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An optimization approach to the similarity criteria of flows and its application*

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Abstract In the present paper, we propose an optimization approach to investigate the similarity criteria of complex flows. With this approach, we may identify the dominant dimensionless variables governing complex flows by numerical sensitivity analysis. Firstly, we define the sensitivity factor and examine its dependence on the dimensionless variables. Then, we apply this approach to study the similarity criteria of porous media flow in a presumed oil reservoir. The similarity principle obtained from the numerical sensitivity analysis is in agreement with the theoretical law, thus demonstrating the feasibility of the proposed optimization approach. Further explanation is given by analyzing the deviation of pressure distribution in a model from its prototype. In addition, we examine the effects of flow parameter variation on the sensitivity factors and find that the dominant dimensionless variables may change from different sets of parameters.

Keywords: similarity, optimization, sensitivity analysis, complex flows, model.

Physical modeling is one of the fundamental approaches to studying flow mechanisms. The physical modeling is based on similarity theory. Generally speaking, a model is entirely similar to the prototype if each of its corresponding dimensionless variables is kept identical. However, complex flows, such as multi-scale, multi-phase and/or multi-component flows in industrial and environmental engineering, tend to involve many parameters[1,2] associated with physical, chemical or even biological processes[3], thus involving a number of dimensionless parameters. Most importantly, it is impossible to keep all the nondimensional parameters identical. For instance, it is hard to simultaneously satisfy the similarity of both settling and incipience of sediment when modeling the sediment-laden water flow in nature. Ship motion modeling requires no difference of both Froude and Reynolds numbers between the model and the actual motion, which is not practical at all[4]. Another paradigm is the porous media flows, particularly the complex ones in the second or the third developed oil reservoirs, which involve the basic formation parameters and the properties of the fluids, and all kinds of processes between phases and interfaces[5,6]. Thus, the dimensionless variables can be up to ten or even more, and inconsistencies between these dimensionless constraints often occur.

Cheng and Tan[7] have pointed out that it is practical to carry out partial modeling when strict similarity of all dimensionless parameters is infeasible. This means that the dominant dimensionless parameters are modeled while the insignificant ones are relaxed. The distorted river model is the very case, in which geometry similarity law is not kept[8]. Now, we are at the point of how to choose the dominant dimensionless parameters. The conventional approach is based on the analysis of physical mechanisms of flows as adopted by Taylor in dealing with the problem of an intense explosion[9]. Also in this way, Cheng[10] successfully derived the similarity law of explosion formation. Doan et al.[11] developed equations for the three-phase and non-isothermal flow in the vicinity of a horizontal well, and obtained the scaling criteria for designing laboratory experiments. Sedov[4] investigated the theory of this approach and applied it in classical mechanical problems. Another new way to select dominant dimensionless parameters is based on numerical sensitivity analysis. Based on multiphase, multicomponent transport theory, Mackinnon et al.[3] presented a general mathematical model describing subsurface aerobic biotransformation of organic

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chemical species in a multiphase setting, and carried out a series of numerical sensitivity studies to examine the impact of the selected dimensionless groups on the overall system biotransformation rates. Peng et al.[12] investigated the sensitivity of oil recovery to the dimensionless variables. Nonetheless, the definition of the sensitivity factor is not reasonable in that its dependence on the dimensionless variables is not considered. In comparison, the conventional approach needs in-depth understanding of the physical mechanisms of flows in advance. The numerical one is more universal in application, though it is far from perfect at present and needs further improvement.

In this paper, we will propose an optimization approach to the principal similarity criterion by numerical sensitivity analysis. The sensitivity factor of a target function to a dimensionless variable is defined. The dependence of the sensitivity factor on the dimensionless variable itself and the necessity of using a dimensionless mathematical model are discussed. We also apply the proposed approach to a case of porous media flow and explore the dependence of the sensitivity factors on the dimensionless parameters themselves.

1 Generalization of the optimization approach

For a complex system with N dimensionless variables, a target function f can be expressed as

$$f = f(\pi_1, \pi_2, \pi_3, \cdots, \pi_N), \tag{1}$$

where π_k is the dimensionless variable ($k = 1, 2, 3, \cdots, N$).

In order to design a practical model of a complex flow, it is crucial to manifest the important ones from all of the dimensionless variables, which can be done by investigating the sensitivity of the target function with respect to each independent variable defined as

$$s_k = \left| \frac{\partial f}{\partial \pi_k} \right|, \quad k = 1, 2, \cdots, N. \tag{2}$$

Then, we may say that the larger the sensitivity factor, the more important the dimensionless variable. In this way, we may further determine the principal similarity conditions by identifying the dimensionless variables with larger sensitivity factors.

Apparently, the sensitivity factor is also the function of all the dimensionless variables. That is to say, the sensitivity factor is not constant unless the target function varies linearly with the dimensionless parameters, which we should pay particular attention to. Therefore, different sets of the independent π_k will definitely lead to different sensitivity factors. For one set of independent π_k of a flow system, we can obtain a group of major dimensionless variables. Yet for another set of independent π_k of the same system, we may analogously obtain another group of major dimensionless variables. In other words, the dominant dimensionless variables are dependent on π_k.

For practical purpose, relative variation is of more physics. Therefore, it is better to redefine the sensitivity factor as

$$s_k = \left| \left. \frac{\partial (f/f_{\mathrm{p}})}{\partial (\pi_k/\pi_{\mathrm{p},k})} \right|_{\pi_k = \pi_{\mathrm{p},k}} \right|, \quad k = 1, 2, \cdots, N. \tag{3}$$

Here, s_k measures the sensitivity of the target function to variable π_k for the case of $\pi_k = \pi_{\mathrm{p},k}$, and $f_{\mathrm{p}} = f(\pi_{\mathrm{p},1}, \pi_{\mathrm{p},2}, \cdots, \pi_{\mathrm{p},N})$, where $\pi_{\mathrm{p},k}$ indicates the value of π_k of the prototype.

Generally speaking, the target function cannot be explicitly expressed as a simple formula for complex flows. Therefore, it is not so straightforward to obtain the sensitivity to get the derivatives of an explicit analytical function. Fortunately, we may solve the equation system for complex flows with the aid of numerical methods. However, we should emphasize the necessity to adopt a dimensionless mathematical model for numerical sensitivity analysis. Since there may be more than one dimensionless variable depending on the same dimensional parameter, the sensitivity factor obtained by slightly deviating the dimensional parameter in a dimensional numerical model seems to be nonsense. On the contrary, if we use a dimensionless numerical model, the slight deviation of a dimensionless variable will lead to a meaningful sensitivity factor, which exclusively reflects the effect of this dimensionless variable on the target function. In addition, when we calculate the derivative in Eq. (3) via difference scheme, $\Delta\pi_k = \pi_{\mathrm{m},k} - \pi_{\mathrm{p},k}$ should be small enough because the target function varies nonlinearly with π_k. Here, $\pi_{\mathrm{m},k}$ stands for the value of π_k of the model. In the present paper, we set $\Delta\pi_k/\pi_{\mathrm{p},k} = 1\%$.

2 Case studies: porous media flow in an oil reservoir

Figure 1 illustrates an ideal two-dimensional oil reservoir with two production wells in it. The reservoir is bounded by the impermeable walls BC and OA and the pressure-specified boundaries OB and AC. Hence, the pressure distribution $p(x, y, t)$ in the reservoir is governed by

$$\begin{cases} \dfrac{\partial^2 p}{\partial x^2} + \dfrac{\partial^2 p}{\partial y^2} + \sum_{i=1}^{2} \dfrac{\mu}{K} q_i \delta(x - x_i, y - y_i) = \dfrac{1}{\chi} \dfrac{\partial p}{\partial t}, \\ p\,|_{x=0} = p_0, \quad p\,|_{x=l_x} = p_l, \\ \left.\dfrac{\partial p}{\partial y}\right|_{y=0} = 0, \quad \left.\dfrac{\partial p}{\partial y}\right|_{y=l_y} = 0, \\ p(x, y, 0) = p_i. \end{cases} \tag{4}$$

Here, $\chi = \dfrac{K}{\phi \mu c_t}$, where K is the permeability, ϕ is the porosity of media, μ is the viscosity of oil, and c_t is the total compressibility; (x_i, y_i) is the coordinate of the wells; q_i is the discharge of the wells, which is positive for injection and negative for production; δ is the Dirac Delta function; p_0, p_l and p_i are constant. Apparently, if K, ϕ, μ and c_t are all constants, this problem is linear and can be solved analytically. Both the analytical solution and its apparent scaling law, as seen in the next subsection, may facilitate to validate the present mathematical model and to verify the following numerical results of similarity principle.

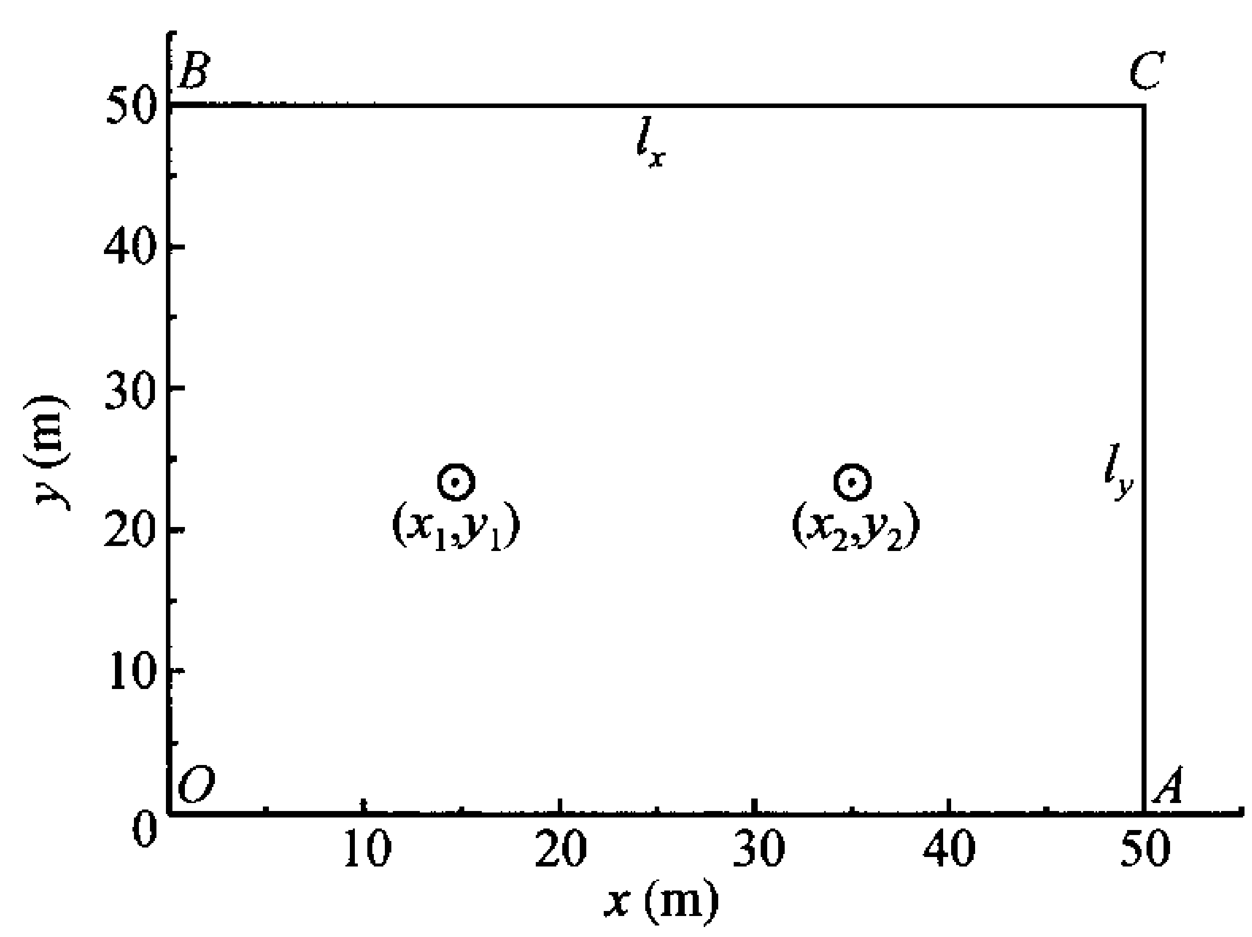

Fig. 1. Sketch of an ideal two-dimensional oil reservoir with two production wells at (x_1, y_1) and (x_2, y_2).

2.1 Similarity law

In order to get the similarity criteria for this problem, we normalize Eq. (4) using the following dimensionless parameters:

$$p_D = \frac{p - p_0}{p_l - p_0}, \quad x_D = \frac{x}{L}, \quad y_D = \frac{y}{L}, \quad t_D = \frac{t}{T} = \frac{\chi}{L^2} t, \tag{5}$$

where L and T are respectively the space and time scales. Then, the dimensionless pressure equation becomes

$$\frac{\partial^2 p_D}{\partial x_D^2} + \frac{\partial^2 p_D}{\partial y_D^2} + \frac{\mu}{K(p_l - p_0)} \sum_{i=1}^{2} q_i \cdot \delta(x_D - x_{iD}) \delta(y_D - y_{iD}) = \frac{\partial p_D}{\partial t_D}. \tag{6}$$

Now, we obtain two dimensionless variables, $\pi_1 = \dfrac{\mu q_1}{K(p_l - p_0)}$ and $\pi_2 = \dfrac{\mu q_2}{K(p_l - p_0)}$, which are the dimensionless oil production of the two wells, namely the ratio of the oil production of the two wells to the average permeating discharge.

To completely duplicate the pressure distribution over these two-dimensional oil reservoir indoors, we must design a similar model with the two dimensionless variables $\pi_{m,1}$ and $\pi_{m,2}$, which are strictly equal to the corresponding values of the prototype $\pi_{p,1}$ and $\pi_{p,2}$, where the subscripts m and p indicate variables for the model and the prototype, respectively. However, if we know $\pi_{p,1} \gg \pi_{p,2}$ in advance, according to the above analysis, π_2 obviously exerts much less influence on the similarity of pressure distribution. Then, we may relax or even disregard this less important similar restriction of $\pi_{m,2} = \pi_{p,2}$ and design an approximately similar model, which is only subjected to the major restriction of $\pi_{m,1} = \pi_{p,1}$. In this way, the pressure distribution will be approximately simulated in the laboratory with negligible deviation from the prototype. This is the very way to tackle the similarity issue of complex flows, because there are so many dimensionless variables for complex flows that some of them cannot be kept identical between the prototype and the model simultaneously. In this case, the restriction to the less important dimensionless variables should be relaxed or even neglected. Even if all of the dimensionless parameters could be matched simultaneously, there is still a problem of error control. The more sensitive the parameter is, the more accurate it should be made in experiments. Anyway, optimization for a selected target function is indispensable. Numerical sensitivity analysis may be considerably helpful to find out which criterion can be neglected or what degree of its difference between the prototype and the model is allowed to satisfy approximate similarity of the concerned target function. In the following section, we will conduct this analysis for the presumed two-dimensional reservoir by means of numerical simulation.

2.2 Validation of numerical model

We apply the finite difference method to solve Eq. (4) with forward-time central-space scheme, which is stable as long as inequality $\Delta t \leqslant \frac{(\min(\Delta x, \Delta y))^2}{2\chi}$ holds. A *non-uniformly structured mesh system is adopted so that the grids are more refined near the wells. In the area of* $|x - x_i|/l_x < 0.1$, $\Delta x/l_x = 0.02$, and if $|y - y_i|/l_y < 0.1$, $\Delta y/l_y = 0.02$. Otherwise, $\Delta x/l_x = 0.1$ and $\Delta y/l_y = 0.1$. The time step is so set that the above inequality is matched.

As already mentioned, Eq. (4) is linear if K, ϕ, μ and c_t are all constants. Then, by using the variable-separation method and Duhamel principle, we can easily access to its analytical solution, which reads

$$p = p_0 + \frac{x}{l_x}(p_l - p_0) + \sum_{i=1}^{2} \frac{q_i \mu}{K} \frac{2 l_x}{l_y \pi^2} \sum_{m=1}^{\infty} \frac{1 - \exp\left[-\chi \frac{m^2\pi^2}{l_x^2} t\right]}{m^2} \sin \frac{m\pi}{l_x} x \sin \frac{m\pi}{l_x} x_i$$
$$+ \sum_{i=1}^{2} \frac{q_i \mu}{K} \frac{4 l_x l_y}{\pi^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1 - \exp\left[-\chi \left(\frac{m^2\pi^2}{l_x^2} + \frac{n^2\pi^2}{l_y^2}\right) t\right]}{m^2 l_y^2 + n^2 l_x^2} \sin \frac{m\pi}{l_x} x \cos \frac{n\pi}{l_y} y \sin \frac{m\pi}{l_x} x_i \cos \frac{n\pi}{l_y} y_i$$
$$+ 2 \sum_{m=1}^{\infty} \exp\left[-\chi \frac{m^2\pi^2}{l_x^2} t\right] \sin \frac{m\pi}{l_x} x \left[\frac{p_i - p_0}{m}(1 - \cos m\pi) + \frac{p_l - p_0}{m} \cos m\pi\right].$$

We have assumed a case (Case a in Table 1) for code validation. Fig. 2 shows the pressure evolution of both numerical and analytical results at different points in the oil reservoir. It can be seen that the agreement is very good except for those exactly at the wells, which is attributed to the refinement of grids. It can be anticipated that more refined grids will lead to better agreements. Of course, there are some discrepancies at the beginning stage due to the arbitrary initial pressure field given. As the time elapses, the effect of the arbitrary initial pressure gradually vanishes and the pressure field becomes steady. We have compared the numerical steady pressure profiles along different lines that are parallel to the axes with those of analytical results as in Fig. 3. Again, the agreements are satisfactory.

Table 1. Parameters for case studies

Case label	K (D)	μ (mPa·s)	c_t (Pa^{-1})	ϕ	p_0 (MPa)	p_l (MPa)	p_i (MPa)	L (m)
a	1.0	2.0	1.0E-9	0.2	10	30	10	50
b	1.0	2.0	1.0E-9	0.2	1	3	1	5
Case label	q_1 (m^3·s^{-1}·m^{-1})	q_2 (m^3·s^{-1}·m^{-1})	$l_x = l_y$ (m)	x_1 (m)	y_1 (m)	x_2 (m)	y_2 (m)	T (s)
a	−0.010	−0.0010	50	15.0	25.0	35.0	25.0	1000
b	−0.001	−0.0001	5	1.5	2.5	3.5	2.5	10

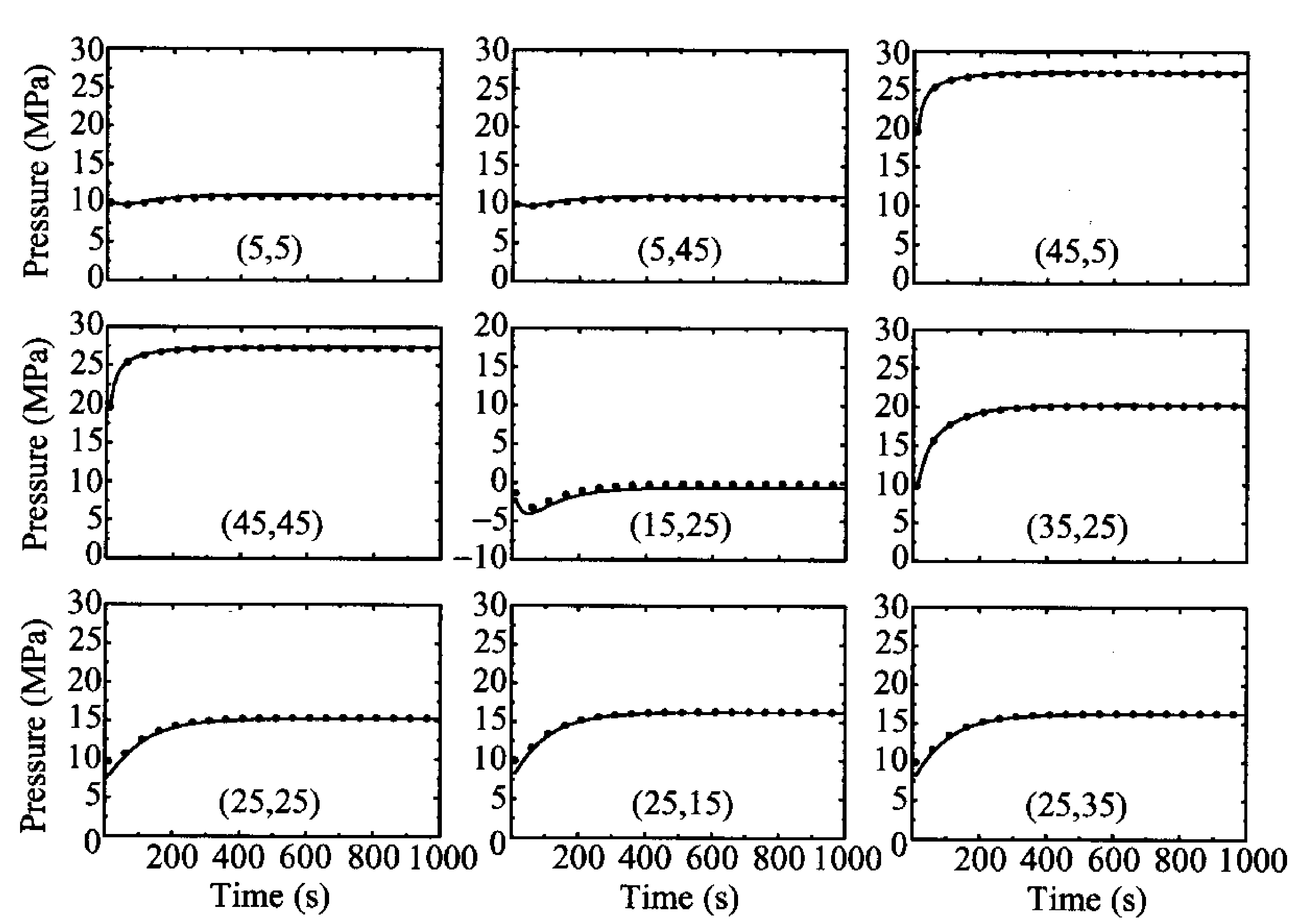

Fig. 2. Comparison of the numerical pressure history (dots) with the corresponding analytical results (solid lines) at different points in the reservoir, which are identified by the coordinates in the parentheses. The two wells are at (15,25) and (35,25), respectively.

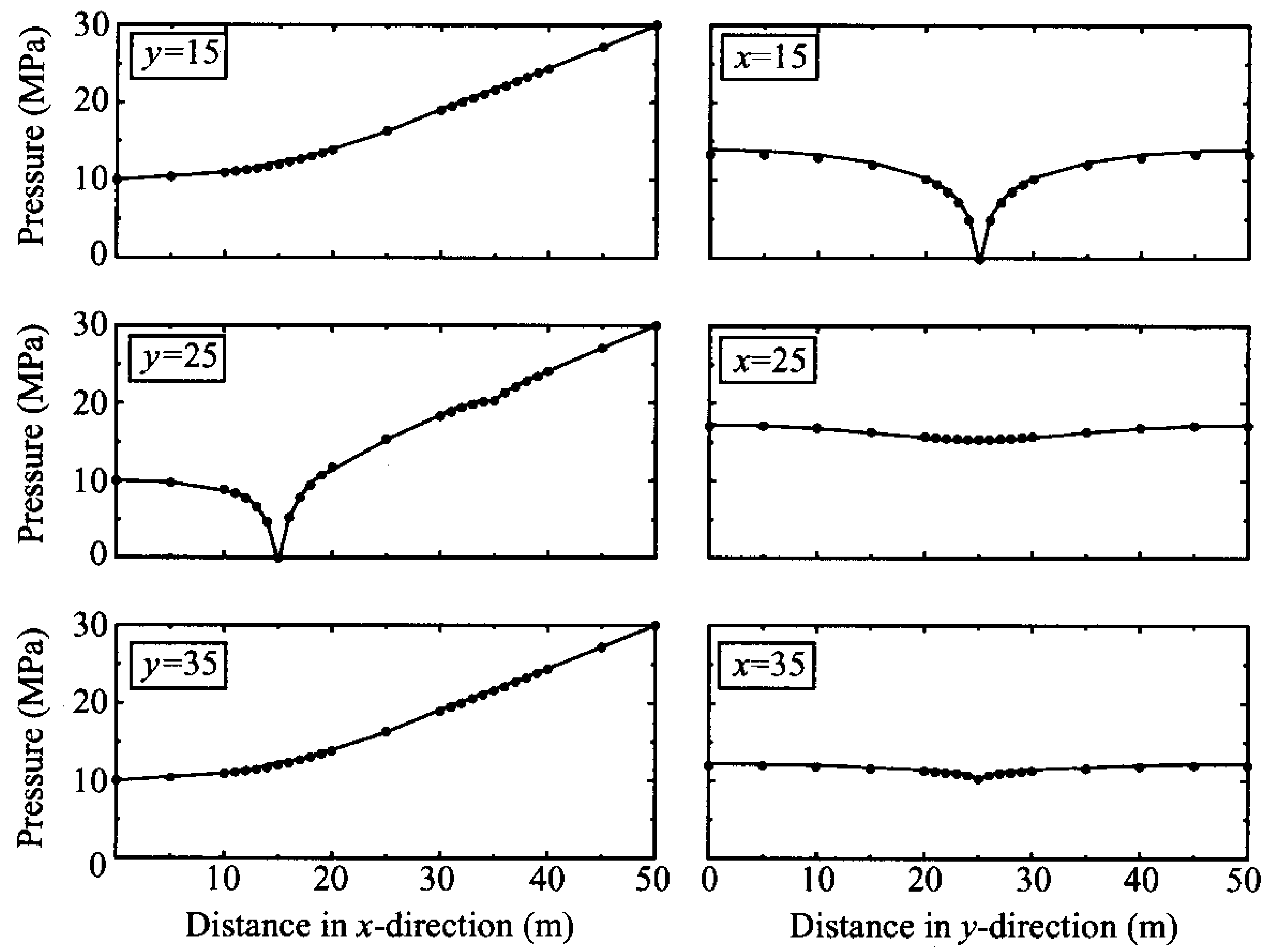

Fig. 3. Comparison between the numerical (dots) and analytical (solid lines) pressure profiling along different lines in x- and y-direction. The two wells are on the profile $y = 25$.

2.3 Sensitivity of pressure to the dimensionless variables

Now, design a small model reservoir (Case b in Table 1) strictly similar to the prototype (Case a in Table 1), in which the production of well 1 is one order higher than well 2, implying $\pi_{p,1} \gg \pi_{p,2}$. Let us investigate the sensitivity of the pressure distribution to the two dimensionless variables. Hence, the target function is the pressure. According to Eq. (3), the sensitivity factors of the pressure at point (x_i, y_j) are written as

$$s_{ijk} = \left| \frac{(p_{m,ij} - p_{p,ij})/p_{p,ij}}{(\pi_{m,k} - \pi_{p,k})/\pi_{p,k}} \right|,$$

$$i = 1,2,\cdots,n_x;\ j = 1,2,\cdots,n_y;\ k = 1,2. \quad (7)$$

Here, n_x, n_y are grid numbers in the x- and y-direction. In order to examine the pressure distribution over the whole domain, we calculate the spatial average of s_{ijk},

$$s_k = \frac{\frac{1}{n_x n_y}\sum_{i=1}^{n_x}\sum_{j=1}^{n_y} |(p_{m,ij} - p_{p,ij})/p_{p,ij}|}{|(\pi_{m,k} - \pi_{p,k})/\pi_{p,k}|},$$

$$k = 1,2. \quad (8)$$

The numerator is simply the mean relative error of pressure over the whole space between the model and the prototype, and the denominator the relative deviation of the dimensionless variables of the model from the prototype. In the present calculation, we let this relative deviation be 1%. From the sensitivity calculation for case b, we obtain $s_1 = 0.69$, $s_2 = 0.024$. Obviously, s_1 is one order higher than s_2, implying that the pressure distribution is more sensitive to the production of well 1. Thus, by means of numerical analysis, we can prove the conclusion of the similarity theory in Section 2.1.

2.4 Error analysis of the partially similar model

We may further explain the physical mechanism in another way of sensitivity analysis. As already mentioned, the numerator of Eq. (8),

$$E = \frac{1}{n_x n_y}\sum_{i=1}^{n_x}\sum_{j=1}^{n_y} |(p_{m,ij} - p_{p,ij})/p_{p,ij}|, \quad (9)$$

represents the mean relative error of pressure between the model and the prototype over the whole reservoir. Apparently, if both $\pi_{m,1} = \pi_{p,1}$ and $\pi_{m,2} = \pi_{p,2}$ hold, we have $E = 0$. Now, keep one of these two dimensionless variables unchanged between the model and the prototype, but the other of the model is allowed some deviation from the prototype. In this case, the pressure field of the model will be different from the prototype and the mean error E will be greater than zero. Fig.4 shows the variation of E with a sensitivity coefficient C as defined in the caption below the figure. Clearly, the mean error is far more sensitive to the first parameter π_1 than to the second one π_2, implying the dominant effect of the well with much larger oil production on the pressure distribution, which demonstrates the theoretical conclusion in Section 2.1 once again. In addition, it can be seen from Fig.4 (a) that π_2 can be as large as 2.8 times of its prototype's value if 5% error of the target function is acceptable.

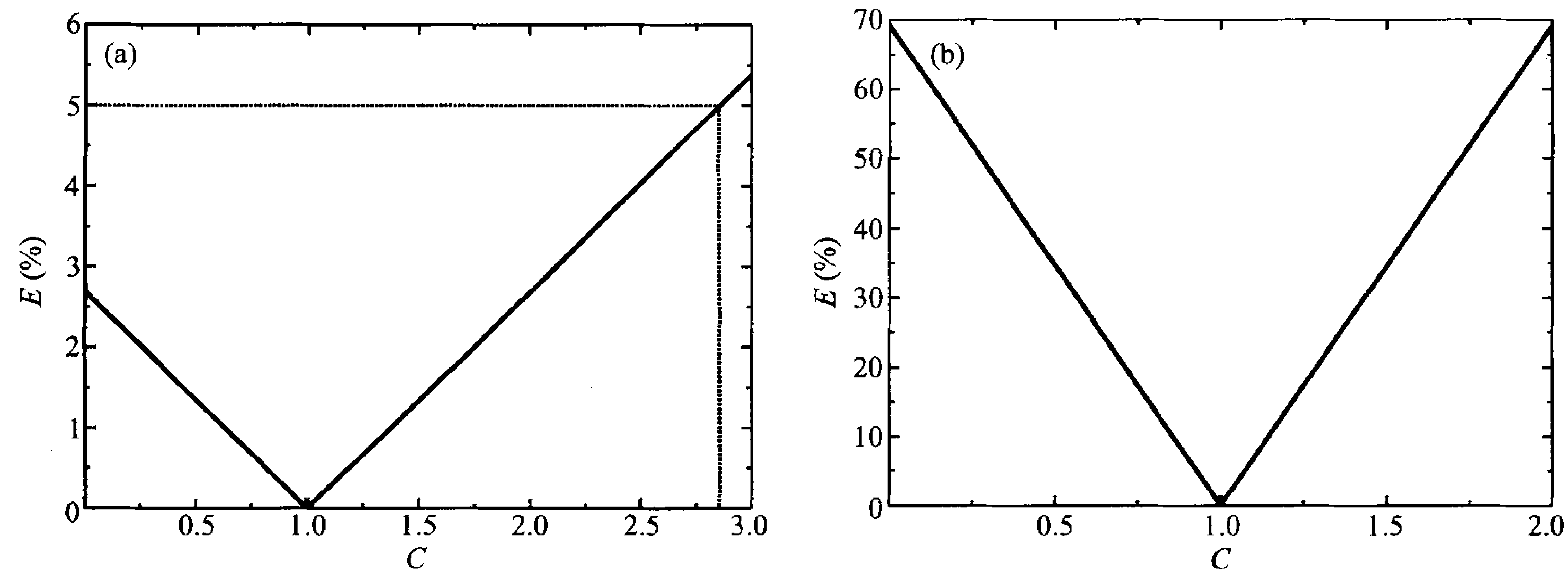

Fig. 4. Variation of E with the sensitivity coefficient C for the cases of (a) $\pi_{m,1}=\pi_{p,1}$ and $\pi_{m,2}=C\pi_{p,2}$; (b) $\pi_{m,1}=C\pi_{p,1}$ and $\pi_{m,2}=\pi_{p,2}$.

In this way of numerical optimization, we can not only single out the more important similarity criteria, which should be totally identical between the model and the prototype, but also display the acceptable degree of the less influential parameters' deviation.

2.5 Influence of flow parameters on the sensitivities

As we know, complex flows in practice involve many parameters, which may cover a large range even for the same kind of flow system. Take again the porous media oil flow as an example, the viscosity of oil ranges from 1 to 10000 mPa·s and the permeability may be as high as 1 D and as low as 1 mD. According to the previous analysis, the dominant parameters may be different from one case to another.

To demonstrate the effects of flow parameters on the sensitivity factor, we have assumed eleven prototypes of porous media flow with different parameters. The dimensionless variables of the eleven cases are listed in Table 2. All the cases are of the same total compressibility, porosity, geometry, boundary conditions, oil production, and dimensionless flow time, and the ratio between the two dimensionless variables is fixed ($\pi_1/\pi_2=10$) as well. However, they are of different permeability or oil viscosity and flow time scale T ($T=L^2/\chi$). Case 1 in Table 2 is exactly case a in Table 1. The permeability and oil viscosity of it are 1 D and 2 mPa·s, respectively. From case 1 to case 11, the permeability decreases or equivalently the oil viscosity increases. For instance, case 6 represents the permeation flow with permeability of 0.1 D and oil viscosity of 2 mPa·s (or equivalently with permeability of 1 D and oil viscosity of 20 mPa·s), and case 11 with permeability of 0.0001 D and oil viscosity of 2 mPa·s (or equivalently with permeability of 1 D and oil viscosity of 20000 mPa·s). Case 11 can be considered to be a reservoir of low permeability or heavy oil.

Table 2. Sensitivity factors of cases with different parameters

case	π_1	π_2	T(s)	s_1	s_2	s_1/s_2
1	1	0.1	1000	0.69	0.024	28.70
2	2	0.2	2000	2.11	0.089	23.70
3	4	0.4	4000	5.39	0.350	15.40
4	6	0.6	6000	8.09	1.070	7.56
5	8	0.8	8000	5.66	1.130	5.01
6	10	1.0	10000	5.27	1.050	5.02
7	15	1.5	15000	2.55	0.460	5.54
8	20	2.0	20000	1.47	0.240	6.12
9	100	10.0	10^5	0.96	0.140	6.86
10	1000	100.0	10^6	0.89	0.120	7.42
11	10000	1000.0	10^7	0.88	0.120	7.33

Performing analogous numerical analysis to case a, we can obtain sensitivity factors of the pressure to the two dimensionless variables for the eleven cases, which are listed in Table 2 and plotted in Fig. 5. It is shown that the sensitivity of the pressure distribution is obviously influenced by the reservoir properties or flow parameters (here referring to the permeability and oil viscosity). Owing to the identical ratio of π_1 and π_2 between all cases, the dominance relationship between the two dimensionless parameters is not changed. However, the dominance degree of π_1 over π_2 varies with their values. This can be apparently seen in Fig. 6, which shows the dependence of the ratio between the two sensitivity factors on the values of the dimensionless variables. From the physical point of view, the degrees of relaxation of the insignificant similarity criterion may differ from models with different flow parameters. For the low permeability and heavy oil reservoirs, s_1/s_2 is smaller. The models are more subjected to the unimportant dimensionless variable π_2. In this case, the permeating discharge is smaller, and the same deviation of oil production will result in a larger pressure error. Or equivalently, the modeled oil production will deviate more from the field under the same laboratory conditions. In contrast, for the high permeability and light oil reservoir, s_1/s_2 is greater. The model error is less sensitive to the unimportant dimensionless variable π_2. In this case, the permeating discharge is larger, and the simulated oil production is more accurate.

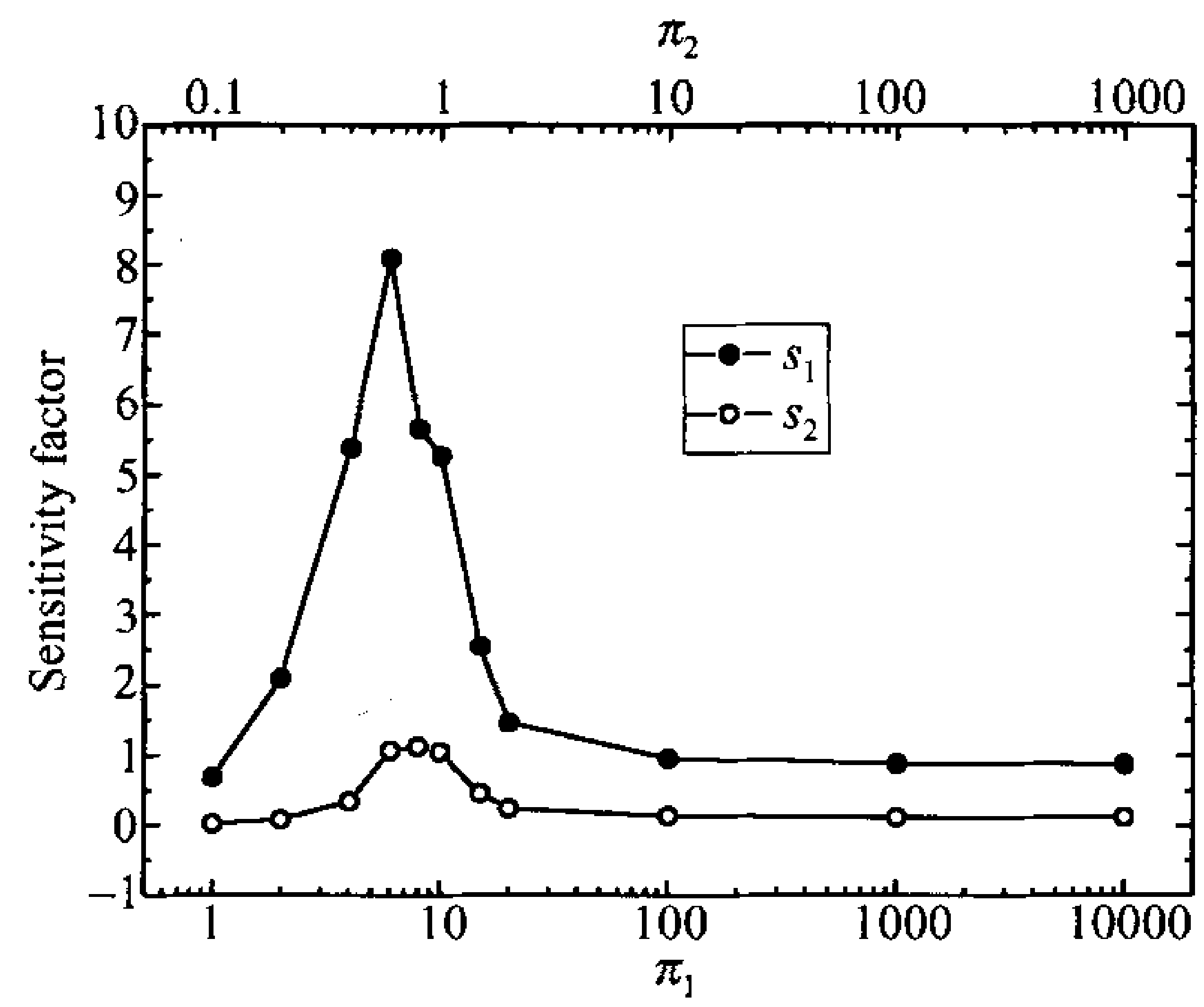

Fig. 5. Sensitivity factors of the pressure to the two dimensionless variables for different values of themselves.

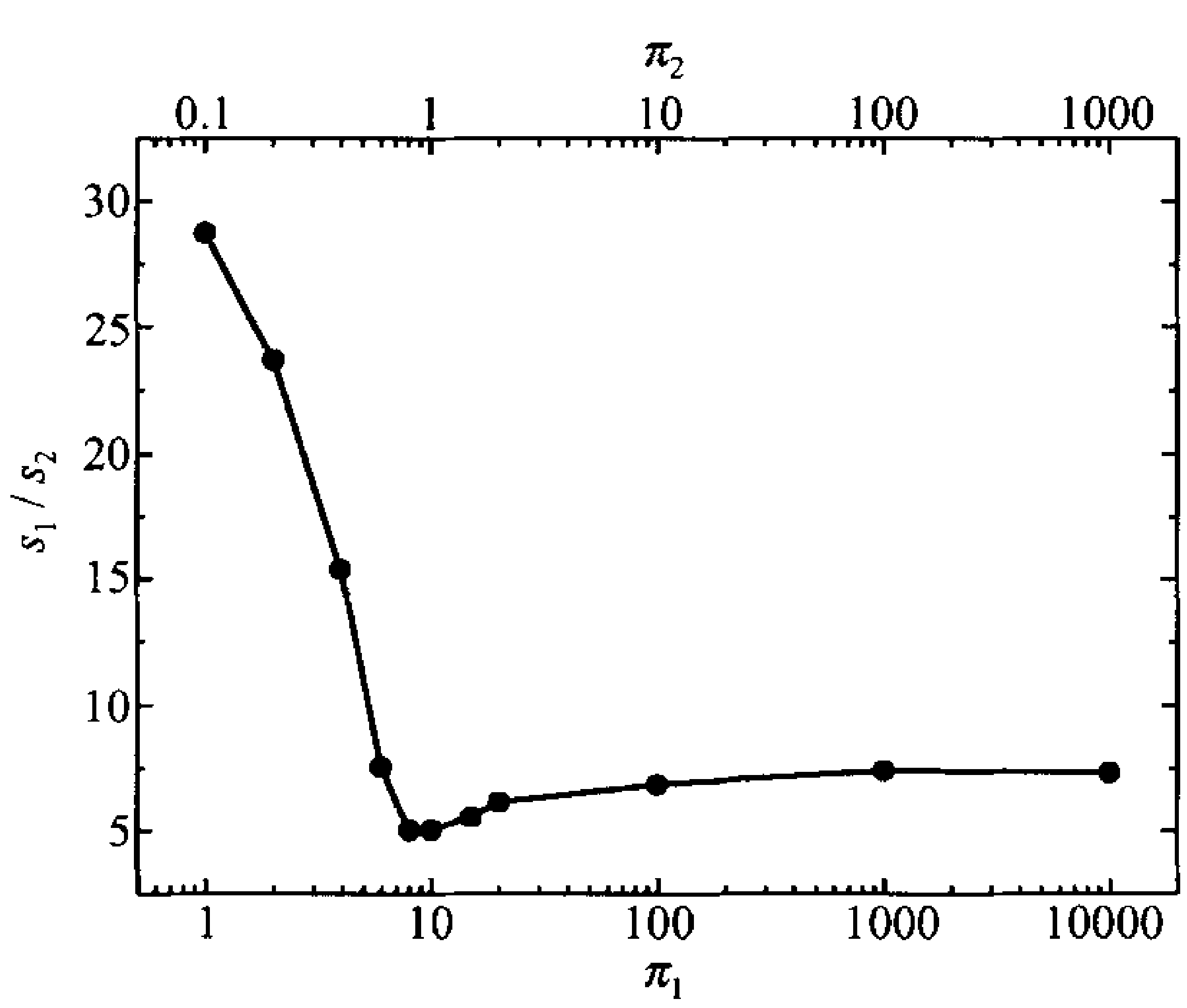

Fig. 6. Dependence of the ratio between the two sensitivity factors on the dimensionless variables themselves.

For more complex flows involving tens of flow parameters, one parameter may be in the expressions of more than one dimensionless variable. Hence, the variation of this parameter will lead not only to the change of the dominance degree of the dimensionless variables, but also to the difference of the similarity laws, which is more essential to physical modeling. In other words, it is very possible that a group of dominant dimensionless parameters turns to be another group when some of the flow parameters change.

3 Conclusion

In reality, completely modeling complex flows involving a number of dimensionless variables are almost impossible. The way out is to identify the dominant similarity criteria and design a partial similar model. In the present paper, we have generalized an optimization approach to finding the dominant dimensionless variables of complex flows via numerical sensitivity analysis. We have particularly given the precise definition of sensitivity factor with physical implication, and discussed the dependence of the sensitivity factor on the dimensionless variables themselves.

By applying the proposed optimization approach to a presumed porous media flow, we have found that the similarity principle obtained from the numerical sensitivity analysis conforms to the theoretical law. In addition, the foregoing analyses show that the proposed approach can not only single out the dominant similarity criteria but also manifest the extent of relaxing some trivial dimensionless variables so long as the approximate similarity of the target function is concerned. The effect of the parameters on the sensitivity factors demonstrates that the dominant dimensionless variables may vary with the flow parameters. Therefore, some of the dimensionless flow variables dominant in one case may become negligible in another case.

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