldsymbol{V}^n}{\Delta t} = -\frac{1}{2}(3\boldsymbol{N}_c^n - \boldsymbol{N}_c^{n-1}) + \frac{1}{2}(3\boldsymbol{T}_s^n - \boldsymbol{T}_s^{n-1}) + \frac{1}{2}\boldsymbol{L}_v^n$$
(4)

where the superscripts refer to time step, N_c , T_s and L_v represent the convective term, turbulent stress term and viscous term, respectively. The velocity \hat{V} is corrected by pressure to obtain a second intermediate velocity, \tilde{V} , from

$$\frac{\tilde{\boldsymbol{V}} - \hat{\boldsymbol{V}}}{\Delta t} = -\nabla \Phi^{n+1/2} \tag{5}$$

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Finally, the velocity at time step n + 1, is obtained from

$$\frac{\boldsymbol{V}^{n+1} - \boldsymbol{\tilde{V}}}{\Delta t} = \frac{1}{2} \boldsymbol{L}_{v}^{n+1} \tag{6}$$

In this equation, the pressure head, Φ , is unknown, and is determined by using the continuity equation. Using the continuity equation for every time step, n or n + 1, and taking the divergence of Eq.(5), we finally get

$$\nabla^2 \Phi^{n+1/2} = \frac{\nabla \cdot \tilde{V}}{\Delta t} \tag{7}$$

After $\Phi^{n+1/2}$ is found from by Eq.(7) with appropriate boundary conditions, \tilde{V} and V^{n+1} can be calculated from Eqs.(5) and (6), respectively. No boundary conditions are necessary for either of the two intermediate velocities.

To control this time-splitting error, the consistent scheme developed by Karniadakis et al.^[21] is employed on boundary conditions for pressure on solid boundaries where velocity vanishes, i.e.

$$\frac{\partial \Phi}{\partial n} = -\frac{2}{Re} \boldsymbol{n} \cdot \nabla \times (\nabla \times \boldsymbol{V}) \tag{8}$$

where *n* refers to the direction normal to the wall, and *n* is the unit normal vector. Because only one direction (i.e. radial direction) does not have a periodic boundary condition, we use the Fourier spectral approximation in the circumferential and radial directions, and a finitedifference approximation in the radial direction to discretize the governing equations. As found by Breuer^[10] for flows past a circular cylinder, LES calculations with central difference schemes were in better agreement with the experimental data. So, in the present study, a central difference is employed. On the other hand, Thomas and Williams^[22] also found that turbulent behaviors can be reasonably predicted without the wall-damping model in their LES calculation by using the Smagorinsky SGS model if the grid resolution is adequate near the wall. Thus a coordinate transformation is used to generate a finer mesh near the wall of the cylinder in the radial direction.

4 RESULTS AND DISCUSSION

To illustrate the computational procedure, the evolution of the wake, and turbulence statistical behaviors in the near wake of the cylinder, some typical results calculated by 3D LES approach will be mainly discussed. Meanwhile, both 3D and 2D direct calculations are also performed to compare with the results calculated by LES.

The number of mesh points in the present LES calculation is $129 \times 129 \times 65$ in the θ , r and z directions, respectively. The computational domain is 33R in the radial direction and 2π in the axial direction, and the time step is 0.001. During this calculation, a threedimensional flow is first calculated for non-dimensional time period of $t = 0 \sim 100$ without using the SGS model. Then the SGS model is used in the computed procedure for LES calculation at t = 100. Further some time-average quantities are obtained from t = 200 to remove the initial effect.

It is verified that the computed results are independent of the time steps and the grid sizes. Some validations for 3D calculations have been performed by Lu et al.^[8,9]. As an example, here, only a 2D calculation validation is shown. Results calculated by two grid

systems: $(r, \theta) = 128 \times 128$ with $\Delta t = 0.001$ and $(r, \theta) = 256 \times 256$ with $\Delta t = 0.0005$ for the calculation of flow past the cylinder at Re = 500 are compared. They produced virtually identical results as shown in Fig.1; the only difference being a small phase lag for the fine grid. The phase lag is attributed to the onset of the wake instability leading to vortex shedding. The Strouhal number defined usually based on the length scale of the cylinder diameter, as determined from the lift coefficient plot, is 0.22 (or $f_o = 0.11$ based on the length scale of the cylinder radius). The averaged drag coefficient is 1.4. Both the Strouhal number and the averaged drag coefficient compare quite well with the values calculated by Henderson^[12] for the 2D calculation, but show discrepancies with experimental values.



Fig.1 Lift and drag coefficients calculated by 2D laminar flow versus time at Re = 500. Solid lines: mesh number 256×256 and time step 0.0005, and dashed lines: mesh number 128×128 and time step 0.001

The force coefficients calculated by LES are shown in Fig.2, and the corresponding power spectrum density (PSD) of the lift coefficient is given in Fig.3. The highest peak corresponds to the vortex shedding frequency 0.101, or the Strouhal number 0.202, which



by LES versus time at Re = 500



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agrees well with the experimental value of approximately 0.2. The averaged drag coefficient is 1.07, which is also in quite a good agreement with the experimental value of approximately 1.0 shown in [12]. These results validate quantitatively the present calculation. Figure 4 shows the streamwise timeaverage velocity profile at x = 2.0. To validate quantitatively the present LES result, DNS velocity profile at Re = 500 calculated by Karniadakis and Triantafyllou^[23] is also plotted in Fig.4. It is found that both LES and DNS results are in good agreement, except for a little difference in the regions of the peak velocity, where the maximum velocity value calculated by LES is smaller than that by DNS.



Fig.4 Streamwise time-average velocity profile at x = 2.0 and Re = 500. Solid line: present LES result; dot: DNS result calculated by Karniadakis and Triantafyllou^[29]

The time-average pressure coefficients, obtained from 3D LES computations as well as 2D and 3D direct calculations, are shown in Fig.5 for Re = 500. Some experimental values



Fig.5 Distributions of time average pressure coefficient. Results calculated by using LES, 3D and 2D laminar flow calculations at Re = 500

from Norberg^[24] for Re = 3000 and 8000, and Yokuda and Ramaprian^[25] for Re =20000 are also shown; the experimental values in the base region of the cylinder locate in a narrow flat band due to different Reynolds numbers and experimental discrepancies. Although the Reynolds number in this calculation is lower than those given experimentally, the mean pressure coefficient calculated by LES becomes much flat in the separation region, which is an important behavior of the wake flow, qualitatively in good agreement with the experimental values. However, the 2D calculated pressure coefficient distributions clearly see gross errors after the separation.

A three-dimensional direct calculation is performed and the results are used as an initial condition for LES calculation and to investigate the transition to three dimensionality of a cylinder wake. Note that the initial velocity is taken as the 2D potential flow velocity. In this study, because turbulent behaviors in the near wake of the cylinder are mainly investigated, only the onset of the mode occurred through a supercritical bifurcation, i.e., mode B, as originally found by Williamson^[26,27], is discussed. Detailed results have been reported in [11,12] based on a direct numerical simulation of transition from two-dimensional to threedimensional states due to secondary instability in the wake of a circular cylinder. As we concern only the mode B, which corresponds to a family of short-wavelength instabilities with a wavelength of approximately 1.644R and a critical Reynolds value of about 260, the computational domain π in the axial direction is chosen here. We then use 32 point uniform grid over the length π , which gives an adequate resolution of the axial structures as suggested by Jordan and Ragab^[3] and Henderson^[12]. To keep this paper in a manageable size, only vorticity patterns are shown in Fig.6. Spatially periodic streamwise and circumferential vorticity structures, as shown in Fig.6(a) and Fig.6(b), are identified along the spanwise direction. The 3D structure of the flow evolves at Re = 500 as mode B develops in the wake with a wavelength of approximately 1.507R (i.e., two wavelengths along the axial direction within a computed domain of π), that lies in the short-wavelength instabilities region as shown in [12]. As time progresses, the flow shows the transition to turbulence in the near wake of the cylinder, and the 3D structure becomes irregular (not shown here). This behavior is consistent with the results calculated by Persillon and Braza^[11] and Henderson^[12]. Meanwhile, time-average pressure distribution is shown in Fig.5.



(a) Radial component, contours from -8 to 8 with increment 0.4; and solid lines for positive values; dashed lines for negative values



(b) Circumferential component, contours from -4 to 4 with increment 0.2



(c) Axial component, contours from -8 to 8 with increment 0.4 Fig.6 Vorticity components contours calculated for 3D flow without SGS model in r-z plane ($\theta = 8.4^{\circ}$) at Re = 500 and t = 50

Turbulent statistics of the region spatially averaged along the axial direction in terms of the total Reynolds stresses is examined here. Following the approach used by Jordan and Ragab^[3] to study the turbulent statistical behaviors in the near wake of the cylinder,

the instantaneous velocity V_i (i = 1, 2, 3, or u, v and w) is divided into three parts and written as $V_i = \bar{V}_i + \tilde{V}_i + V'_i$, where \bar{V}_i , \tilde{V}_i , V'_i are the mean, periodic and random quantities, respectively. Based on the velocity decomposition, the Reynolds stresses can be calculated. By comparing with the previous results of the Reynolds stresses, the cylindrical velocity components are transformed into the Cartesian components, for convenience, denoted as u, vand w, corresponding to the streamwise, transverse and spanwise components, respectively. Figure 7 shows the contours of the streamwise stress $(\langle \tilde{u}^2 + u'^2 \rangle)$, the transverse stress $(\langle \tilde{v}^2 + v'^2 \rangle)$, and the shear stress of $\langle \tilde{u}\tilde{v} + u'v' \rangle$, where the angular bracket $\langle \rangle$ denotes the mean value. The mean quantities represent the results obtained by time-average of six shedding cycles as shown in Fig.2 for the force coefficients and by space-average along the axial direction. Figure 7 shows negligible levels of stresses within the upstream laminar regimes and along the cylinder periphery, suggesting again that the Smagorinsky model can perform well by responding correctly to the local strain-rates and by contributing reasonably



- (a) Results calculated in this study. Top: $\langle \tilde{u}^2 + u'^2 \rangle$ contours from 0.01 to 0.2 with increment 0.01; middle: $\langle \tilde{v}^2 + v'^2 \rangle$ contours from 0.02 to 0.42 with increment 0.02; bottom: $\langle \tilde{u}\tilde{v} + u'v' \rangle$ contours from -0.12 to 0.12 with increment 0.012
- (b) Results calculated by Jordan and Ragab^[2] at $Re = 5\,600$. Top: $\langle \tilde{u}^2 + u'^2 \rangle$ contours with maximum of 0.2 and increment of 0.02; middle: $\langle \tilde{v}^2 + v'^2 \rangle$ contours with maximum of 0.42 and increment of 0.042; bottom: $\langle \tilde{u}\tilde{v} + u'v' \rangle$ contours from -0.12 to 0.12 with increment 0.012

Fig.7 Turbulent stress statistics contours in the formation region behind the cylinder

negligible eddy viscosity in regions of known laminar flow conditions or where the turbulence is adequately resolved by the local grid spacing.

According to the argument of Jordan and Ragab^[2], the patterns may serve a sufficient indication to reach statistical steady-state in the formation region by evidence of their distinct symmetric character. Correspondingly, the patterns in Fig.7 show such a character, and the present results have reached the statistical steady-state. A visual inspection of the present total Reynolds stress results shown in Fig.7 reveals similar distribution reported in Cantwell and Coles^[28] within the formation region. Moreover, the DNS data at subcritical Reynolds numbers of 525 in [13] and 3400 in [29] as well as the LES results at 3900 in [3] and 5600 in [2] by using dynamic SGS model are also qualitatively in good agreement with the present observation. As a typical case, the contours of the stresses calculated by Jordan and Ragab^[2] based on LES coupled with dynamic SGS model at Re = 5600 are also shown in Fig.7 to compare with the present results. Although the Reynolds numbers are different, the patterns are very similar. The shear layer shed from the cylinder is stronger for higher Reynolds number.

The peak magnitudes of the stresses, $\langle \tilde{u}^2 + u'^2 \rangle$, $\langle \tilde{v}^2 + v'^2 \rangle$ and $\langle \tilde{u}\tilde{v} + u'v' \rangle$, and some previous experimental and computational results^[2,3,13,27,28] for a regime $500 \leq Re \leq 140\,000$ are listed in Table 1. As the Reynolds regime is below the critical value of about 3×10^5 , these flows should qualitatively share similar dynamics. So, we can find the peak magnitudes of the stresses are in similar orders. In particular, the present calculated peak magnitudes of the stresses agree well with the DNS data at Re = 525 (Mittal and Balachandar^[13]). Based on the above comparisons, the turbulent statistical behaviors in the near wake of the cylinder can be reasonably predicted in the subcritical Reynolds regime.

Re	500	$525^{[12]}$	$3400^{[14]}$	3 900 ^[3]	5 600[2]	140 000 ^[27]
$\langle \tilde{u}^2 + u'^2 \rangle$	0.20	0.21	0.17	0.18	0.20	0.22
$\langle ilde{v}^2 + v'^2 angle$	0.42	0.40	0.35	0.40	0.42	0.43
$\langle ilde{u} ilde{v}+u'v' angle$	0.12	0.15	0.10	0.12	0.12	0.19

 Table 1 Peak normal and shear Reynolds total stresses in the formation region

From Fig.7, the cylinder wake regions are similar if based on Reynolds stresses reaching similar order of 10^{-1} of magnitude with their maximums for $\langle \tilde{u}^2 + u'^2 \rangle$, $\langle \tilde{v}^2 + v'^2 \rangle$ and $\langle \tilde{u}\tilde{v} + u'v' \rangle$. Figure 8 shows the velocity components (u, v, w) variations versus time at two locations inside and outside of wake, respectively. These velocity plots demonstrate the effect of turbulence on flow at in-wake location and laminar flow behavior at out-of-wake location. The corresponding power spectra are also shown in Fig.9. The spectra show the most energy at the vortex shedding frequency with notable peaks, but with less energy, at subharmonics of the vortex shedding frequency. Also the distribution of energy at the in-wake location is much broader banded than that at the out-of-wake location.

A useful quantity in visualizing the flow structure in the near wake is the magnitude of vorticity and its components. Figure 10 shows the vorticity contours in several sections at different spanwise locations, and Fig.11 displays the vorticity components patterns in r-zplane of $\theta = 8.4^{\circ}$. The patterns represent that random small-scale eddies appear in the large-scale structures, compared with the 3D structure shown in Fig.6 in transition regime, connecting the alternating shed vortices. Some random eddies with different scales exist in the near wake of the cylinder.







5 CONCLUDING REMARKS

Three-dimensional viscous flow around a circular cylinder is investigated using large



Fig.10 Vorticity magnitude contours from 0.25 to 10 with increment 0.25 at t = 265(axial length $L_z = 2\pi$)



Fig.10 Vorticity magnitude contours from 0.25 to 10 with increment 0.25 at t = 265(axial length $L_z = 2\pi$) (continued)

eddy simulation coupled with the Smagorinsky subgrid-scale model. A fractional step method and a combined finite-difference/spectral approximations are used to solve the threedimensional filtered incompressible Navier-Stokes equations. The direct calculation without SGS model can predict the mode B of transition from two-dimensional to three-dimensional due to secondary instability in the wake of a circular cylinder. Furthermore, LES calculation is performed to deal with turbulent statistical behaviors and flow structures in the near wake of the cylinder at lower subcritical Reynolds number Re = 500.



- (a) Radial component contours from -8 to 8 with increment 0.4; solid lines for positive values; dashed lines for negative values
 - Fig.11 Vorticity components contours in a plane at $\theta = 8.4^{\circ}$ and t = 265



(b) Circumferential component contours from -4 to 4 with increment 0.2



(c) Axial component contours from -8 to 8 with increment 0.4 Fig.11 Vorticity components contours in a plane at $\theta = 8.4^{\circ}$ at t = 265 (continued)

Computed results of lift and drag coefficients, vortex shedding frequency, time-average velocity profile, and peak magnitudes of the stresses are in good agreement with the experimentally and computationally determined values in the previous studies. According to the present results, the Smagorinsky model can perform well by responding correctly to the local strain-rates and by contributing reasonably negligible eddy viscosity in regions of known laminar flow conditions or where the turbulence is adequately resolved by the local grid spacing. The computational approach used in this study can reasonably predict the global features of the flow and some turbulent statistical behaviors in the near wake of the cylinder.

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