Stochastic response analysis of visco-elastic slit shear walls

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Abstract. Slit shear walls are reinforced concrete shear wall structures with purposely built-in vertical slits. If the slits are inserted with visco-elastic damping materials, the shear walls will become visco-elastic sandwich beams. When adequately designed, this kind of structures can be quite effective in resisting earthquake loads. Herein, a simple analysis method is developed for the evaluation of the stochastic responses of visco-elastic slit shear walls. In the proposed method, the stiffness and mass matrices are derived by using Rayleigh-Ritz method, and the responses of the structures are calculated by means of complex modal analysis. Apart from slit shear walls, this analysis method is also applicable to coupled shear walls and cantilevered sandwich beams. Numerical examples are presented and the results clearly show that the seismic responses of shear wall structures can be substantially reduced by incorporating vertical slits into the walls and inserting visco-elastic damping materials into the slits.

Key words: shear wall; sandwich beam; vibration; damping; modal analysis; stochastic response analysis.

1. Introduction

The response of a structure under dynamic excitation is largely determined by the relative amounts of the energy fed in and the energy dissipated. Therefore, to reduce its dynamic response, resonance should be avoided in order to keep the energy input low and sufficient damping should be incorporated to dissipate the vibration energy. However, there are many cases in which the spectra of the input are so wide that resonance cannot really be avoided and for such cases, it is crucial to introduce extra damping into the structure so as to limit the dynamic amplification effect due to resonance.

It is well known that when properly designed, extra damping can be added to a beam by incorporating a visco-elastic sandwich layer into the structure (Narka 1976). Herein, it is

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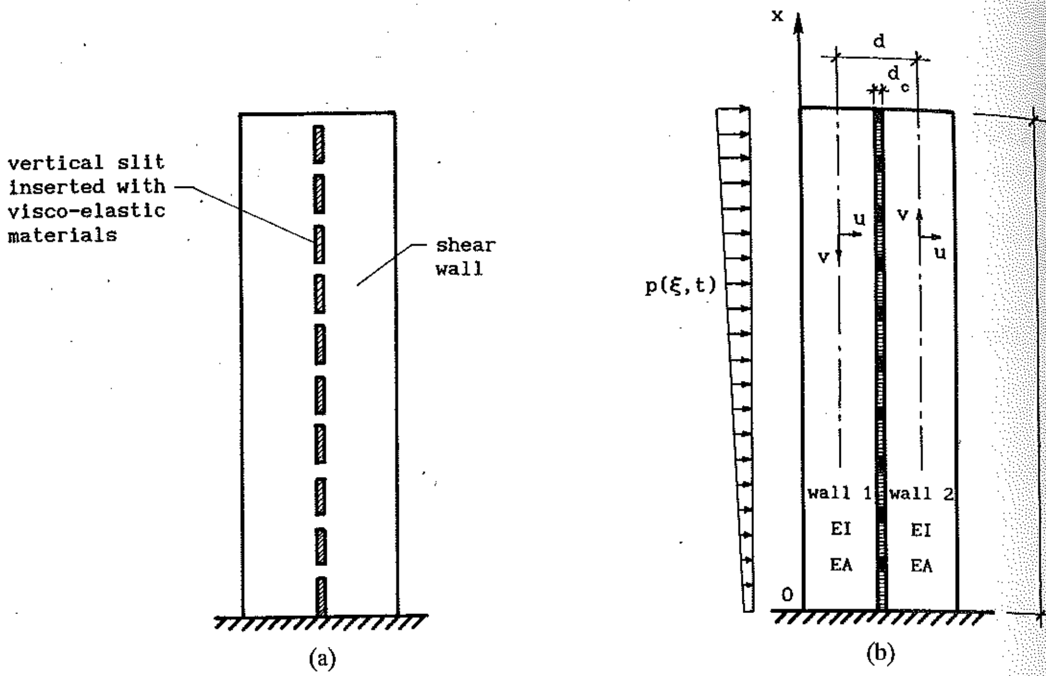


Fig. 1 (a) A typical slit shear wall structure; (b) slit shear wall treated as two solid walls sandwiching a visco-elastic core

suggested that this same technique may also be applied to concrete shear walls which are essentially vertical cantilevered beams. Concrete shear walls are widely used in the design and construction of tall buildings in seismic regions. However, although they can have fairly high lateral strength, they also generally have much higher lateral stiffness than other structural forms. Due to their high stiffness, they tend to attract large amounts of seismic energy, and because of their low damping, the energy fed in may build up very quickly and eventually cause severe damage to the structures. Visco-elastic sandwich layers may be incorporated into shear walls by casting vertical slits near the centroidal axis of the walls and inserting visco-elastic damping materials into the slits (see Fig. 1(a)). Such shear walls with purposely built-in vertical slits are called "slit shear walls". It is hoped that by transforming the solid shear walls into slit shear walls which are really cantilevered sandwich beams, the seismic resistances of the shear walls can be greatly increased.

As shown in Fig. 1(a), a slit shear wall may be treated as the limiting case of a pair of coupled shear walls with very short connecting beams. A simple yet accurate approach for analysing coupled shear wall structures is to replace the connecting beams by equivalent continuous shear connections and then apply the so called continuous connection method to derive and solve the governing differential equations. Use of this method for static analysis have been studied by many investigators and a detailed review of the method was given by Coull and Stafford Smith (1967). It was extended for dynamic analysis by Tso and Chan (1971), Tso and Biswas (1972), Jennings and Skattum (1973), Coull and Mukherjee (1973), and Mukherjee and Coull (1973), among others in the 70s'. A recent study by Kwan *et al.* (1993) confirmed that the continuous connection method may also be applied to slit shear walls provided that the local deformations at the beam-wall joints and the shear deformations of the short connecting beams are appropriately taken into account.

It has been pointed out by Jennings and Skattum (1973) during their study of the dynamic behaviour of coupled shear walls that coupled shear walls and sandwich beams are similar in many of their essential features. In fact, if the shear modulus of the equivalent shear connection in the coupled shear wall structure and that of the sandwiched layer in the sandwich beam are equal, the motion equations of the two structural systems will be identical. Sandwich beams with visco-elastic cores are effective vibration control devices and have been studied quite extensively in the field of mechanical engineering. A complete set of motion equations governing the vibration of damped sandwich beams have been derived and solved exactly for several important boundary conditions by Rao (1978). The results so obtained on natural frequencies and loss factors have been presented in both the forms of design graphs and approximate formulae but they are not sufficient for response analysis. Finite element methods with visco-elastic damping considered (Bogner and Soni 1981, Holman and Tanner 1981, Johnson *et al.* 1981, Soni 1981) have also been developed for the analysis of damped sandwich structures. However, owing to the fact that the shear modulus of the damping core is complex, the computer storage and time required will generally be several times higher than those of undamped analysis.

For the purpose of evaluating the stochastic responses of slit shear walls and in fact also of coupled shear walls and sandwich beams, a simplified analysis method based on the Rayleigh-Ritz method is developed in this paper. Two families of eigen-mode functions, namely the flexural and axial vibration modes of an undamped cantilever beam, which already satisfy the geometric boundary conditions, are used as the approximate mode shapes of the slit shear wall structure. Energy formulation is used to derive the governing equations and a standard computer routine EISPACK, developed by Wilkinson (1965), is used to determine the eigenvalues and eigenvectors of the system equations. The transfer functions are then obtained by Fourier transformations and the stochastic responses of the structure evaluated by a complex modal superposition method. The proposed method is computationally very efficient and is particularly useful for parametric studies.

2. Derivation of system equations

Consider a typical slit shear wall whose geometry is shown in Fig. 1(a). The slit shear wall structure may be treated as consisting of two solid walls with a visco-elastic sandwich core bonding them together, Fig. 1(b). For simplicity, only symmetric slit shear walls, i.e., slit shear walls with the vertical joints located at the centroidal axis, are considered in the present study. Since the slit shear wall is symmetric, the two solid walls sandwiching the core have the same flexural and axial rigidities which are denoted by EI and EA respectively. The visco-elastic properties of the core are described by the complex shear modulus $G_c=G(1+i\eta_c)$ in which G is the elastic shear modulus, i is the square root of -1, and η_c is the core loss factor (for readers who are not familiar with complex stiffness and damping, please refer to Appendix A for more detailed explanation).

The following assumptions are made in the formulation:

(1) The lateral displacements are small and uniform across any section.
(2) The axial displacements are continuous.
(3) The walls bend as per Euler hypothesis.

(4) The core layer deforms mainly through shear and does not carry any axial load.
(5) Rotatory inertia are negligible.

These assumptions are the same as those used by Rao (1978) except that the longitudinal inertia was neglected in Rao's analysis but is taken into account in the present method.

Let the flexural response u and axial response v of the two solid walls be in the forms of limit series as follows:

$$u(\xi, t)=\sum_{j=1}^{m} \phi_j(\xi)\,\mu_j(t) \tag{1}$$

$$v(\xi, t)=\sum_{k=1}^{n} \psi_k(\xi)\,\upsilon_k(t) \tag{2}$$

where $\xi=x/l$ is the nondimensional coordinate, and $\phi_j(\xi)$ and $\psi_k(\xi)$ are the j-th flexural and the k-th axial vibration modes respectively of an undamped cantilever beam. The flexural vibration modes are given by (Clough and Penzien 1975):

$$\phi_j(\xi)=\cos a_j\xi-\cosh a_j\xi+b_j(\sin a_j\xi-\sinh a_j\xi) \tag{3}$$

in which a_j is the root of the frequency equation:

$$\cos a_j \cosh a_j+1=0 \tag{4}$$

and b_j is related to a_j by:

$$b_j=-\frac{\cos a_j+\cosh a_j}{\sin a_j+\sinh a_j} \tag{5}$$

On the other hand, the axial vibration modes are given by Clough and Penzien (1975):

$$\psi_k(\xi)=\sqrt{2}\,\sin c_k\xi \tag{6}$$

where

$$c_k=\frac{2k-1}{2}\pi \tag{7}$$

Note that all the mode shape functions already satisfy the geometric boundary conditions of the structure.

The governing differential equations for the generalized deflections $\mu_j(t)$ and $\upsilon_k(t)$, i.e., the dynamic equilibrium equations, are derived by means of energy formulation through the application of the Lagrange equations. Let $\{y\}^T=\{\mu_1\ \mu_2\ \cdots\ \mu_m\ \upsilon_1\ \upsilon_2\ \cdots\ \upsilon_n\}$. The Lagrange equations are given by:

$$\frac{d}{dt}\left(\frac{\partial \Gamma}{\partial \dot{y}_j}\right)+\frac{\partial \Lambda}{\partial y_j}=f_j \qquad j=1, 2, \cdots, m+n \tag{8}$$

where Γ and Λ are the kinetic energy and strain energy respectively of the structure, and f_j is the generalized force. The kinetic energy Γ of the structure is given by:

$$\Gamma=\frac{1}{2}\rho l \int_0^1 \left\{\left(\frac{\partial u}{\partial t}\right)^2+\left(\frac{\partial v}{\partial t}\right)^2\right\} d\xi \tag{9}$$

where ρ is the mass per unit length or height of the structure, while the strain energy Λ of the structure is given by:

$$\Lambda = \Lambda_1 + \Lambda_2 + \Lambda_3 \tag{10}$$

in which

$$\Lambda_1 = \frac{1}{2}\, l \int_0^1 2EA \left(\frac{1}{l} \frac{\partial v}{\partial \xi} \right)^2 d\xi \tag{11}$$

$$\Lambda_2 = \frac{1}{2}\, l \int_0^1 2EI \left(\frac{1}{l^2} \frac{\partial^2 u}{\partial \xi^2} \right)^2 d\xi \tag{12}$$

$$\Lambda_3 = \frac{1}{2}\, l \int_0^1 G_c A_c \left(\frac{d}{l d_c} \frac{\partial u}{\partial \xi} + \frac{2v}{d_c} \right)^2 d\xi \tag{13}$$

When $j \leq m$, the generalized force f_j is equal to:

$$f_j = l \int_0^1 p(\xi, t)\, \phi_j(\xi)\, d\xi \qquad j = 1, 2, \cdots, m \tag{14}$$

where $p(\xi, t)$ is the distributed horizontal load acting on the structure. When $j > m$, f_j is equal to zero because no vertical dynamic load is applied. If the wall structure is subjected to horizontal earthquake movement with ground acceleration $\ddot{u}_o(t)$, then $p(\xi, t) = -\rho \ddot{u}_o(t)$ and the generalized forces would become:

$$f_j = -\rho\, l\, \ddot{u}_o(t)\, g_j \qquad j = 1, 2, \cdots, m \tag{15}$$

in which

$$g_j = \int_0^1 \phi_j(\xi)\, d\xi \qquad j = 1, 2, \cdots, m \tag{16}$$

Substituting the equations for $u(\xi, t)$ and $v(\xi, t)$ into the energy expressions and then differentiating the energy expressions as per the Lagrange equations, the dynamic equilibrium equation of the slit shear wall is obtained as follows:

$$[M]\{\ddot{y}\} + [K]\{y\} = \{f\} \tag{17}$$

where the matrices $[M]$ and $[K]$ are as given in Appendix B, and the force vector is defined by $\{f\}^T = \{f_1\ f_2\ \cdots\ f_{m+n}\}$. Note that the mass matrix $[M]$ is a diagonal matrix, while the complex stiffness matrix $[K]$ is a symmetric matrix.

Stochastic response analysis

The eigenvalues and eigenvectors of the system equation are calculated by the computer program EISPACK (the eigen-solver developed by Wilkinson) and the eigenvector matrix $[Z]$ so obtained is normalized such that:

$$[Z]^T\, [M]\, [Z] = [I] \tag{18a}$$

$$[Z]^T\, [K]\, [Z] = [\Omega] \tag{18b}$$

in which $[I]$ is an identity matrix and $[\Omega]$ is a diagonal matrix of natural frequencies. Introducing the coordinate transformation $\{y\}=[Z]\{q\}$ into Eq. (17) and then pre-multiplying the equation by $[Z]^T$, a set of uncoupled equations is obtained:

$$\ddot{q}_r+\omega_r^2(1+i\,\eta_r)\,q_r=p_r \qquad r=1,2,\cdots,m+n \tag{19}$$

where ω_r is the modal frequency, η_r is the modal damping factor, and p_r is the r-th component of $[Z]^T\{f\}$. Denoting the r-th eigenvector, i.e., the r-th column of $[Z]$, by $\{z_r\}$, then $p_r=\{z_r\}^T\{f\}$.

Applying Fourier transformation to Eq. (19), the following set of equation is obtained:

$$\boldsymbol{Q}_r(\omega)=H_r(\omega)\boldsymbol{P}_r(\omega) \qquad r=1,2,\cdots,m+n \tag{20}$$

where $\boldsymbol{Q}_r(\omega)$ and $\boldsymbol{P}_r(\omega)$ are the Fourier transforms of q_r and p_r respectively, and $H_r(\omega)$ is the r-th transfer function given by:

$$H_r(\omega)=\frac{1}{\omega_r^2(1+i\,\eta_r)-\omega^2} \tag{21}$$

The Fourier transform $\boldsymbol{P}_r$ is, in turn, given by:

$$\boldsymbol{P}_r(\omega)=\{z_r\}^T\,\{\boldsymbol{F}(\omega)\} \tag{22}$$

in which $\{\boldsymbol{F}(\omega)\}$ is the Fourier transform of $\{f\}$. If the wall structure is subjected to earthquake movement with ground acceleration $\ddot{u}_o(t)$, then:

$$\boldsymbol{F}_r(\omega)=-\rho\,l\,g_r\,\ddot{\boldsymbol{U}}_o(\omega) \tag{23}$$

where $\ddot{\boldsymbol{U}}_o(\omega)$ is the Fourier transform of $\ddot{u}_o(t)$. Substituting the value of $\boldsymbol{F}_r(\omega)$ into Eq. (22),

$$\boldsymbol{P}_r(\omega)=-\rho\,l\,\{z_r\}^T\,\{g\}\,\ddot{\boldsymbol{U}}_o(\omega) \tag{24}$$

in which $\{g\}$ is the vector defined by $\{g\}^T=\{g_1\ g_2\ \cdots\ g_m\ 0\ 0\ \cdots\ 0\}$. Let λ_r denote $\rho\,l\,\{z_r\}^T\{g\}$, then $\boldsymbol{P}_r(\omega)=-\lambda_r\ddot{\boldsymbol{U}}_o(\omega)$. Substituting this value of $\boldsymbol{P}_r(\omega)$ into Eq. (20), the Fourier transform $\boldsymbol{Q}_r(\omega)$ is obtained as:

$$\boldsymbol{Q}_r(\omega)=-\lambda_r H_r(\omega)\,\ddot{\boldsymbol{U}}_o(\omega) \tag{25}$$

Generally, any response R of the system can be expressed as a linear function of $\{y\}$ or $\{q\}$. Let the response R, which may be the lateral deflection of the structure, bending moment in one of the walls or shear stress in the sandwich core, be expanded as a linear combination of q_r as follows:

$$R=\sum_{r=1}^{m+n}\kappa_r q_r \tag{26}$$

Applying Fourier transformation to this equation, the Fourier transform $\boldsymbol{R}(\omega)$ of the response is obtained as:

$$\boldsymbol{R}(\omega)=\sum_{r=1}^{m+n}\kappa_r\boldsymbol{Q}_r(\omega) \tag{27}$$

If only earthquake loading is considered, then substituting the value of $\boldsymbol{Q}_r(\omega)$ as given by Eq. (25) into the above equation, $\boldsymbol{R}(\omega)$ can be expressed directly in terms of the seismic input as follows:

$$\boldsymbol{R}(\omega)=\sum_{r=1}^{m+n}-\kappa_r\lambda_r H_r(\omega)\ddot{U}_o(\omega) \tag{28}$$

Theoretically, the variance of the response R can be calculated by (Tian *et al.* 1982):

$$\sigma_R^2=\frac{1}{4\pi}\left\{\int_{-\infty}^{\infty}\sum_r\sum_s \kappa_r\lambda_r H_r(\omega)\,\kappa_s^*\lambda_s^* H_s(\omega)^* S_{\ddot{u}\ddot{u}}(\omega)\,d\omega\right\} \tag{29}$$

where κ_s^*, λ_s^* and $H_s(\omega)^*$ are the complex conjugates of κ_s, λ_s and $H_s(\omega)$ respectively, and $S_{\ddot{u}\ddot{u}}(\omega)$ is the spectral density of $\ddot{u}_o(t)$. However, the transfer functions are not Hermitian, in other words, $H_r(-\omega)\neq H_r(\omega)^*$ and $H_s(-\omega)\neq H_s(\omega)^*$. As a result, in the negative frequency domain, the damping would become negative and the structure would be unstable. The overcome this problem, Tsushima *et al.* (1976) and Bronowicki (1981) proposed that the imaginary part of the complex stiffness (the damping component) should be modified by the following factor:

$$\mathrm{sign}(\omega)=\begin{cases}1\,;\ \omega>0\\ 0\,;\ \omega=0\\ -1\,;\ \omega<0\end{cases} \tag{30}$$

so that the damping would remain positive in the negative frequency domain. Applying their method to the present analysis, the complex stiffness of the sandwich core becomes:

$$G_c=G(1+i\ \mathrm{sign}(\omega)\,\eta_c) \tag{31}$$

With the complex stiffness so modified, the transfer function becomes:

$$H_r(\omega)=\frac{1}{\omega_r^2(1+i\ \mathrm{sign}(\omega)\,\eta_r)-\omega^2} \tag{32}$$

which is now Hermitian. A stable solution in the whole frequency range can then be obtained as:

$$\sigma_R^2=\frac{1}{4\pi}\left\{\int_0^{\infty}\sum_r\sum_s\frac{\kappa_r\lambda_r\kappa_s^*\lambda_s^*S_{\ddot{u}\ddot{u}}(\omega)}{[\omega_r^2(1+i\eta_r)-\omega^2][\omega_s^2(1-i\eta_s)-\omega^2]}\,d\omega\right.$$
$$\left.+\int_{-\infty}^{0}\sum_r\sum_s\frac{\kappa_r^*\lambda_r^*\kappa_s\lambda_sS_{\ddot{u}\ddot{u}}(\omega)}{[\omega_r^2(1-i\eta_r)-\omega^2][\omega_s^2(1+i\eta_s)-\omega^2]}\,d\omega\right\} \tag{33}$$

A general solution to this equation has been given by Tian (1982). However, for most structures including cantilevered structures of which the natural frequencies are well separated, the cross terms in the integrand (the terms with $r\neq s$) are generally very small and may be neglected so that the above equation can be simplified to:

$$\sigma_R^2=\frac{1}{4\pi}\int_{-\infty}^{\infty}\sum_r\frac{|\kappa_r|^2|\lambda_r|^2S_{\ddot{u}\ddot{u}}(\omega)}{[\omega_r^2(1+i\eta_r)-\omega^2][\omega_r^2(1-i\eta_r)-\omega^2]}\,d\omega \tag{34}$$

Although there may be large fluctuations of $S_{\ddot{u}\ddot{u}}(\omega)$ as a function of ω, the frequency intervals over which the power spectral densities are substantial are usually much larger than the bandwidths of the oscillator at the various modal frequencies (Miles and Thomson 1976). Hence, the variations of $S_{\ddot{u}\ddot{u}}(\omega)$ within the modal bandwidths may be treated as small. Assuming that $S_{\ddot{u}\ddot{u}}(\omega)$ varies slowly within the modal bandwidths of the oscillator, the above integration can be carried out by the calculus of residues as follows. The integrand has a pair of poles $\omega_{r,1}=\omega_r\sqrt{(1+i\eta_r)}$ and $\omega_{r,2}=-\omega_r\sqrt{(1-i\eta_r)}$ above the real axial in the positive and negative frequency domain respectively which can be expressed more simply in the following forms:

$$\omega_{r,1}=\omega_r(\alpha_r+i\beta_r) \tag{35}$$

$$\omega_{r,2}=\omega_r(-\alpha_r+i\beta_r) \tag{36}$$

where α_r and β_r are given by:

$$\alpha_r=\frac{1}{\sqrt{2}}[(1+\eta_r^2)^{1/2}+1]^{1/2} \tag{37}$$

$$\beta_r=\frac{1}{\sqrt{2}}[(1+\eta_r^2)^{1/2}-1]^{1/2} \tag{38}$$

Assuming $S_{\ddot{u}\ddot{u}}(\omega_{r,1})=S_{\ddot{u}\ddot{u}}(\omega_{r,2})=S_r$, and applying the calculus of residues, the equation for the variance of R becomes:

$$\sigma_R^2=\sum_r\frac{|\kappa_r|^2|\lambda_r|^2\alpha_r S_r}{4\eta_r\omega_r^3(\alpha_r^2+\beta_r^2)} \tag{39}$$

When $\eta_r<0.1$, this equation may be approximated by:

$$\sigma_R^2\simeq\sum_r\frac{|\kappa_r|^2|\lambda_r|^2 S_r}{4\eta_r\omega_r^3} \tag{40}$$

It is noteworthy that the form of the above equation agrees well with that of Eq. (2. 130) in the reference by Warburton (1976).

4. Numerical examples

4.1. Example 1

In order to check and evaluate the accuracy of the proposed method, the first example is taken to be the cantilevered sandwich beam shown in Fig. 2, which has been studied quite extensively by Johnson *et al.* (1981) and by Soni (1981) in various analysis and experiments. Four different values of core loss factor (η_c=0.1, 0.2, 0.6 and 1.0) are considered in the analysis. Five flexural and two axial eigen-function terms are used (i.e., m=5 and n=2) in the numerical calculations. The modal frequencies and the loss factor parameters (loss factor parameter=modal damping factor/core loss factor) so obtained by the proposed approximate method are tabulated in Table 1 where for comparison, the exact analytical results obtained by Soni (1981) using sixth order analysis are also shown. It is evident from the tabulated results that the proposed

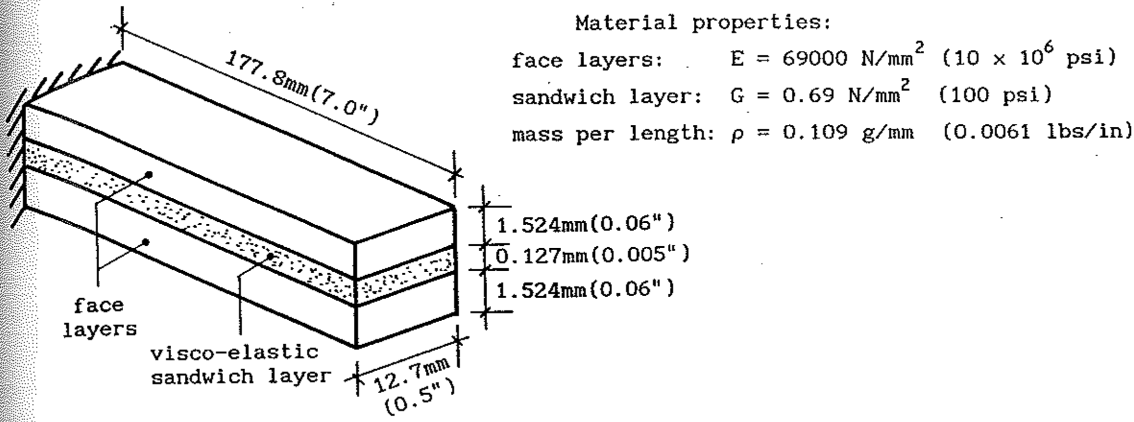


Fig. 2 Example 1 – cantilevered sandwich beam

Table 1 Example 1 – natural frequencies and loss factors of a sandwich beam for various core loss factors

Core loss factors	Modes	Natural frequencies (H_z)		Loss factor parameters (η_r/η_c)	
		Proposed approximate method	Exact analytical method	Proposed approximate method	Exact analytical method
η_c=0.1	1	64.2	64.1	0.282	0.282
	2	297.5	296.4	0.246	0.242
	3	746.1	743.7	0.164	0.154
	4	1400.1	1393.9	0.095	0.089
	5	2269.0	2261.1	0.060	0.057
η_c=0.2	1	64.4	64.2	0.278	0.273
	2	297.9	296.6	0.245	0.242
	3	746.4	743.9	0.164	0.154
	4	1400.2	1394.0	0.095	0.089
	5	2268.7	2261.2	0.060	0.057
η_c=0.6	1	66.0	65.5	0.246	0.246
	2	301.4	298.9	0.236	0.232
	3	748.9	745.5	0.163	0.153
	4	1412.4	1394.9	0.095	0.089
	5	2269.6	2261.7	0.060	0.057
η_c=1.0	1	68.3	67.4	0.202	0.202
	2	307.2	302.8	0.222	0.218
	3	754.1	748.6	0.161	0.150
	4	1403.5	1396.6	0.095	0.088
	5	2271.0	2262.9	0.060	0.057

ethod agrees very closely with the exact theory for the fairly wide range of modal frequency d core loss factor considered.

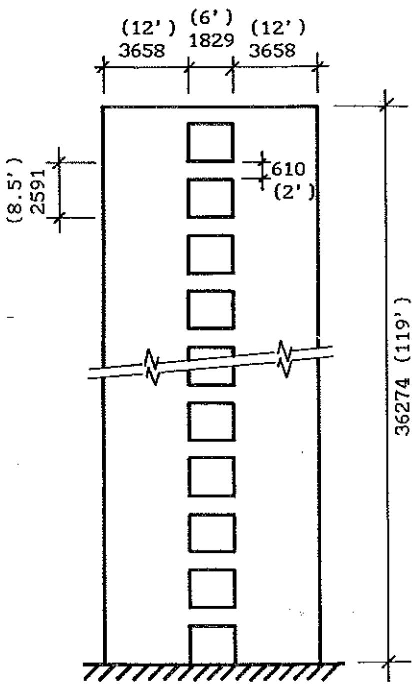


Fig. 3 Example 2 – coupled shear wall structure (note: all dimensions are in mm except otherwise stated)

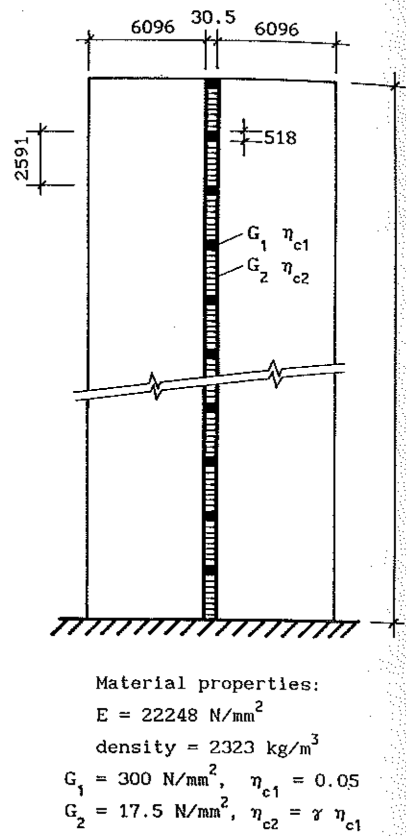


Fig. 4 Example 3 – slit shear wall structure (note: all dimensions are in mm)

4.2. Example 2

To evaluate the applicability of the proposed method to coupled shear walls, the coupled wall structure in Fig. 3, which resembles a real building structure and has been studied by Jennings and Skattum (1973), is analysed. The spandrel beams of the coupled wall structure are modelled as an equivalent continuous laminae and the whole structure is treated as a cantilevered sandwich beam with the shear modulus of the sandwiched core taken as the equivalent shear modulus of the continuous laminae. In the analysis, 10 flexural eigen-function terms (m=10) are used, but as the axial vibrations are generally less important than the flexural vibrations, only 2 axial eigen-functions terms (n=2) are used. The eigenvalues for the lateral vibrations calculated by the present method are compared with those calculated by Jennings and Skattum in Table 2. Good agreement between the two sets of results up to the sixth mode is obtained.

4.3. Example 3

The proposed method is applied to a slit shear wall structure in this example. The strucure analysed resembles a real and typical shear wall structure and has overall dimensions as shown

Table 2 Example 2 – eigenvalues of a symmetric coupled shear wall structure under lateral motion

Methods of analysis	Eigenvalues for lateral vibrations					
	Mode numbers					
	1	2	3	4	5	6
Proposed approximate method	8.8	40.0	102.7	178.6	261.2	363.6
Exact analytical method	8.9	41.9	97.1	185.1	263.7	363.0
% Difference between above methods	−1.1	−4.5	5.8	−3.5	−0.9	0.2

Fig. 4. It is modelled as a cantilevered sandwich beam with a visco-elastic sandwich core. As illustrated in the figure, the sandwich core is nonhomogeneous because the short connecting beams in the core and the visco-elastic insert at the slits are in general made of different materials. The short connecting beams which are made of reinforced concrete generally have higher shear stiffness and lower damping while the visco-elastic material inserted into the slits would have somewhat lower shear stiffness and higher damping. As a result, the values of G and η_c are not uniform along the height of the structure. This, however, poses on particular difficulties for the present method of analysis. To allow for variations of G and η_c with height, numerical integration is employed when using Eqs. (B5-B7) to derive the stiffness matrix $[K]$ of the structural system. As in the first example, 5 flexural eigen-function terms (m=5) and 2 axial eigen-functions terms (n=2) are used. In the response analysis, a typical earthquake spectrum (Kanai-Taijimi spectrum) as given by the following equation:

$$S_{\ddot{u}\ddot{u}}(\omega)=\frac{1+4\zeta^2\left(\dfrac{\omega}{\omega_o}\right)^2}{\left(1-\left(\dfrac{\omega}{\omega_o}\right)^2\right)^2+4\zeta^2\left(\dfrac{\omega}{\omega_o}\right)^2}S_o \tag{41}$$

where ζ=0.642 and ω_o=15.5 rad. s^{-1} (typical values adopted by Wen (1979) in his theoretical studies) is used. To study the effects of core damping on the dynamic responses of the structure, the structure is re-analysed for several different values of η_{c2}. The root mean square values of the maximum combined stress in the wall and the maximum deflection of the structure versus the multiplying factor γ (defined by $\eta_{c2}=\gamma\eta_{c1}$) are plotted in Fig. 5. From the results, it can be clearly seen that by increasing the core damping, both the maximum combined stress in the walls and the maximum deflection of the structure can be reduced by more than 50%.

4.4. Example 4

The previous example shows that the increase of core damping can significantly reduce the dynamic response of a slit shear wall structure. This example, in which the same slit shear wall structure is re-analysed using many different combinations of core stiffness and core damping so to allow the example to serve as a parametric study, shows, moreover, that the dynamic response of the structure can also be reduced by adjusting the shear stiffness of the core layer to

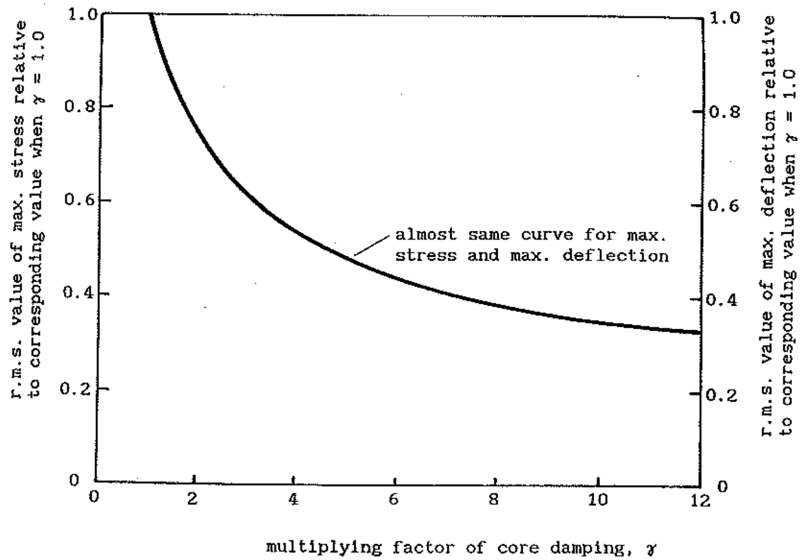


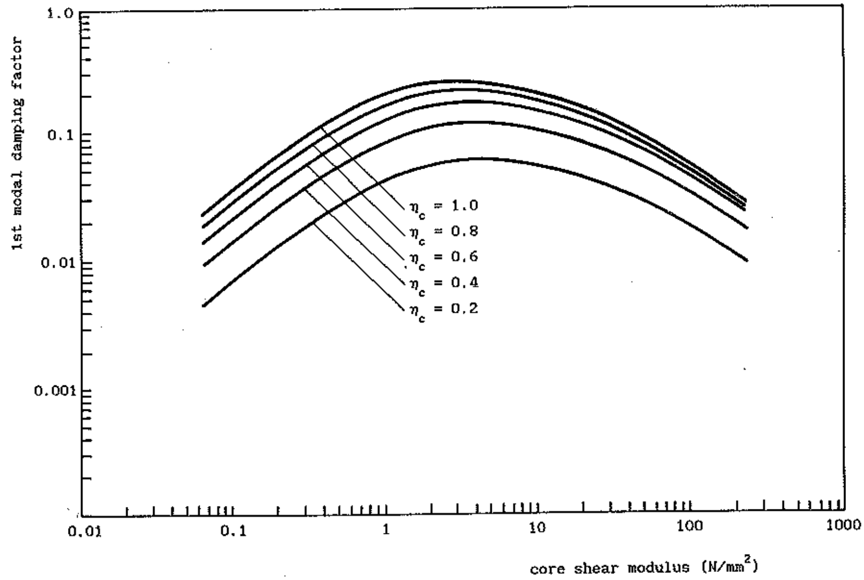
Fig. 5 Example 3 – effects of increasing core damping on maximum stress and maximum deflection of structure

a certain optimum value. In actual engineering practice, it should be quite possible to adjust the shear stiffness of the core layer by inserting different visco-elastic materials into the slit and/or modifying the width of the slit so that the core stiffness is close to the optimum value and consequently the dynamic response of the structure is minimized.

In order to reduce the number of parameters to be studied and in fact also to render the results more generally applicable, the visco-elastic core is taken to be homogeneous in this study rather than nonhomogeneous. This would not reduce the generality of the results, as nonhomogeneous cores can always be treated as equivalent homogeneous cores by using the equivalent mean properties in the analysis. Several different earthquake spectra, including the one used in the previous example, have been considered and the results are presented in the following.

The combined effects of core stiffness and core damping on the overall damping of the structure are depicted in Fig. 6 where the modal damping factor of the 1st mode of vibration of the structure is plotted against the shear modulus of the visco-elastic core for five different values of core loss factor (η_c=0.2, 0.4, 0.6, 0.8 and 1.0). It is seen from the results that there exists an optimum core shear modulus at which the 1st modal damping factor of the structure reaches a maximum and, as expected, the modal damping factor of the structure increases with the core loss factor. Similar results but somewhat different optimum values for the core shear modulus are obtained for the higher modes of vibration. Nevertheless, since for a cantilevered structure, the dynamic responses are mainly contributed by the 1st mode of vibration, the optimum value of core shear stiffness for maximum 1st modal damping factor may be taken as the optimum value for maximum overall damping of the structure. The modal damping factors are intrinsic properties of the structure which are independent of the earthquake spectrum and, therefore, the results presented in Fig. 6 are generally applicable regardless of the actual frequency spectrum of the earthquake excitation.

The existence of an optimum core shear stiffness for maximum overall damping may be



ig. 6 Example 4 – combined effects of core stiffness and core damping on 1st modal damping factor of structure

plained by the fact that the energy dissipation through the visco-elastic core is proportional to e strain energy absorbed by the core. When the core shear stiffness is high, the shear stress duced in the core would also be high but the corresponding shear strain would be rather low ading to relatively small strain energy absorbed by the core. On the other hand, when the core ear stiffness is low, the shear stress in the core would become low despite high shear strain ing induced and as a result the strain energy absorbed by the core would again be relatively w. Therefore, at both extremes of high and low core shear stiffness, the damping capacity of

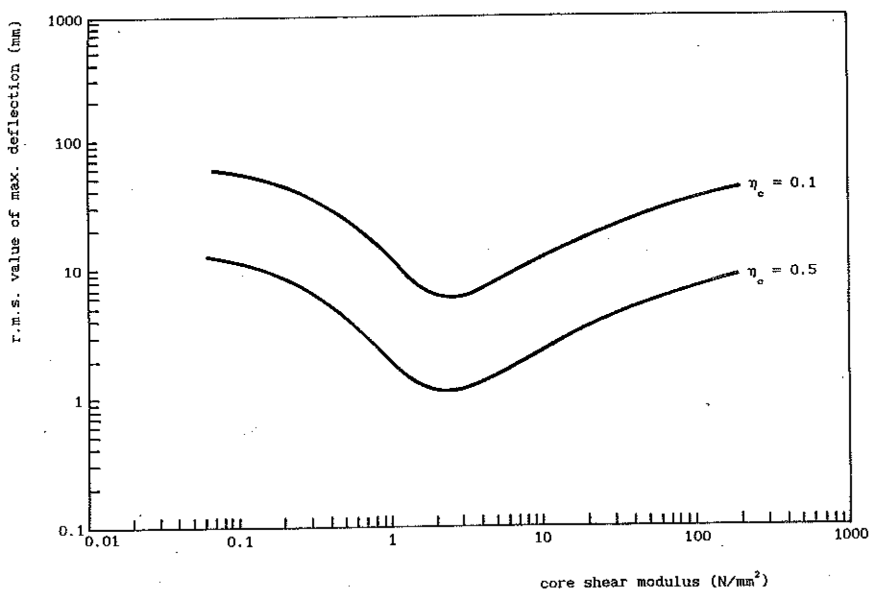


Fig. 7 Example 4 – effects of core stiffness on deflection response of structure for η_c=0.1 and η_c=0.5

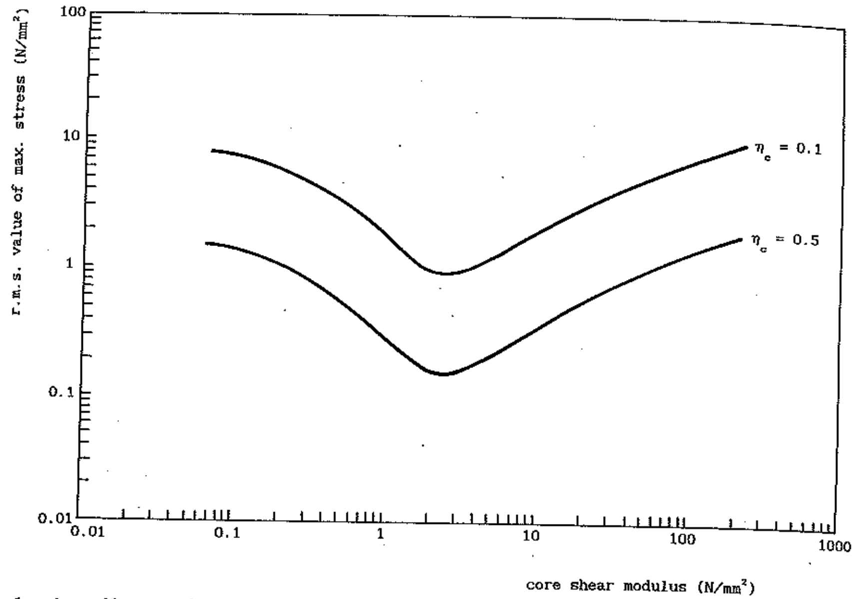


Fig. 8 Example 4 – effects of core stiffness on stress response of structure for η_c=0.1 and η_c=0.5

the structure would remain relatively low and at certain intermediate value of core shear stiffness, in which case neither the shear stress nor the shear strain in the core is too low, the damping capacity of the structure reaches a maximum.

The deflection and stress responses of the structure are shown in Fig. 7 and Fig. 8 respectively. Unlike the modal damping factors, the dynamic responses of the structure are dependent on the frequency spectrum of the earthquake excitation. Although several different earthquake spectra have been considered in the study, due to limited space, only the results for one earthquake spectrum are presented; the results shown in the two figures are for the same earthquake spectrum as that applied in the previous example which is given by Eq. (41). Two values of core loss factor, namely: η_c=0.1 and η_c=0.5, have been used in the analysis. From the results, it can be seen that the dynamic responses of the structure are at their minimum when the core shear modulus is approximately equal to its optimum value for maximum 1st modal damping factor. The results for the other earthquake spectra applied are similar except that the optimum value of core shear modulus for minimum dynamic response of the structure is slightly different for different earthquake spectra. For all the earthquake spectra considered in this particular study, the optimum core shear modulus for minimum dynamic response is never too far away from the optimum value for maximum 1st modal damping factor of the structure. It may be said, therefore, that in most cases, setting the core shear stiffness near its optimum value for maximum overall damping would more or less lead to minimum dynamic response of the structure.

It is noteworthy from the results of Fig. 8 that the stress response of the structure is highest when the core shear modulus is very high in which case the slit shear wall structure acts more like a solid shear wall. In other words, the dynamic response of a slit shear wall structure is generally lower than that of a solid shear wall which is equivalent to a slit shear wall structure with very high core stiffness. Hence, it may be concluded that by introducing vertical slits to a solid shear wall so as to convert it into a slit shear wall structure with reduced core stiffness, it is possible to reduce the stress response of the structure during earthquakes.

Conclusions

The idea of incorporating extra damping into shear wall structures by casting vertical slits near the centroidal axis of the walls and inserting visco-elastic damping materials into the slits so that the shear walls are effectively transformed into visco-elastic sandwich beams is proposed. This results in a novel type of earthquake resistant structures- the slit shear wall system. It is hoped that by so doing, the energy dissipation capacities and hence the seismic resistances of shear wall structures can be greatly increased.

An approximate method for the stochastic response analysis of slit shear walls is developed. This response analysis method is applicable not only to slit shear walls but also to coupled shear walls and cantilevered sandwich beams because their governing equations are actually identical. In the proposed method, the system equations are derived by Rayleigh-Ritz method while the stochastic responses of the structure are evaluated using a complex modal superposition method. Nonhomogeneous sandwich core can also be dealt with by employing numerical integration to evaluate the stiffness matrix of the sandwich core. Before applying the proposed method to slit shear walls, the accuracy of the method is verified by analysing a cantilevered sandwich beam and a coupled shear wall structure and comparing the results to those obtained by other researchers.

Numerical examples of applying the proposed method to slit shear wall structures are presented. In the examples, the effects of core stiffness and core damping on the dynamic responses of the structure are studied by re-analysing the structure many times using different combinations of core parameters. The results show that (1) both the deflection and stress responses of the structure can be substantially reduced by incorporating extra core damping into the structure; (2) there exist a certain optimum value of core stiffness at which the overall damping of the structure is maximum; (3) in most cases, the dynamic responses of the structure would be minimum when the core stiffness is approximately equal to its optimum value for maximum overall damping; and (4) the stress responses of slit shear wall structures are generally lower than those of solid shear walls which are equivalent to slit shear walls with very high core stiffness. Hence it is demonstrated that a slit shear wall system with visco-elastic material of high damping capacity inserted into the slits may be a good solution for earthquake resistant building structures.

It should, however, be borne in mind that so far only theoretical studies have been carried out. Experimental investigations are still lacking. Particularly, suitable visco-elastic materials for insertion into the slits are yet to be identified before the material and installation costs can be determined and the feasibility of the proposed system evaluated. Further researches on these are recommended.

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Notations

A	sectional area of one face layer (or that of one wall)
A_c	sectional area of sandwiched core
E	Young's modulus of face layer (or that of wall)
G	shear modulus of sandwiched core
G_c	complex shear modulus of sandwiched core
H_r	transfer function of r-th mode
$p(\xi, t)$	lateral load acting on structure

moment of inertia of one face layer (or that of one wall)
sqare root of -1
length of cantilever beam (or height of wall)
response of structure being evaluated
spectral density of ground acceleration
lateral displacement (flexural deflection)
ground acceleration
longitudinal displacement (axial deflection)
kinetic energy of structure
strain energy of structure
mass per unit length or height of structure
modal frequency of r-th mode
core loss factor
modal damping factor of r-th mode

Appendix A

Structural damping mechanisms are complicated in nature and difficult to evaluate. In general, viscous damping forces, which are proportional to the velocities of motion but opposite in direction, are easiest to deal with mathematically. For this reason, damping forces are usually replaced for the purpose of analysis by equivalent viscous damping. This equivalent damping is generally determined from experiment by equating the dissipation of energy per cycle to that of a viscous damper and expressed in terms of the damping ratio ζ (ratio of damping to critical damping).

The energy dissipated per cycle by a viscous damper increases linearly with the frequency of vibration when the amplitude of vibration is constant. However, experimental observations cannot corroborate this intrinsic frequency dependent characteristic. A mechanism that leads to the energy dissipation per cycle being independent of the frequency of vibration is hysteretic damping which is a much better approximation than viscous damping for most engineering structures. Hysteretic damping can be incorporated using the concept of complex stiffness by expressing the structural stiffness as $k(1+i\eta)$ where k is the elastic stiffness, i is the square root of -1 and η is a loss factor. For a single-degree-of-freedom system, the relation between the loss factor η and the equivalent damping ratio ζ is:

$$\eta=2\zeta \tag{A1}$$

For a multi-degree-of-freedom system, however, there is no simple relation between the loss factors of the structural components and the damping ratios of the system as above. Generally, the loss factors of the various structural components have to be determined experimentally by testing the structural components as individual single-degree-of-freedom systems and then the damping factor of each vibration mode of the system determined through complex eigen analysis of the system equations as in Sections 2 and 3 of the present paper. In cases where testing of the individual structural components is not possible, the loss factors of the structural components may have to be estimated by back analysis of the system behaviour of similar structures or even by engineering judgement.

Appendix B

The mass matrix $[M]$ is a diagonal matrix given by:

$$[M]=\rho l\,[I] \tag{B1}$$

where $[I]$ is an identity matrix. On the other hand, the stiffness matrix $[K]$ is a symmetric matrix of the following form:

$$[K]=\begin{bmatrix}[A]+[C] & [E] \\ [E]^T & [B]+[D]\end{bmatrix} \tag{B2}$$

in which $[A]$ and $[C]$ are $m \times m$ matrices, $[B]$ and $[D]$ are $n \times n$ matrices, and $[E]$ is a $m \times n$ matrix. The submatrices $[A]$ and $[B]$ are diagonal matrices and their diagonal elements are given respectively by:

$$A_{jj}=2a_j^4\frac{EI}{l^3} \tag{B3}$$

$$B_{kk}=2c_k^2\frac{EA}{l} \tag{B4}$$

Finally, the elements of $[C]$, $[D]$ and $[E]$ are given respectively by the following equations:

$$C_{jk}=\int_0^1 G_c\frac{A_c d^2}{l d_c^2}\frac{\partial\phi_j}{\partial\xi}\frac{\partial\phi_k}{\partial\xi}\,d\xi \tag{B5}$$

$$D_{jk}=\int_0^1 4G_c\frac{A_c l}{d_c^2}\psi_j\psi_k\,d\xi \tag{B6}$$

$$E_{jk}=\int_0^1 2G_c\frac{A_c d}{d_c^2}\frac{\partial\phi_j}{\partial\xi}\psi_k\,d\xi \tag{B7}$$