

Vapour Recoil Effect on a Vapour–Liquid System with a Deformable Interface *

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A new two-sided model of vapour–liquid layer system with a deformable interface is proposed. In this model, the vapour recoil effect on the Marangoni–Bénard instability of a thin evaporating liquid layer can be examined only when the interface deflexion is considered. The instability of a liquid layer undergoing steady evaporation induced by the coupling of vapour recoil effect and the Marangoni effect is analysed using a linear stability theory. We modify and develop the Chebyshev–Tau method to solve the instability problem of a deformable interface system by introducing a new equation at interface boundary. New instability behaviour of the system has been found and the self-amplification mechanism between the evaporation flux and the interface deflexion is discussed.

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Evaporation process has attracted a great deal of attention because of various kinds of industrial applications, such as thin-film evaporators, boiling equipments and heat pipes. When temperature drop across the liquid layer exceeds a critical value, the interfacial instability results in evaporative convection where a flat surface becomes undulated.

In previous works,^[1,2] most studies on the Marangoni–Bénard instability were carried out for single liquid phase systems, in which the vapour phase adjacent to the liquid layer was considered to be passive. In this case, the dynamics of the thermal and mechanical perturbations in the vapour can be neglected. Miller^[3] examined the instabilities of an isothermal evaporating interface associated with a moving boundary. Burelbach *et al.*^[4] investigated the nonlinear stability of evaporating and condensing liquid films. Vapour recoil, thermocapillary and rupture instabilities are discussed in their works. In general, the local surface depressions can be produced by the rapidly departing vapour and this mechanism is called the ‘differential vapour recoil’. Recently, Ozen and Narayanan^[5] proposed a two-sided model that consists of a liquid layer and its own vapour. In their theoretical analysis, the Marangoni–Bénard instabilities of the system only at onset of evaporating were considered. In that case, the evaporative flux of unperturbed state was assumed to be zero, and the differential vapour recoil effect could not be taken into account. In Ozen and Narayanan’s model,^[5] the Hertz–Knudsen equation^[6] was not introduced under the interface boundary conditions and the convective instability of the system in the case of non-evaporation can not be deduced when the evaporation accommodation coefficient is zero. Zhang *et al.*^[8] and Chai and Zhang^[9] experimentally studied the effects of liquid

evaporation on the Marangoni–Bénard convection in thin liquid layers evaporating at room temperature.

In our previous works,^[10] we discussed the instabilities of the two-sided model with a non-deformable vapour–liquid interface. Since the classical Chebyshev–Tau method^[11] we used before can only solve the eigenvalue problem of the system without interface deflexion, we develop the Chebyshev–Tau method specifically for interface deflexion and solve the problem with interface deflexion accurately in the present study.

Here we propose a theoretical model of two-layer evaporating system with a deformable interface, as shown schematically in Fig. 1. The physical model consists of a liquid of depth d_l underlying its own vapour of depth d_v . Both the top and bottom walls are considered as rigid perfectly conducting boundaries. The top wall is assumed as a porous medium to vapour, through which the vapour phase can pass at a certain velocity. The phase change rate at the interface can be controlled by adjusting the vapour pressure.

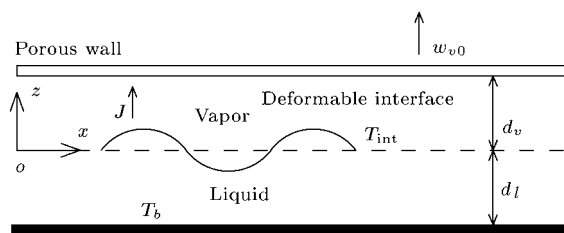


Fig. 1. Schematic diagram of a definite depth liquid–vapour layer system with a deformable interface.

In an unperturbed state, the liquid evaporates at a certain steady evaporating rate, and it is assumed that there is no convection in both the vapour layer

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and the evaporating liquid layer. However, the local evaporating velocity is not constant and it may change upon the perturbation. The interfacial tension at the interface is considered to be a linear function of temperature: $\sigma = \sigma_0 - \sigma_T(T - T_0)$, where T_0 is the reference temperature of interface. At the evaporating interface, the mass flux J is assumed to obey the Hertz–Knudsen equation:^[6,7]

$$J = \beta \sqrt{\frac{M}{2\pi RT}} (p_s(T) - p_0(T)), \quad (1)$$

where β is the evaporation accommodation coefficient; M is the molecular weight of vapour; $p_s(T)$ is the saturation pressure at surface temperature T ; $p_0(T)$ is the vapour pressure just beyond the interface; and R is the universal gas constant.

The governing equations for each fluid layer are the continuity equation, the energy equation and the Navier–Stokes equations with the Boussinesq approximation,^[12–14] i.e., only the densities ρ_i ($i = v, l$) are dependent on the temperature. The treatment adapted in the present study is different from that developed by Pearson,^[15] since a set of simplified equations with elimination of the horizontal velocity u and the pressure p were applied in Pearson’s analysis, whereas a set of original equations (2)–(9) are applied in our analysis. In our model, the boundary conditions applied at the deformable interface is much more complex than that with a flat interface. As the pressure and the interface deflexion variable are introduced in the normal stress balance condition at the interface, the pressure and the velocity can not be decoupled under the boundary conditions in the governing equation system of the problem, and all primary variables are used in the present analysis.

We introduce two-dimensional spatiotemporal normal perturbations proportional to $\exp[\lambda t + ikx]$ into the linearized full governing equations and boundary conditions (see Ref. [11]). The dimensionless perturbations are written as

$$[u'_i, w'_i, T'_i, \eta'] = [U_i(z), W_i(z), \Theta_i(z), \eta] \exp[\lambda t + ikx],$$

where u'_i, w'_i, T'_i , and η' with $i = v, l$ are the perturbations of the horizontal velocity, vertical velocity, temperature and interface deflexion; U_i, W_i, Θ_i , and η are the amplitudes of the horizontal velocity, vertical velocity, temperature and interface deflexion, respectively; λ is the complex time growth rate; and k is the real dimensionless wavenumber. We use $\nu_l/d_l, d_l^2/\nu_l, d_l$ and ΔT as the scaling factors for velocity, time, length and temperature, respectively. The depth ratio $h = d_v/d_l$. The dimensionless ratio of the physical properties are $\kappa^* = \kappa_v/\kappa_l$ (thermal diffusivity), $\beta^* = \beta_v/\beta_l$ (volumetric expansion coefficient), $\chi^* = \chi_v/\chi_l$ (thermal conductivity), $\mu^* = \mu_v/\mu_l$ (dy-

namic viscosity), $\rho^* = \rho_v/\rho_l$ (density) and $\nu^* = \nu_v/\nu_l$ (kinematical viscosity), respectively. The subscript v refers to the vapour, while l to liquid.

$$ikU_v + DW_v = 0, \quad (2)$$

$$\lambda\rho^*U_v + w_{v0}DU_v = -ikP_v + \nu^*\rho^*\nabla^2U_v, \quad (3)$$

$$\lambda\rho^*W_v + w_{v0}DW_v = -DP_v + \nu^*\rho^*\nabla^2W_v + \rho^*\beta^*\frac{Ra}{Pr}\Theta_v, \quad (4)$$

$$\lambda\Theta_v + \frac{dT_{v0}}{dz}W_v + w_{v0}D\Theta_v = \frac{\kappa^*}{Pr}\nabla^2\Theta_v, \quad (5)$$

$$ikU_l + DW_l = 0, \quad (6)$$

$$\lambda U_l = -ikP_l + \nabla^2U_l, \quad (7)$$

$$\lambda W_l = -DP_l + \nabla^2W_l + \frac{Ra}{Pr}\Theta_l, \quad (8)$$

$$\lambda\Theta_l + \frac{dT_{l0}}{dz}W_l = \frac{1}{Pr}\nabla^2\Theta_l, \quad (9)$$

$$D\eta = 0. \quad (10)$$

Boundary conditions at the top wall ($z = h$):

$$U_v = W_v = \Theta_v = 0, \quad (11)$$

and the bottom wall ($z = -1$):

$$U_l = W_l = \Theta_l = 0. \quad (12)$$

Boundary conditions at the interface ($z = 0$):

$$\rho^*(W_v - \lambda\eta) = W_l - \lambda\eta, \quad (13)$$

$$2J_0(W_l - W_v) + P_l - P_v - 2DW_l + 2\mu^*DW_v = \left[k^2 \frac{1}{PrCa} + (1 - \rho^*) + \frac{Ga}{Pr} \right] \eta, \quad (14)$$

$$Pr(DU_l + ikW_l) - Pr\mu^*(DU_l + ikW_l) = -ikMa \left(\Theta_l + \frac{dT_{l0}}{dz}\eta \right), \quad (15)$$

$$E(W_l - \lambda\eta) - \chi^* \frac{dT_v}{dz} + \frac{dT_l}{dz} = 0, \quad (16)$$

$$U_l = U_v, \quad (17)$$

$$\Theta_l + \frac{dT_{l0}}{dz}\eta = \Theta_v + \frac{dT_{v0}}{dz}\eta, \quad (18)$$

$$W_v - W_l = E_2[E_1^{-1}(\Theta_l + \frac{dT_{l0}}{dz}\eta) - P_v], \quad (19)$$

where D is the dimensionless differential operator d/dz , ∇^2 is the operator $D^2 - k^2$, and dT_{i0}/dz is the temperature gradient of fluid- i at the unperturbed state; w_{v0} is the dimensionless evaporation velocity of vapour leaving the interface in the unperturbed state. Ma is the Marangoni number defined as $\sigma_T\Delta T d_l/(\mu_l\kappa_l)$, where ΔT is the temperature differences in the liquid layer, Ra is the Rayleigh number defined as $Ra = g\beta_l d_l^3\Delta T/\kappa_l\nu_l$, Ca is the capillary number defined as $Ca = \mu_l\kappa_l/\sigma_0 d_l$, Ga is defined as

$Ga = gd_l^3/\nu_l\kappa_l$, E is defined as $E = \frac{\rho_l\nu_l\mathcal{L}}{\chi_l\Delta T}$, where here \mathcal{L} is the evaporation latent heat. E_1 is defined as $E_1 = \frac{\rho_l\nu_l^2T_0}{\rho_v h_l^2\mathcal{L}\Delta T}$, and E_2 as $E_2 = \beta\sqrt{\frac{M}{2\pi RT}\frac{\rho_l - \rho_v}{\rho_v}\frac{\nu_l}{h_l}}$. Pr is the Prandtl number of the liquid defined as ν_l/κ_l .

The linear equations (2)–(10) together with its boundary conditions (11)–(19) are discretized by using the spectral numerical method (Tau–Chebyshev).^[11] In addition, we extend the original method by using a additional equation to describe the interface deformation. For treatment of the boundary conditions at deformable interface, the relation (10) is introduced into the governing equations system of the problem, since the perturbed component of the interface deformation for a certain wavenumber is a constant. By introducing the relation (10), the number of the governing equations of system, 13, is the same as that of the corresponding boundary conditions. It is remarkable that after using the modified method, we can obtain completely the numerical results in the system with interfacial deformation.

The alcohol with its own vapour at 25°C is selected in the present study and the depth of the liquid layer is 1 mm. The ratios of physical properties (see Ref. [16]) and dimensionless numbers of the liquid–vapour system are $\nu^* = 34.7$, $\rho^* = 2.3 \times 10^{-4}$, $\chi^* = 8.5 \times 10^{-2}$, $\kappa^* = 6.1 \times 10^2$, $Pr = 14.9$, and $Ra = 0$, respectively. The depth ratio of vapour and liquid layers is $h = 1.0$. In present study, first we do not consider the buoyancy effect on the system when $Ra = 0$. The Rayleigh effect on the convection instability of the liquid–vapour system will be discussed later in the end of paper.

When the system is perturbed, it responds according to the dynamical equations. This responds may carry it still further from the original unperturbed state. The neutral curves of Marangoni–Bénard instability in the system with $\beta = 0$ have been shown in Fig. 2. It is obvious that at all wavenumbers the system without evaporation ($w_{v0} = 0$) is more stable than that with non-zero evaporation velocity at the free vapour interface ($w_{v0} \neq 0$). For example, the critical Marangoni number of the system in the case of $w_{v0} = 0$ is greater than that in the case of $w_{v0} = 1.0$. This means the vapour recoil effect can obtain the vapour–liquid Marangoni–Bénard system more unstable.

In fact, upon the perturbation the temperature profile along a deflected interface is not uniform (see in Fig. 1). When the vapour layer heated from the bottom, the temperature in the valley of the vapour surface is higher than that at the crest since the valley is close to the heat source (hot bottom wall). In the valley, the temperature is higher than the unperturbed reference temperature and the evaporation will take

place here at the perturbed state. At the crest where the temperature is lower than that of the reference state, the vapour will condense into its own liquid. A disturbance inducing a local increase in surface temperature will increase the local evaporation rate and decrease the local surface tension. The increasing of evaporation rate produces a local increase in normal force and at the same time it results a corresponding local depression at the interface. At this point of interface, the liquid becomes hotter and its evaporation rate becomes greater than before so the local interface will continue to depress. This process produces auto-amplification of the original disturbance and makes the vapour–liquid Marangoni–Bénard system more unstable.

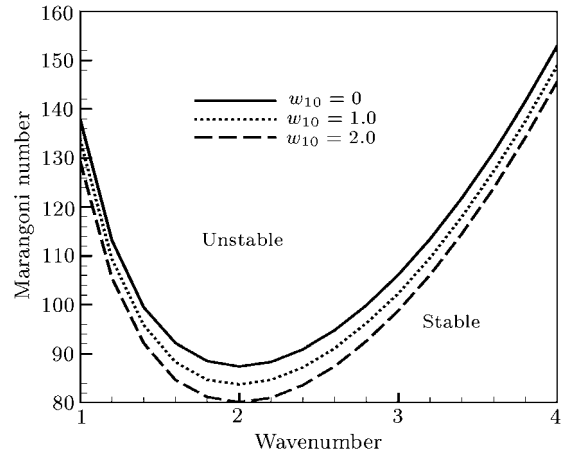


Fig. 2. Marangoni number of the liquid layer versus dimensionless wavenumber for different evaporation velocities with the evaporating coefficient $\beta = 0$.

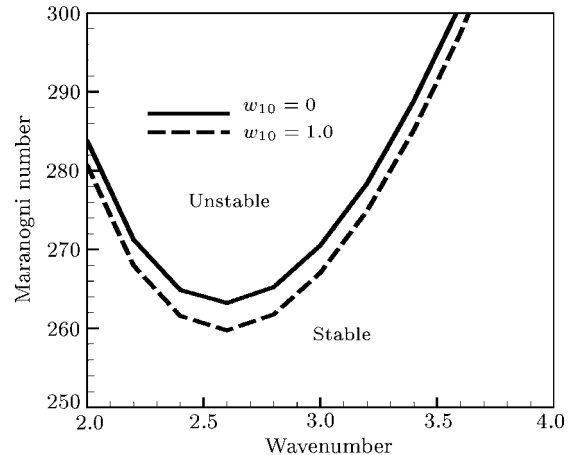


Fig. 3. Marangoni number of the liquid layer versus dimensionless wavenumber for different evaporation velocities with the evaporating coefficient $\beta = 0.001$.

In Fig. 3 when the accommodation evaporation ef-

ficient of the system, $\beta = 0$, is 0.001, the Marangoni-Bénard system with zero evaporation velocity ($w_{v0} = 0$) is more stable than that at $w_{v0} = 1.0$. Compared Fig. 2 to Fig. 3, we find that the system with $\beta = 0$ is more unstable than that with $\beta = 0.001$, at the same evaporating velocity.

In many laboratory situations, gravity plays an important role. The Rayleigh effect makes the system unstable. However, this effect can be avoid when the liquid layer is very thin. In this case, gravity force simply pulls the perturbed interface back to its original position, and the gravitational effect on the interface makes the system more stable. If the steady evaporation rate is high enough to overcome the stabilizing effect of the gravity and the cooling effect of the evaporation, the vapour recoil effect is capable to produce convection in the liquid layer. As we know, the accommodation evaporation efficient presents the cooling ability. The larger the value of β , the larger the temperature difference across the liquid layer needed to drive the convection in it.

In summary, new behaviour of the Marangoni-Bénard instability in a vapour-liquid layer system have been found in our present study when considering the coupling of interface deflexion to the Marangoni effect at the evaporating interface. The vapour recoil effect has destabilization mechanism to the system,

and the cooling effect of the evaporation has stabilization mechanism to the system.

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