On supersonic combustion *

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Abstract Some basic concepts and features of supersonic combustion are explained from the view point of macroscopic aerodynamics. Two kinds of interpretations of supersonic combustion are proposed. The difference between supersonic combustion and subsonic combustion is discussed, and the mechanism of supersonic combustion propagation and the limitation of heat addition in supersonic flow are pointed out The results of the calculation of deflagration in supersonic flow show that the entropy increment and the total pressure loss of the combustion products may decrease with the increase of combustion velocity. It is also demonstrated that the oblique detonation wave angle may not be controlled by the wedge angle under weak underdriven solution conditions and be determined only by combustion velocity. Therefore, the weak underdriven solution may become self-sustaining oblique detonation waves with a constant wave angle.

Keywords: supersonic combustion, deflagration, oblique detonation, hypersonic, ramjet.

The use of oxygen in the air by a space vehicle is an attractive technical issue. Due to the wide range of the vehicle speed, it is important for the oxygen to be utilized in a speed range as high as possible, otherwise the loss will outweigh the gain by the increase of operational weight brought into orbit. Thus a supersonic combustion ramjet, which may also work under hypersonic flight condition, is interesting. Therefore, the discussion of supersonic combustion is almost always related to supersonic combustion ramjets[1—3].

With a supersonic combustion ramjet, various factors must be considered, such as inlets (forebody), fuel injectors (for mixing enhancement), combustor (including ignition, flame propagation, flame holding and combustion products) and nozzles, which are in turn related to material construction, energy management, hypersonic dynamics, combustion, experimental facility and measuring instruments. It is a very complicated engineering project. For the design of a combustor one must consider not only combustion, but also the total efficiency, special impulse, thrust and so on. To equate the supersonic combustion with Scramjet will make it difficult for one to better understand supersonic combustion phenomena.

Generally speaking, combustion is a violent chemical reaction process, accompanied by sound, light and electric effects, and by orderly and disorderly moving processes (aerodynamics and thermodynamics). Besides, supersonic combustion includes hyper velocity motion and coupling processes between the motion and combustion. Sometimes, kinetic energy of molecules is higher than their thermal energy released in chemical reaction. In addition to macroscopic transition processes of mass and energy by diffusion, convection, turbulence and radiation, supersonic combustion also involves molecular random collision, reaction and transition of mass and

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energy in sub-macroscopic and microcosmic scale. Furthermore, non-linear effects caused by changes in pressure and temperature make it impossible to obtain a reliable extrapolation of kinetic data of elementary reactions obtained experimentally at low states to high states. Elementary reactions, particularly three-body recombination reactions, are difficult to be isolated from each other, so the values of the rate constants obtained from experiments are with a large uncertainty[4]. The precise rate constants under high pressure and high temperature conditions are still not available, so the perfect complete numerical simulations of actual supersonic combustion processes are almost impossible. A simplified calculation or numerical simulation with reasonable assumptions will be desirable. Consequently, it is necessary to find the nature of the processes from phenomenal observation in order to simplify and model the processes.

In this paper, some basic concepts and features of supersonic combustion are explained from the view point of macroscopic aerodynamics, two kinds of interpretations of supersonic combustion are proposed; the difference between supersonic combustion and subsonic combustion is discussed, and the mechanism of supersonic combustion propagation and the limitation of heat addition in supersonic flow within a constant area combustor are pointed out.

0.1 The concept of supersonic combustion

Builder[5] put forward a concept of optimal compression for an airbreathing engine based on the Brayton cycle analysis, and pointed out that one may deduce an optimal amount of compression for the air flow entering a combustor to obtain the highest cycle efficiency under a certain efficiency condition of compression and expansion. It is shown that when the flight speed is low, the kinetic energy of the air is not enough to be used for the optimal compression. Further compression by machines is needed in order to obtain a higher efficiency. For example, a turbojet employs a turbine machine for further compression. When the flight speed is higher than a certain value, the air flow entering a combustor will remain to be supersonic after the optimal compression. With a further compression (i.e. deceleration), the efficiency of the engine will decrease. Therefore the combustion has to take place under the supersonic flow condition. This kind of airbreathing engine, which works under hypersonic flight condition, is called the supersonic combustion ramjet (Scramjet). The term of "supersonic combustion" applied here means the combustion in a supersonic flow. However, supersonic combustion phenomena do not take place in Scramjets only. It is, therefore, necessary to clarify the concept of supersonic combustion for a better understanding of the supersonic combustion phenomena and also for the development of Scramjets.

What does supersonic combustion mean? There are two kinds of definitions for its literal meaning. Namely, supersonic combustion can be defined as the combustion in supersonic flow, or as the combustion, whose velocity relative to unburned gases is higher than the sound speed of the unburned gases. According to the former definition, all combustion in supersonic flow is the supersonic combustion regardless of supersonic or subsonic combustion velocity. In this way, the subsonic combustion should be understood as the combustion in subsonic flow or in a medium at a standstill. Obviously, this definition does not address the physical substance of combustion processes. Because of the relativistic nature of movement, the flow velocity may change as the reference system changes. However, a physical process should not depend on a changeable reference system. The latter definition takes the relativistic nature of movement into account and addresses the physical substance of combustion processes. The former definition emphasizes the flow in general, and is related to a scope of combustion investigation.

So more strictly in the viewpoint of combustion, one should take the latter definition of the supersonic combustion. Namely, the supersonic combustion is a combustion where the velocity of a combustion zone relative to unburned gas is higher than the sound speed of the unburned gas. Thus the concepts of subsonic and supersonic combustion correspond to the concepts of deflagration and detonation. The difference between them lies in the fact that one is in accordance with wave combustion and the other is not only a wave combustion (constant area) but also a zone combustion (variable area). In this way, the knowledge obtained for the detonation combustion may be used as a reference in the supersonic combustion investigation.

To distinguish the above two kinds of definition, the former is named as the combustion in supersonic flow, and the latter the supersonic combustion[1)].

0.2 Mechanism of supersonic combustion propagation

Since disturbing speed could by no means be higher than the local sound speed, the mechanism of supersonic combustion propagation is different from that of the subsonic combustion sustained directly by combustion itself. In supersonic combustion, the source of the combustion must be at the upstream, and does not come directly from the combustion propagation itself. Even for real self-sustaining detonation waves the combustion is caused by sustaining upstream shock waves. Shock waves may be induced by the combustion, and may exist in flow fields. Surely, the supersonic combustion should be self-ignition processes without external ignition actions, such as shock waves and high speed particles produced by combustion. That is the reason why supersonic combustion experiments need high enthalpy conditions. In other words, to realize a supersonic combustion, the combustion condition must be satisfied before the combustion or ignition sources located continuously at upstream. Figuratively speaking, the supersonic combustion is being pulled in its propagation by an action outside the combustion.

For subsonic combustion, the combustion can be supported by the combustion itself, and also by an upstream resource. Figuratively speaking, the subsonic combustion can be pulled (piloted) or pushed by the combustion itself.

1 The nature of one-dimensional combustion

1.1 Characteristics of steady combustion in one-dimensional pre-mixture flow

This subject is a simplified classical model presented in literature. Some of the results are merely summarized here.

Some basic relations between parameters are as follows for constant specific heat and molecular weight without friction and mass change[6]:

$$\frac{(M^2-1)}{\gamma M^2}\frac{\mathrm{d}P}{P} = -\frac{\mathrm{d}A}{A} + \frac{\mathrm{d}T_t}{T}, \tag{1}$$

$$(M^2-1)\frac{\mathrm{d}M}{M} = \left(1+\frac{\gamma-1}{2}M^2\right)\frac{\mathrm{d}A}{A} - \frac{\gamma M^2+1}{2}\frac{\mathrm{d}T_t}{T}, \tag{2}$$

$$\frac{\mathrm{d}P_t}{P_t} = -\frac{\gamma}{2}M^2\frac{\mathrm{d}T_t}{T_t}, \tag{3}$$

1) Here the combustion velocity (speed) or burning velocity is relative to the unburned gas.

where A, P, T, P_t and T_t are section area, pressure, temperature, total pressure and total temperature, respectively.

$$T_t = T\left(1 + \frac{\gamma - 1}{2}M^2\right), \quad P_t = P \cdot T_t^{[\gamma/(\gamma-1)]},$$
$$\mathrm{d}T_t = \mathrm{d}Q/c_p,$$

where Q is the heat addition per unit mass of fluid due to combustion.

So we come to a conclusion that: There is a factor $(M^2 - 1)$ in the above equations, so the distinction between supersonic combustion and subsonic combustion is clear. $M > 1$ is supersonic combustion, and $M < 1$ is subsonic combustion.

Since area variable is independent of heat addition, it is difficult to integrate the equations in a general case for arbitrary area variation, except for some special processes, for example, for constant pressure ($\mathrm{d}P = 0$), constant area ($\mathrm{d}A = 0$), or constant Mach number ($\mathrm{d}M = 0$) processes. However, the common and different characteristics for the supersonic combustion and subsonic combustion can be found from above equations also.

The common characteristics include: (i) for a constant pressure combustion ($\mathrm{d}P = 0$), the section area should increase. (ii) For a constant area combustion ($\mathrm{d}A = 0$), it is necessary to make M approach 1, which may induce choke. (iii) When Mach number is close to 1, the variation of Mach number becomes very sensitive to heat addition.

The different characteristics will be described in the next paragraph.

1.2 The differences between supersonic combustion and subsonic combustion.

According to the above equations, some differences are shown in table 1.

Table 1 The differences between supersonic combustion and subsonic combustion

	Flame propagation	Pressure	Fluid velocity away from flame	Increased area	Total pressure loss
Supersonic combustion	pull	increase ↑	M ↓, →1	M ↑	big
Subsonic combustion	pull or push	decrease ↓	M ↑, →1	M ↓	small

Supersonic combustion causes the pressure to increase and the Mach number to decrease towards the limit value of 1. Meanwhile, Mach number increases with the increase of the section area. On the contrary, for subsonic combustion, the tendency of the above variation is just the opposite. Subsonic combustion causes the pressure to decrease, and the Mach number to increase towards the limit value of 1, and Mach number to decrease with the increase of the section area.

Since the total pressure loss caused by heating is directly proportional to an exponential function of Mach number squared, the loss by supersonic combustion is much larger than that by subsonic combustion. However, the total pressure loss by practicable diffusion from a hypersonic state to a subsonic state should be much higher than that by supersonic combustion.

1.3 Supersonic combustion in a constant area duct

Assume that two parallel steady supersonic flows enter into a duct of constant area and react with each other when mixing. Within a distance the exothermic reaction is completed, as shown in figure 1

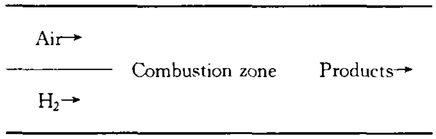

Fig. 1. Supersonic combustion controlled by mixing within a constant area duct

When this case is treated as the one-dimensional combustion, the parameters at each position are taken by the average across the section. The combustion is controlled by mixing, so it takes place gradually. Without any shock wave and choke, the parameters are continuous, so the above equations can be integrated. When shock waves appear, the parameters will become discontinuous. Thus it will be more difficult to integrate those equations. Therefore, integral equations can be employed to leave out internal flow details. The average parameters on each section satisfy conservation equations of mass, momentum and energy. Taking the average parameters before combustion at the entrance as the initial parameters, we can get two steady solutions according to the classical combustion theory. The one without shock waves corresponds to the weak detonation branch. After combustion, the flow remains to be supersonic, so the flow is not able to be affected by disturbance from down stream. Strehlow pointed out that weak detonation is supersonic combustion[7]. The other one with shock waves corresponds to the strong detonation branch. The flow after combustion is subsonic, so the flow can be affected by disturbance from downstream.

From the classical detonation theory, we know that the amount of heat-addition permitted is limited by the Mach number in a supersonic constant area combustor. The higher the Mach number is, the greater the amount of heat addition will be. In other words, in a constant area supersonic combustor with low Mach number, it is difficult to increase the amount of heat addition.

Since the entropy increment and the total pressure loss are the minimum for a constant area combustor under C-J condition, the C-J condition is the optimal one in supersonic combustion[8]. With or without shock waves supersonic combustion can approach the C-J condition[1)].

There has been no literature on an arbitrary contour combustor at the moment. So far, Billig[9] investigated a type of supersonic combustors, and came to the conclusions that the total pressure loss with shock waves was less than one without shock waves. So the loss induced by shock waves in supersonic combustion is not detrimental.

2 A simplified combustion flow field in supersonic flow

2.1 Subsonic combustion in supersonic flow

Let a steady supersonic flow pass over an ideal wedge. The wedge's tip only serves as an ignition source. It is assumed that an idealized subsonic combustion wave is formed on the tip of the wedge. Since the wall controls the flow direction, the flow will be compressed and an oblique shock wave will form at the upstream of the combustion wave as shown in figure 2 (where the angle of the wedge is set to 0).

That is an idealized subsonic combustion in supersonic flow, where the combustion takes place in zone 2 compressed by the shock wave. The temperature and pressure in zone 2 will increase to some extent, but will not be enough to induce combustion in this part. The combustion

1) The C-J detonation means a detonation satisfying the Chapman-Jouguet condition

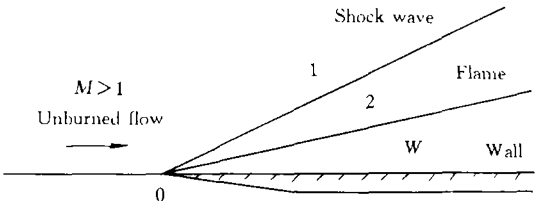

Fig. 2. Idealized planar subsonic combustion in supersonic flow.

velocity is determined by the physical and chemical properties and states of the gaseous flow (including species, temperature, pressure and turbulent statuses). Since the velocity does not have a consequential relation with the amount of heat addition, it may be treated as a variable to investigate its influence on the states of the products.

When the state of coming flow, the amount of heat addition, the angle of the wedge and the combustion velocity are given, this flow field can be solved.

To simplify the calculation, the following assumptions are made as usual in the one-dimensional combustion calculation: steady, inviscid, and non-heat conducting flow; ideal gas behavior with constant c_p; combustion be equivalent to heat addition; neglect the momentum change along wave tangent direction.

In fig. 2, it is shown that the whole flow field is divided into three zones by two discontinuous lines, and the direction of the product flow is parallel to the wall. So the flow field can be determined.

The amount of heat addition and unknown parameters are measured by the static enthalpy and the parameters of the coming flow, respectively:

$$q = Q/c_pT_1,\ P = P_w/P_1, M_f = u_f/a_2, v = \rho_1/\rho_w,\ q_f = qT_1/T_2,$$

where u_f is the combustion velocity, M_f is the combustion velocity measured by the sound speed in front of combustion.

The condition for stable shock waves is $\beta \leqslant 90°$. The steady subsonic combustion condition is

$$M_f^2 \leqslant 1 + q_f + \gamma q_f - \sqrt{(1+\gamma)q_f(2 + q_f + \gamma q_f)}.$$

The right side is a decreasing function of q_f. Thus, the larger the q_f is, the less the maximum value $M_{f\max}$ of M_f will be. For example, when $q_f = 5$, $\gamma = 1.4$, $M_{f\max} \cong 0.196$.

Some calculation results are shown in figs. 3 and 4, where both the solutions of oblique shock waves and of combustion waves take the value of the weak solutions and the angle of the wedge takes temporally the value of zero.

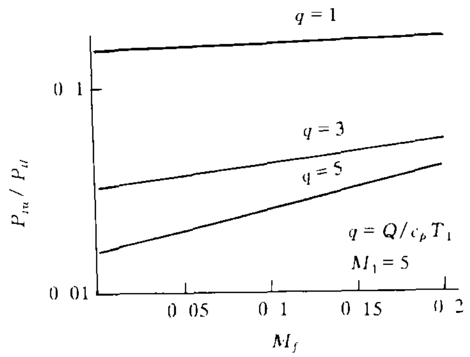

Fig. 3. Total pressure vs. combustion velocity.

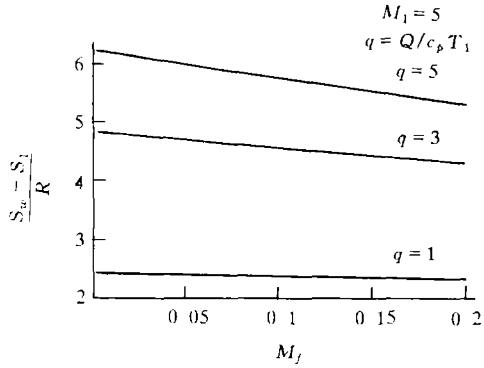

Fig. 4. Entropy vs. combustion velocity.

These figures show the variation of the total pressure and entropy increment versus the combustion velocity under the condition of flow direction controlled by a flat plate: the total pressure loss and the entropy increment increase with the increase of the amount of heat addition; the total pressure loss and the entropy increment decrease with the increase of the combustion velocity. The latter conclusion seems to be contradictory to the combustion theory. The reason is that the shock waves ahead of the combustion waves may not be neglected under many boundary conditions.

2.2 Supersonic combustion in supersonic flow

For the above flow field, when the combustion velocity $M_f > 1$ (namely for supersonic combustion), combustion waves will catch up with the shock waves and the structure of the flow field will change. The zone 2 vanishes, therefore $M_f = u_f / a_1$. Since the flow direction of the product compressed by the supersonic combustion has already deflected upwards, the flow will pass through a Prandtl-Meyer expansion fan before being parallel to the wall as shown in fig. 5. Since the expansion waves are treated as an isentropic process, only the solutions of the combustion waves will be dealt with here.

Steady supersonic combustion condition is

$$M_f^2 \geqslant 1 + q + \gamma q + \sqrt{(1+\gamma) q (2 + q + \gamma q)} \text{ and } \beta \leqslant 90^\circ .$$

For a given q, M_f has the minimum value M_{cj}.

According to the combustion theory, when $M_f > M_{cj}$, we can obtain two solutions:

(1) The weak solution is one without shock waves and with the supersonic normal velocity behind the combustion wave. The head of expansion waves is located at the downstream of the combustion wave, as shown in fig. 5. The combustion waves are not controlled by the wall of wedge (or flat plate).

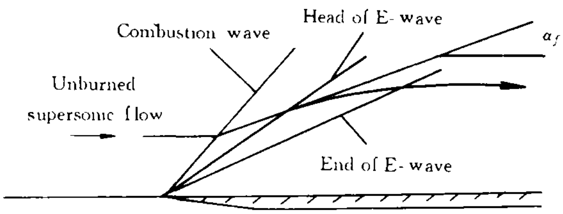

Fig 5 The structure of supersonic combustion flow field at a moment.

(2) The strong solution is one with shock waves and with subsonic normal velocity behind the combustion wave. The head of expansion waves coincides with the combustion wave. The combustion waves are controlled by the wall of wedges.

For both weak and strong solutions, the entropy increment of the combustion products increases with the increase of combustion velocity[10]. Therefore, the velocity of spontaneous supersonic combustion waves will decrease to that of the C-J detonation wave, if it is higher than the C-J velocity.

2.3 The concept of an oblique detonation

An oblique detonation wave is induced by an oblique shock wave, which is formed by a wedge in an incoming unburned mixture at a supersonic velocity. One assumes that the flow angle deflected behind the wave is equal to the wedge angle. Thus the relation between the detonation wave angle and the wedge angle can be determined as shown in figure 6[1,11].

The weak underdriven oblique detonation branch poses some questions. Why does the wave angle increase with the decrease of wedge angle? This branch is corresponding to the weak detonation branch. The increase of the wave angle means the increase of the combustion velocity. According to combustion theory, the compression amount on a weak detonation branch decreases with the increase of combustion velocity increase, which leads to the decrease of the flow deflected angle. The curve of the branch shows only the relationship between the combustion velocity (or wave angle) and the flow deflected angle. However, as mentioned in the above paragraph, the wedge angle does not have a consequential relation with the flow deflected angle. The small wedge angle may not be able to control the flow deflected angle for the underdriven branch. Therefore, the curve does not necessarily show the relationship between the wave angle and the wedge angle.

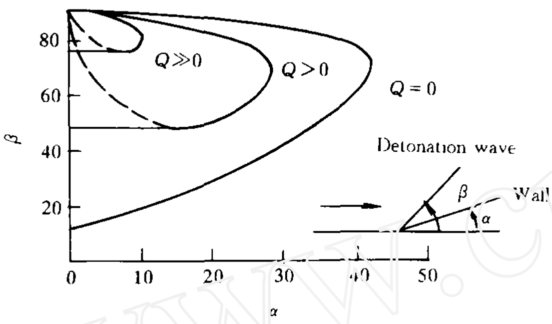

Fig. 6. Oblique detonation wave angle vs. wedge angle. The up-part of an oblique detonation curve is a strong solution, and the under-part a weak solution. Along the weak solution curve, the point, where $\alpha > \alpha_{cj}$, is a weak overdriven solution, whereas a weak underdriven solution.

The C-J detonation wave is a steady supersonic combustion wave with constant velocity, at which the flow deflected angle is symblized as α_{cj}. Under the underdriven condition the wedge angle $\alpha < \alpha_{cj}$, so the wave angle can be independent of the wedge angle. Thus a weak underdriven oblique detonation obtainable in practice may become the C-J oblique detonation with a constant wave angle as shown by the horizontal solid line in fig. 6. In this case, the head of expansion waves is coincidentally parallel to the detonation wave.

In the case of weak overdriven oblique detonation, the normal velocity after combustion is subsonic, so the wave angle is controlled by the wedge angle.

3 Conclusions

The design of scramjet as an engineering research project requires a better understanding of some fundamental concepts and features of supersonic combustion, which is even essential for the optimization of combustion performance of scramjets.

Some basic combustion patterns in supersonic flow have been obtained with idealized combustion flow analysis. There are some essential differences between supersonic and subsonic combustion, so the results obtained for subsonic combustion could not readily be used in supersonic combustion.

Real combustion phenomena in supersonic flow are very complicated. Owing to the limitation of experimental facilities and simulation conditions, some physical and chemical features of combustion under hypersonic flow condition have not been fully understood. Some old concepts may need to be updated. Supersonic combustion is not only technical problems, but also special theoretical problems, as far as the hypersonic aerodynamics and combustion are coupled. There are many unknowns and issues in supersonic combustion, which pose as a very big challenge. The investigation of supersonic combustion should begin with a study on basic phenomena by experimental and theoretical methods, only by which, a better understanding of supersonic combustion is possible.

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