RESEARCH PAPERS

Heat transfer in supersonic dusty-gas flow past a blunt body with inertial particle deposition effect*

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Abstract Heat transfer in a supersonic steady flow of a dilute dusty-gas past a sphere is considered at large and moderate Reynolds numbers. For the regime of inertial particle deposition on the frontal surface of the body, a parametric study of maximum increase in the particle-induced heat flux at the stagnation point is performed over a wide range of the Reynolds number, the particle inertia parameter, the ratio of the phase specific heats, and the body surface temperature.

Keywords: heat transfer, intensification mechanism, supersonic flow, dusty gas, blunt body, inertial particle deposition.

The heat transfer of a body immersed in a high-speed dusty-gas flow is a complex and multi-parameter problem, since in the near-wall region different flow patterns may take place. The first experimental studies of high-speed two-phase flows past bodies were performed using fairly coarse particles, which were deposited on the frontal surface of the body[1,2]. This dusty-gas flow is called the inertial particle deposition regime. Numerical simulation of the flow regimes with and without inertial particle deposition requires essentially different mathematical models. This paper continues the previous study[3] and devotes to a detailed parametric investigation of the relative stagnation heat flux caused by the particles in the free stream. In applications (for example, a motion of a high-speed vehicle in a dusty atmosphere of the Earth or Mars), the Mach and Reynolds numbers of the flow past the vehicle vary on a fairly wide range. In our study, of practical interest is the range of variation of the Mach and Reynolds numbers: $M=2\sim10$ and $Re=10^2\sim10^8$.

1 General formulation of the problem

Consider a steady supersonic flow of a uniform dusty gas past an axisymmetric blunt body. For dilute dusty gases, we use the two-fluid model[4], in which the particle volume fraction is neglected. For this two-phase medium, the basic assumptions are as follows: the carrier phase is a viscous perfect gas with constant specific heats c_p, c_v and the dispersed phase consists of solid spherical particles of identical radius σ and mass m. Below, the subscripts s, ∞ and c refer to the dispersed-phase parameters, the free-stream parameters and the adiabatic stagnation parameters corresponding to the limiting hypersonic flow velocity. The asterisk denotes a dimensional variable to distinguish it from the corresponding nondimensional variable, when necessary. For the continuous flow regime around the particles, the expressions for the interphase momentum and energy exchange (per particle) are written in the form[5]:

$$f_s = 6\pi\sigma\mu^*(V^* - V_s^*)G,$$

$$q_s = 4\pi\sigma\lambda^*(T^* - T_s^*)D, \qquad (1)$$

$$G = \left(1 + \frac{1}{6}Re_s^{2/3}\right)\Phi_1(M_s, Re_s),$$

$$D = (1 + 0.3Pr^{1/3}Re_s^{1/2})\Phi_2(M_s, Re_s),$$

$$M_s = \frac{|V^* - V_s^*|}{a^*},$$

$$Re_s = \frac{2\sigma|V^* - V_s^*|\rho^*}{\mu^*},$$

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$$Pr = \frac{c_p \mu^*}{\lambda^*}.$$

Here, V^* and T^* are the velocity and temperature, μ^* and λ^* the gas viscosity and thermal conductivity in the power-law temperature dependence, a^* is the gas speed of sound while Φ_1 and Φ_2 are the following correction function[6]:

$$\Phi_1 = (1 + \exp(-0.427 M_s^{-4.63} - 3 Re_s^{-0.88}))/\varphi, \tag{2}$$

$$\Phi_2 = \left(1 + 3.42 \frac{M_s}{Re_s} \frac{1 + 0.3 Re_s^{1/2} Pr^{1/3}}{Pr}\right)^{-1},$$

$$\varphi = 1 + (M_s / Re_s) \cdot [3.82 + 1.28 \exp(-1.25 (Re_s / M_s))].$$

We introduce a curvilinear coordinate system (x, y) with the origin at the stagnation point on the body and the x- and y- axes directed respectively along the generator and the normal to the body surface. For the dusty-gas flow past the blunt body, the following nondimensional parameters are introduced:

$$x = \frac{x^*}{L}, \quad y = \frac{y^*}{L}, \quad u = \frac{u^*}{U_\infty}, \quad v = \frac{v^*}{U_\infty},$$

$$\rho = \frac{\rho^*}{\rho^*_\infty}, \quad n_s = \frac{n_s^*}{n^*_{s\infty}}, \quad p = \frac{p^*}{\rho^*_\infty U^2_\infty},$$

$$T = \frac{2T^* c_p}{U^2_\infty}, \quad \mu = \frac{\mu^*}{\mu^*_c}.$$

Here, u and v are the velocity components respectively in x- and y- direction, p and ρ the gas pressure and density, n_s is the particle number concentration, L the radius of curvature of the body at the stagnation point and U_∞ the free-stream velocity. Then the model equations of dusty-gas flows can be given in the nondimensional form[4]:

$$\mathrm{div}(\rho V) = 0, \quad \mathrm{div}(n_s V_s) = 0, \tag{3}$$

$$\rho (V \nabla) V + \nabla p + \alpha\beta\mu n_s (V - V_s) G = \frac{2\varepsilon}{3\kappa} [-\nabla(\mu \mathrm{div} V) + 3 \mathrm{div}(\mu \Lambda)],$$

$$(V_s \nabla) V_s = \beta\mu G (V - V_s),$$

$$(V_s \nabla) T_s = \frac{2 c_p}{3 c_s Pr} \beta\mu D (T - T_s),$$

$$\rho (V \nabla) T = 2 (V \nabla) p + \frac{2\varepsilon}{\kappa} \left[2\mu S^2 - \frac{2}{3} \mu (\mathrm{div} V)^2\right] + \frac{\varepsilon}{\kappa Pr} \mathrm{div}(\mu \nabla T) + 2\alpha\beta\mu n_s G \mid V - V_s \mid^2 + \frac{2}{3} \frac{\alpha\beta\mu}{Pr} n_s D (T_s - T),$$

$$p = \frac{\gamma - 1}{2\gamma} \rho T, \quad \mu = T^\omega, \quad \kappa = \frac{\gamma - 1}{\gamma + 1},$$

$$Re = \frac{U_\infty \rho^*_\infty L}{\mu^*_\infty}, \quad \alpha = \frac{m n^*_{s\infty}}{\rho^*_\infty}, \quad \varepsilon = \frac{1}{Re} \frac{\mu^*_c}{\mu^*_\infty},$$

$$\beta = \frac{6\pi\sigma\mu^*_c L}{m U_\infty}.$$

Here, Λ is the carrier-phase strain rate tensor, γ the gas specific heat ratio, c_s the material specific heat of the particles, α the particle mass loading rate and β the particle inertia parameter.

On the body surface, a constant temperature T_w and the no-slip condition are specified for the gas and no-rebound condition is assumed for the particles. Ahead of the bow shock, the gas and particles are in equilibrium. Taking into account the fact that in the known applications the free-stream particle mass loading is generally below several percent ($\alpha \ll 1$), in next sections we will neglect the effect of particles on the carrier-phase parameters and give a simplified formulation of the problem.

2 Dusty-gas flows past a sphere at $\alpha \ll 1$

As a typical example of a blunt body, we will consider a sphere of radius L. The carrier phase is a gas with constant thermodynamic parameters: $\gamma = 1.4$, $Pr = 0.7$, and $\omega = 0.5$. In the $\alpha \ll 1$ case, the problems of finding the parameters of the gas and the particles are separated: we can calculate first the carrier-phase parameters and then the dispersed-phase motion in the given gas velocity and temperature field.

For $Re = 10^2 \sim 10^5$, the gas parameters near the frontal surface of the sphere were found numerically by solving the complete Navier-Stokes equations (with shock capturing) on a non-uniform grid with grid point clustering toward the body surface. An implicit finite-difference scheme based on a finite-volume method was used in computations. However, in the limiting case of high Reynolds numbers ($Re \approx 10^8$), the boundary layer becomes very thin and the use of the complete Navier-Stokes equations meets serious numerical difficulties. For $Re > 10^5$ and high (but finite) Mach numbers, we use an asymptotic approach (or so-called "inviscid shock layer plus boundary layer" scheme) for calculating the gas flow field. Here, the gas parameters in the inviscid shock layer are specified on the basis of the Hayes approximate analytical solution[7]. For a given carrier phase, the particle velocity and concentration fields in the inviscid shock layer depend only on the Mach number and two nondimensional parameters, which could be the scaled particle inertia parameter $\beta_0 = \beta\mu_1 / u_1 \varphi_1$ (u_1 is the nondimensional modulus of the gas velocity gradient) and the particle drag deviation parameter $Rb = Re_{s0}^{2/3}/6$. Here the subscript 1 refers to the nondimensional parameter at the inviscid-flow stagnation point and $Re_{s0} = 2\sigma\rho_c^* U_\infty / \mu_c^*$. Besides, the particle temperature distribution in the shock layer depends also on the ratio of phase specific heats c_s/c_p.

For determining the dispersed-phase parameters we use the complete Lagrangian method[8]. Introduce the Lagrangian variables x_0 and τ, where x_0 is the coordinate of the trajectory origin at the outer edge of the calculation domain $y = y_{sho}$ and $\tau = t^* U_\infty / L$ is the nondimensional time of particle motion from $y = y_{sho}$ along the chosen trajectory. Here y_{sho} is a constant so chosen that the domain boundary lies completely in the undisturbed flow and it was taken equal to the maximum distance to the bow shock in the calculation domain $0 \leqslant x \leqslant 1$. In the Lagrangian variables, the dispersed-phase momentum and energy equations take the form:

$$\frac{dx_s}{d\tau} = \frac{u_s}{1 + y_s}, \quad \frac{dy_s}{d\tau} = v_s,$$
$$\frac{du_s}{d\tau} = \beta\mu G(u - u_s) - \frac{u_s v_s}{1 + y_s}, \tag{4}$$
$$\frac{dv_s}{d\tau} = \beta\mu G(v - v_s) + \frac{u_s^2}{1 + y_s},$$
$$\frac{dT_s}{d\tau} = \frac{2\beta c_p}{3 c_s Pr}\mu D(T - T_s).$$

Based on the boundary conditions for the particles on the surface $y = y_{sho}$, we obtain the continuity equation for the dispersed phase as

$$\frac{1}{n_s(\tau, x_0)} = \frac{(1 + y_s)\sin(x_s)[u_s(\partial y_s/\partial x_0) - v_s(1 + y_s)(\partial x_s/\partial x_0)]}{(1 + y_{sho})^2 \sin(x_0)\cos(x_0)}. \tag{5}$$

In order to use this equation on a given particle trajectory, the following additional unknown variables should be introduced:

$$w_1 = \frac{\partial x_s(\tau, x_0)}{\partial x_0}, \quad w_2 = \frac{\partial u_s(\tau, x_0)}{\partial x_0},$$
$$w_3 = \frac{\partial y_s(\tau, x_0)}{\partial x_0}, \quad w_4 = \frac{\partial v_s(\tau, x_0)}{\partial x_0}.$$

The required equations for the above variables can be derived through differentiating the particle motion equations with respect to the Lagrangian coordinate x_0

$$\frac{dw_1}{d\tau} = \frac{w_2}{1 + y_s} - \frac{u_s w_3}{(1 + y_s)^2}, \tag{6}$$
$$\frac{dw_2}{d\tau} = \beta\mu G\left(w_1 \frac{\partial u}{\partial x} + w_3 \frac{\partial u}{\partial y} - w_2\right) + \beta(u - u_s)\frac{\partial}{\partial x_0}(G\mu) - \frac{u_s w_4 + v_s w_2}{1 + y_s} + \frac{u_s v_s w_3}{(1 + y_s)^2},$$
$$\frac{dw_3}{d\tau} = w_4,$$
$$\frac{dw_4}{d\tau} = \beta\mu G\left(w_1 \frac{\partial v}{\partial x} + w_3 \frac{\partial v}{\partial y} - w_4\right) + \beta(v - v_s)\frac{\partial}{\partial x_0}(G\mu) + \frac{2u_s w_2}{1 + y_s} - \frac{u_s^2 w_3}{(1 + y_s)^2}.$$

Eqs. (5), (6) together with (4) constitute a closed system of ordinary differential equations and the corresponding initial conditions at $\tau = 0$ are specified as

$$x_s = x_0, \quad y_s = y_{sho}, \quad u_s = \sin(x_0),$$
$$v_s = -\cos(x_0), \quad T_s = T_\infty,$$
$$w_1 = 1, \quad w_2 = \cos(x_0), \quad w_3 = 0, \quad w_4 = \sin(x_0).$$

3 Maximum particle-induced heat flux at the stagnation point

We assume that, in the inertial particle deposition regime, the total energy flux of the deposited particles is converted into thermal energy of the body and the effect of rebounding particles is not considered. Accordingly, the dispersed-phase contribution to the heat flux at the stagnation point is

$$Q_s = -m n_{sw}^* v_{sw}^* [c_s(T_{sw}^* - T_w^*) + v_{sw}^{*2}/2]. \tag{7}$$

Here the particle parameters at the wall are denoted by the subscript sw. Introducing nondimensional variables into (7), we obtain the ratio of the heat fluxes in the dusty and pure gases in the following form (Q_0 is the heat flux from the carrier phase):

$$\frac{Q_0 + Q_s}{Q_0} = 1 + \alpha\sqrt{Re} J(M, Re, \beta_0, Rb, c_s/c_p, T_w). \tag{8}$$

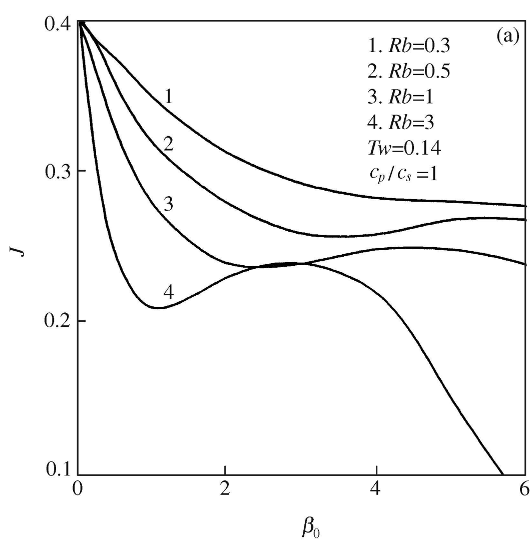
On the right side of (8), the second term represents the heat transfer intensification due to the presence of particles and the function J takes the form:

$$J = \frac{Pr\,|n_{sw} v_{sw}|\,[v_{sw}^2 + (c_s/c_p)(T_{sw} - T_w)]\,T_\infty^{1/2}}{T_w^{1/2}(\partial T/\partial y_1)_w},$$
$$y_1 = y\sqrt{Re}. \tag{9}$$

The above fact that the particle-induced increase in the stagnation-point heat flux has the order of the product $\alpha\sqrt{Re}$, which can be of order unity even for very small free-stream particle loading ratio α, seems to be very important for predicting the maximum thermal loads on high-speed vehicles moving through dusty clouds.

4 Calculation results and discussion

Our main aim is to study the dependence of the intensification function J on the governing parameters in supersonic flow (in the present work, the Mach number M was equal to 6). The calculation results obtained for large Re are presented in Figs. 1 and 2. In accordance with the two-phase boundary-layer theory[8], it was assumed that when the inertial particles move through the boundary layer their parameters do not change. Fig. 1 shows that, the heat flux at the stagnation point, scaled to its value for pure gas flow, may increase by up to $0.4\alpha\sqrt{Re}$ due to the particles. As is clear from the figures, the function J changes significantly with β_0 when the other similarity criteria (Rb, c_p/c_s, T_w and Re) keep constant. In the parameter range under consideration, the dependence of J on β_0 is nonmonotonous. With increasing β_0, J first decreases for small β_0 ($\beta_0 \leqslant 1$), reaches a minimum, and then increases again. For the same T_w but increased Rb, this nonmonotonicity becomes even more pronounced and another turning point at a local maximum appears (see curve 4). For the same Rb but increased T_w, the nonmonotonicity of J also becomes more pronounced. Besides, for small Rb (say, $Rb \leqslant 1$), as T_w increases J decreases except for a small range near $\beta_0 = 0$. These results demonstrate the multi-parameter nature of the heat transfer intensification effect. The effect of the ratio of the specific heats of the phases c_p/c_s on the intensification function J is shown in Fig. 2. In general, for the same β_0, J increases substantially with decrease in c_p/c_s. It is interesting that, for the same values of T_w and Rb, there are different trends in the variations of J as c_p/c_s decreases from 2 to 0.25. When c_p is less than c_s (for example, $c_p/c_s = 0.25$), the function J at large β_0 increases monotonously with β_0 and may attain a value higher than that at $\beta_0 = 0$ (see curve 1). On the contrary, when c_p is greater than c_s (for example, $c_p/c_s = 2.0$), J monotonously decreases with β_0 and does not reach a minimum (see curve 4). The results obtained for moderate Reynolds numbers are given respectively in Fig. 3 (for $Re = 10^5$) and Fig. 4 (for $Re = 10^3$). In these calculations, the drag parameter Rb keeps unchanged ($Rb = 0.5$) and the wall temperature takes two specified values (here the lower temperature $T_w = 0.14$ corresponds to the cold wall case) while the case of variable ratio c_p/c_s corresponds to SiO_2 particles. The obtained results show that, for $c_p/c_s \leqslant 1$, J becomes lower at a higher value of T_w except for the proximity of $\beta_0 = 0$. This phenomenon in the moderate Re case is similar to that in the large Re case as mentioned before. However, when $c_p/c_s > 1$ the value of J may be higher for increased T_w over a quite wide range of β_0 (see curve 3). The comparison of Figs. 3(a) and 4(a) with Figs. 1(a) and 2 shows that, with decrease in Re,



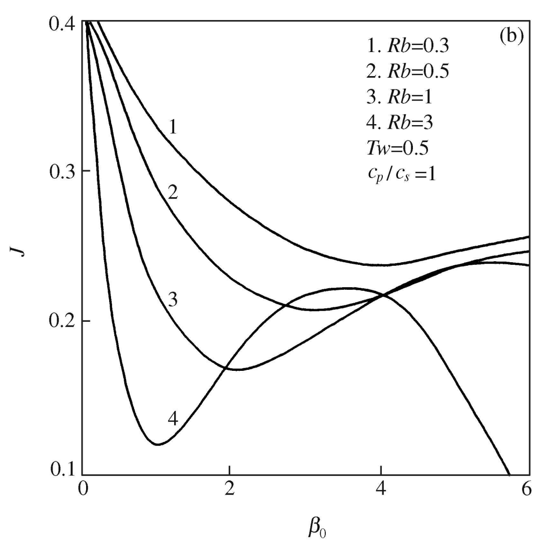


Fig. 1. J vs. β_0 for $c_p/c_v = 1$ at $Re = 10^8$. (a) $T_w = 0.14$; (b) $T_w = 0.5$.

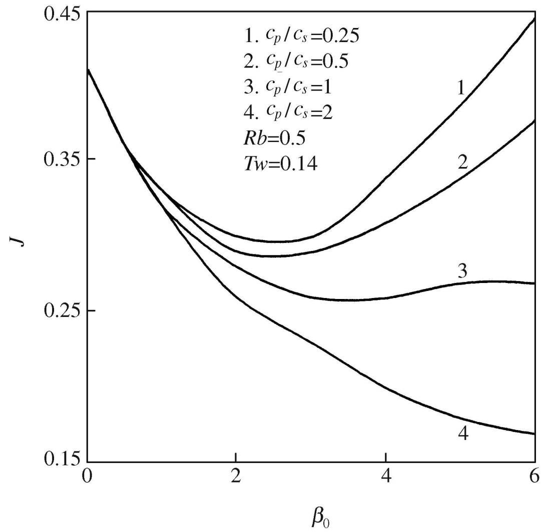


Fig. 2. J vs. β_0 for $T_w=0.14$, $Rb=0.5$ and different c_p/c_v at $Re=10^8$.

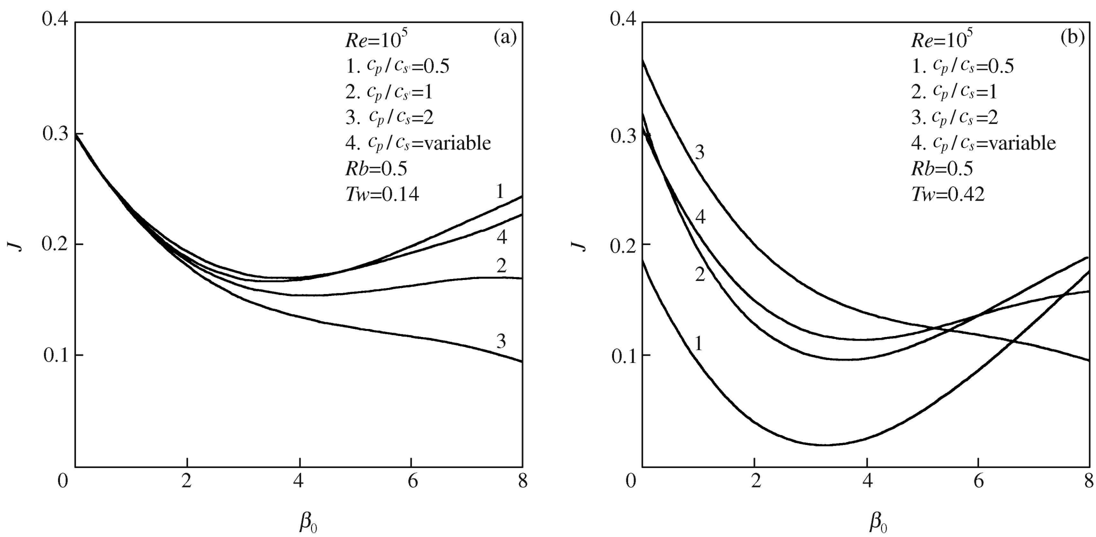


Fig. 3. J vs. β_0 for $Rb=0.5$ at $Re=10^5$. (a) $T_w=0.14$; (b) $T_w=0.42$.

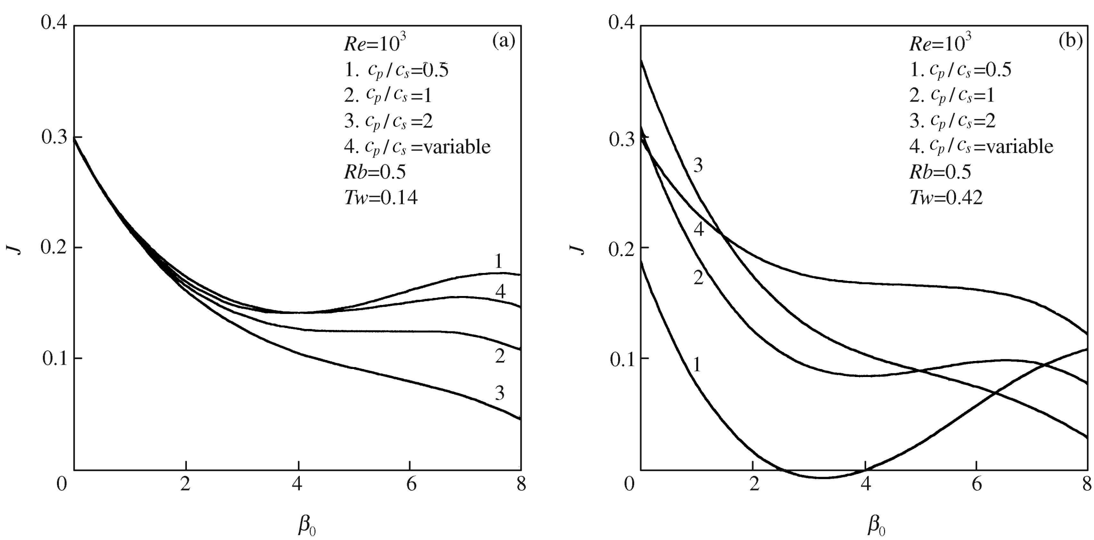


Fig. 4. J vs. β_0 for $Rb=0.5$ at $Re=10^3$. (a) $T_w=0.14$; (b) $T_w=0.42$.

there is a trend of decrease in J (for constant c_p/c_s). Generally speaking, J depends only slightly on Re especially for large Reynolds number flows. Finally, it is worth noting that, for a hot-wall case (say, $T_w = 0.42$), the value of J may become negative when the thermal parameter c_p/c_s takes small values (see curve 1 in Fig. 4(b)). This situation may be realized for particles with moderate inertia properties, which reach the body surface with the temperature much lower than the surface temperature. In this case the particles work as a cooler.

5 Summary

On the basis of numerical calculations, a parametric study of the maximum increase in the particle-induced heat flux at the stagnation point of a sphere in viscous supersonic dusty-gas flows is performed for the inertial particle deposition regime. The parameter range under consideration is of practical interest, and the theoretical estimate shows that the particle-induced increase in the stagnation-point heat flux is proportional to the product of the free-stream particle mass loading rate and the square root of the Reynolds number of the flow past the body. This effect may be substantial even at very low free-stream particle mass concentrations and it depends on many similarity parameters. The calculations show that there is a trend to increase in the particle -induced heat flux with increase in the relative specific heat of the particle material and with decrease in the nondimensional wall temperature.

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