Article ID: 0253-4827(2001)02-0210-10

IDENTIFICATION OF NONLINEAR DYNAMIC SYSTEMS: TIME-FREQUENCY FILTERING AND SKELETON CURVES *

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(Paper from ZHANG Jing-hui, Member of Editorial Committee, AMM)

Abstract: The nonlinear behavior varying with the instantaneous response was analyzed through the joint time-frequency analysis method for a class of S. D. O. F nonlinear system. A masking operator on definite regions is defined and two theorems are presented. Based on these, the nonlinear system is modeled with a special time-varying linear one, called the generalized skeleton linear system (GSLS). The frequency skeleton curve and the damping skeleton curve are defined to describe the main feature of the non-linearity as well. Moreover, an identification method is proposed through the skeleton curves and the time-frequency filtering technique.

Key words: system identification; nonlinear dynamic system; non-stationary signal; time-frequency analysis; Hilbert transform

CLC number: O322 **Document code**: A

Introduction

Identification of nonlinear systems has received considerable attention in recent years because most structures exhibit some degree of nonlinearity. Both parametric and non-parametric techniques have been studied intensively, such as the linearization, higher order spectra, restoring force surface, nonlinear auto-regressive moving average model (NARMAX), neural networks and so on. There seems that the choice of a particular algorithm is closely linked to the objectives of the analysis and that there are no universally applicable procedures[1]. In Refs. [2,3], the nonlinear system is analyzed with a time-varying linear model called the pseudo-linear one. And the backbone curve and instantaneous logarithmic decrement were obtained through the narrow band filtering technique together with the Hilbert transform. Ref. [4] deduced the relationship

* Received date: 2000-03-06; Revised date: 2000-09-19

Foundation item: the National Natural Science Foundation of China (19632001); the Doctor Research Foundation of Xi'an Jiaotong University; the Chinese Postdoctoral Science Foundation

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between the two systems by using a so-called band-pass mapping. But the narrow band filtering technique used in the previous references doesn't work in many cases because the response of the nonlinear system is not a narrow-band signal. Also the bandwidth lacks practical meaning and is difficult to be estimated.

It is well-known that the dynamic nature and behavior of nonlinear systems generally vary with the values of the response and excitation. And this information is usually contained in the non-stationary vibration signals. To study the nonlinearity by using the non-stationary signals, the quadratic time-frequency distribution of Cohen class is adopted due to its ability to express a signal in both time and frequency domain simultaneously.

1 Mathematical Basis

For non-stationary signals, classical methods based on the Fourier transform are not applicable. The Fourier spectrum gives no time-localized information but the total energy at a certain frequency. Thus the instantaneous dynamic characters of the signal can not be obtained. On the contrary, with the conception of time-frequency domain, the joint time-frequency analysis expresses a signal by a function with both time and frequency as the variables. The change with time can be described directly through this method. In particular, the quadratic time-frequency distribution of Cohen class defined as follows can indicate the energy density of a signal in the time-frequency plane roughly:

$$\rho_x(t,\omega) = \frac{1}{4\pi^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\phi(\theta,\tau)\, X^*\left(u-\frac{1}{2}\tau\right)\cdot X\left(u+\frac{1}{2}\tau\right) e^{-j\theta t-j\tau\omega+j\theta u}\,du\,d\tau\,d\theta, \tag{1}$$

where $X(t)$ is the analytic form of the real-valued signal, that is

$$X(t) = x(t) + j\tilde{x}(t),$$

where $\tilde{x}(t)$ is the Hilbert transform of $x(t)$

$$\tilde{x}(t) = \frac{1}{\pi}\, pv\int_{-\infty}^{+\infty}\frac{x(\tau)}{t-\tau}d\tau,$$

and $\phi(\theta,\tau)$ is the kernel function of the quadratic time-frequency distribution.

Generally the high time-frequency resolution and low cross-terms are the most important properties expected for a certain distribution. The Wigner-Vile distribution is the best one with consideration of the time-frequency resolution, but it is of a large cross-term. The quasi-Wigner distribution, the exponential distribution (also called the Choi-Williams distribution) and the cone-shaped kernel distribution are some practical distributions in common use. The distribution used in this paper is one with a exponential-cone-shaped kernel[5].

The asymptotic signal is an important concept that will be referred in this paper. It can be expressed as

$$x(t) = a(t)e^{j\phi(t)}, \tag{2}$$

where

$$\frac{1}{a(t)}\left|\frac{da(t)}{dt}\right| \ll \left|\frac{d\phi(t)}{dt}\right|.$$

In this section, the masking operator within a definite region and the effective time-frequency region of an asymptotic signal are defined and two important theorems are deduced. These provide mathematical basis for the work in section 2.

Definition **1** Suppose $\rho_x(t,\omega)$ is a certain quadratic time-frequency distribution of a continuous signal $x(t)$, and Ω is a finite closed region on the time-frequency plane. Let

$$\rho_x^{\Omega}(t,\omega) = \begin{cases} \rho_x(t,\omega), & (t,\omega) \in \Omega, \\ 0, & (t,\omega) \notin \Omega. \end{cases}$$

If the same quadratic time-frequency distribution of a signal $y(t)$, $\rho_y(t,\omega)$, approaches $\rho_x^{\Omega}(t,\omega)$ best, we call $y(t)$ the projection of $x(t)$ on Ω, and the mapping from $x(t)$ to $y(t)$ the time-frequency masking operator on definite region Ω, denoted by

$$M(\cdot,\Omega): x(t) \to y(t).$$

The masking operator on Ω can be taken as a filtering with pass region Ω in the time-frequency plane.

Definition **2** The effective time-frequency region of an asymptotic signal $x(t) = a(t)\cos\phi(t)$ is defined as

$$\Omega_x(t) = [\omega(t) - \Delta\omega(t)/2, \quad \omega(t) + \Delta\omega(t)/2], \tag{3}$$

where

$$\omega(t) = \dot{\phi}(t), \Delta\omega(t) = |\dot{a}(t)/a(t)|.$$

Theorem **1** Suppose $x(t)$ is an asymptotic signal with effective region Ω_x. The projection of a continuous function $f(x(t))$ on Ω_x can be expressed as

$$M[f(x(t)),\Omega_x] \approx \frac{1}{2\pi}\frac{1}{a(t)}\int_0^{2\pi} f[a(t)e^{j\phi}]\, e^{-j\phi}d\phi \cdot x(t). \tag{4}$$

Proof $f(x(t)) = f(a(t)e^{j\phi(t)})$ is a function of time t. a can be taken as a variable independent of ϕ because $a(t)$ varies slowly compared with $e^{j\phi}$. And thus it can be taken as a constant within any period of $e^{j\phi}$. So $f(x(t)) = f(a(t)e^{j\phi(t)})$ can be taken as a function of two independent variables, a and ϕ. Besides, it is a periodic function of ϕ. Taking the Fourier series expansion with ϕ

$$f[x(t)] = \sum_{k=-\infty}^{+\infty} C_k(t)e^{jk\phi(t)}, \tag{5}$$

$$C_k(t) = \frac{1}{2\pi}\int_0^{2\pi} f[a(t)e^{j\phi}]e^{-jk\phi}d\phi. \tag{6}$$

Let

$$\Omega^k(t,\omega) = \begin{cases} [0.5(2k-1)\omega(t), 0.5(2k+1)\omega(t)], & k = 1,2,3,\cdots, \\ [0, 0.5\omega(t)], & k = 0. \end{cases} \tag{7}$$

Obviously $\Omega_x(t,\omega) \subset \Omega^1(t,\omega)$, and the central line of the two regions coincides with each other. Because $C_k e^{jk\phi}$ is localized around the central line of $\Omega^k(t,\omega)$, the projection of $f[x(t)]$ on $\Omega_x(t,\omega)$ approaches the first term of the Fourier series, that is, $C_1(t)e^{j\phi(t)}$,

$$M[f[x(t)],\Omega_x] \approx M[f[x(t)],\Omega^1] \approx C_1(t)e^{j\phi(t)} = \frac{1}{2\pi}\frac{1}{a(t)}\int f[a(t)e^{j\phi}]e^{-jk\phi}d\phi \cdot x(t). \tag{8}$$

A continuous nonlinear function defined in a close region can be approximated by a polynomial function with arbitrary precision, so the power series plays an important role in the study of nonlinear dynamic systems. For the projection of the symmetric power series of an asymptotic signal on its effective time-frequency region, there yields

$$M[|x|^n \cdot \mathrm{sign}(x), \Omega_x] \approx \frac{2}{\sqrt{\pi}} \frac{\Gamma(n/2+1)}{\Gamma((n+1)/2+1)} a^{n-1} x, \tag{9}$$

where $\Gamma(\cdot)$ denotes the Γ function.

Theorem **2** Suppose $x(t)$ is an asymptotic signal with effective time-frequency region Ω_x. The projections of the first two order derivatives on Ω_x can be expressed as follows

$$M[\dot{x}(t), \Omega_x] \approx \dot{x}(t), \quad M[\ddot{x}(t), \Omega_x] \approx \ddot{x}(t). \tag{10}$$

Proof

$$\dot{x}(t) = [\dot{a}(t) + \mathrm{j}\, a(t)\omega(t)]\mathrm{e}^{\mathrm{j}\phi(t)}, \tag{11}$$

$$\ddot{x}(t) = [\ddot{a}(t) - a(t)\omega^2(t) + 2\mathrm{j}\omega(t)\dot{a}(t) + \mathrm{j}\, a(t)\dot{\omega}(t)]\mathrm{e}^{\mathrm{j}\phi(t)}. \tag{12}$$

If $a(t)$ and $\phi(t)$ are smooth and non-oscillating signals, the terms in the brackets are slow signals compared with $\mathrm{e}^{\mathrm{j}\phi(t)}$. So the instantaneous frequency of both $\dot{x}(t)$ and $\ddot{x}(t)$ equals to $\omega(t)$. Thus $\dot{x}(t)$ and $\ddot{x}(t)$ are all located around Ω_x. Then the desired results hold.

2 Time-Frequency Filtering and the General Skeleton Linear System

Consider the following S.D.O.F. autonomous nonliner system

$$m\ddot{y} + F(y, \dot{y}) = 0. \tag{13}$$

The response is an asymptotic signal with a constant frequency if the system is linear. Generally the response is no longer an asymptotic signal but contains multiple components if there is nonlinearity in the system. But for weak nonlinear systems and several piecewise linear systems, one asymptotic component is dominant. We call this asymptotic component the principal component in this paper, and express it as

$$x(t) = a(t)\cos[\phi(t)], \tag{14}$$

while the instantaneous amplitude, $a(t)$, and instantaneous frequency,

$$\omega(t) = \mathrm{d}\phi(t)/\mathrm{d}t, \tag{15}$$

be calculated as follows

$$a(t) = \sqrt{x^2(t) + \tilde{x}^2(t)}, \tag{16}$$

$$\omega(t) = \frac{x(t)\dot{\tilde{x}}(t) + \tilde{x}(t)\dot{\tilde{x}}(t)}{x^2(t) + \tilde{x}^2(t)}, \tag{17}$$

$a(t)$ and $\omega(t)$ are both variables varying with time slowly.

The response signal of (13) can be expressed as

$$y(t) = x(t) + z(t), \tag{18}$$

where $z(t)$ is the sum of the other components including the sub-harmonic components and the super-harmonic components of $x(t)$. $z(t)$ and its derivatives are of much lower energy compared with $x(t)$ and its derivatives.

Denote the effective time-frequency region of $x(t)$ as Ω_x. Considering the projection of Eq. (13) on Ω_x with the contributions of $z(t)$ and its derivatives be neglected, one has

$$M(m\ddot{x}, \Omega_x) + M(F(x, \dot{x}), \Omega_x) = 0. \tag{19}$$

According to Theorems 1 and 2, $x(t)$ satisfies the following differential equation approximately

$$\ddot{x} + 2h_0\dot{x} + \omega_0^2 x = 0, \tag{20}$$

where

$$\omega_0 = \left[\frac{1}{\pi m a}\int_0^{2\pi} F(a\cos\phi, -a\omega\sin\phi)\cos\phi \mathrm{d}\phi\right]^{1/2}, \tag{21}$$

$$h_0 = -\frac{1}{2\pi m a\omega}\int_0^{2\pi} F(a\cos\phi, -a\omega\sin\phi)\sin\phi \mathrm{d}\phi. \tag{22}$$

This is a time-varying linear system with slowly varying instantaneous parameters ω_0 and h_0. The response equals to the principal component of that of system (20) approximately. And the variation of the instantaneous parameters with the instantaneous response can indicate the main nonlinear property of system (13). So the time-varying linear system is called the general skeleton linear system of system (13) in this paper, denote by GSLS for short, while ω_0 and h_0 are called the instantaneous undamped inherent frequency and the instantaneous decay coefficient of GSLS, respectively.

For a S. D. O. F. nonlinear non-autonomous system excited by an asymptotic signal, when the harmonic component is dominant in the response, GSLS has the same form as (20) - (22). It should be noted that (20) - (22) no longer works if the harmonic component is not dominant.

GSLS developed above has the same expression as the equivalent linear system deduced through the classical methods such as the KBM. The concept of GSLS is adaptable to the non-stationary vibrations of both linear and nonlinear systems. Moreover, the response signal of GSLS can be obtained from that of the corresponding nonlinear system with the time-frequency filtering technique. So GSLS together with skeleton curves discussed in the next section can be identified directly with just one sample of non-stationary vibration data. As time-varying signals, the responses of GSLS can not be obtained through common methods based on the Fourier transformation.

3 The Skeleton Curves

The regressive curve $\omega_0(a, \omega)$ and $h_0(a, \omega)$ can describe the nonlinear stiffness and damping of system (13) in visual forms, respectively. The main nature of the nonlinearity can be learned immediately at a glance of them. So we call them the frequency skeleton curve and the damping skeleton curve, respectively. Generally, the skeleton curves are both three-dimensional curves. But for several special systems, they have more concise forms.

Consider a system with the following form

$$\ddot{y} + P(y) + Q(\dot{y}) = 0, \tag{23}$$

where $P(y)$ and $Q(\dot{y})$ are real-valued odd functions. The instantaneous parameters of GSLS are

$$\omega_0(a, \omega) = \omega_0(a) = \left[\frac{1}{\pi a}\int_0^{2\pi} P(a\cos\phi)\cos\phi \mathrm{d}\phi\right]^{1/2}, \tag{24}$$

$$h_0(a,\omega) = h_0(a\omega) = -\frac{1}{2\pi a\omega}\int_0^{2\pi} Q(-a\omega\sin\phi)\sin\phi d\phi. \tag{25}$$

The skeleton curves can be expressed by two planar curves, $\omega_0(a)$ and $h_0(a\omega)$. Obviously, the skeleton curves of a linear system are both horizontal straight lines. There exists nonlinearity in stiffness or damping whenever the corresponding skeleton curve is not a horizontal straight line.

Suppose that $P(\cdot)$ and $Q(\cdot)$ can be asymptotically expanded with polynomial functions, then Eq. (23) becomes

$$\ddot{y} + \sum_{j=1}^{n} c_i |\dot{y}|^j \cdot \text{sign}(\dot{y}) + \sum_{i=1}^{m} k_i |y|^i \cdot \text{sign}(y) = 0, \tag{26}$$

with the skeleton curves

$$\omega_0(a) = \left[\sum_{i=1}^{m} \frac{2}{\sqrt{\pi}} \frac{\Gamma(i/2+1)}{\Gamma((i+1)/2+1)} k_i \cdot a^{i-1}(t)\right]^{1/2}, \tag{27}$$

$$h_0(a\omega) = \sum_{j=1}^{n} \frac{1}{\sqrt{\pi}} \frac{\Gamma(j/2+1)}{\Gamma((j+1)/2+1)} c_{ji} \cdot [a(t)\omega(t)]^{i-1}. \tag{28}$$

4 Identification of Skeleton Curves

In this section we discuss the identification of the skeleton curves concisely. The process includes three steps as follows.

4.1 Choice of the excitation signal

First choose an appropriate excitation to guarantee that Eqs. (20) - (22) hold. An impact signal can be used but it is with poor precision for systems with large damping, because the effective data number is limited due to the quick decay of the response. To avoid this shortage a forced response can be adopted. To guarantee that the harmonic component is dominant in the response the instantaneous frequency of the excitation should approach the resonance frequency. Besides, the forced response signal may be of large amplitude even by using excitation with small energy in the primary resonance case. A prior knowledge about the system is needed for the identification in this case.

4.2 Extraction of the response signal of GSLS

Because the asymptotic signal $x(t)$ is dominant in $y(t)$, Ω_x can be estimated using the modulus of $\rho_y(t,\omega)$. $|\rho_y(t,\omega)|$ takes the maximum value at $\omega(t)$ at any moment t. With a cut off point $\alpha(0 < \alpha < 1)$, we can take the maximum zone with midline $\omega(t)$ as the estimation of Ω_x which satisfies

$$|\rho_y(t,\omega)| \geq \alpha |\rho_y(t,\omega(t))|.$$

Then $x(t)$ can be extracted from $y(t)$ using the time-frequency masking operator on Ω_x. Because

$$x(t) \approx M(y(t), \Omega_x), \tag{29}$$

where $M(\cdot,\cdot)$ denotes the time-frequency masking operator on a definite region.

$$\rho_x(t,\omega) = \begin{cases} \rho_y(t,\omega), & (t,\omega) \in \Omega_x, \\ 0, & (t,\omega) \notin \Omega_x. \end{cases} \tag{30}$$

Taking the inverse transform of $\rho_x(t,\omega)$, one yields

$$x(\tau) = \frac{1}{2\pi \cdot x^*(0)} \iiint \frac{\rho_x(t,\omega)}{\phi(\theta,\tau)} e^{j t\omega + j\theta(t-\tau/2)} \mathrm{d}t\mathrm{d}\omega\mathrm{d}\theta, \tag{31}$$

where $\phi(\theta,\tau)$ is the kernel of the quadratic time-frequency distribution of Cohen class.

There are singularities in (31) that will bring difficulties in calculation. In Ref. [6] another time-frequency filtering technique based on the wavelet transform along the wavelet ridge is proposed.

It needs special notice that most amount of the random noise in the response is filtered out after the previous procedure because Ω_x is a narrow band in the time-frequency plane. So the method proposed here is excellent with consideration of precision.

4.3 Calculation of the instantaneous parameters of GSLS

For GSLS, a special time-varying linear system whose instantaneous parameters are slowly varying functions of time, the parameters can be expressed by analytic functions of the response as follows[2,3,6]

$$\ddot{x}(t) + 2h_0(t)\dot{x}(t) + \omega_0^2(t)x(t) = m^{-1}u(t), \tag{32}$$

the instantaneous parameters can be calculated as follows

$$\omega_0^2(t) = \omega^2(t) + \frac{\alpha(t)}{m} - \frac{\beta(t)\dot{a}(t)}{ma(t)\omega(t)} - \frac{\ddot{a}(t)}{a(t)} + 2\frac{\dot{a}^2(t)}{a^2(t)} + \frac{\dot{a}(t)\dot{\omega}(t)}{a(t)\omega(t)}, \tag{33}$$

$$h_0(t) = \frac{\beta(t)}{2m\omega(t)} - \frac{\dot{a}(t)}{a(t)} - \frac{\dot{\omega}(t)}{2\omega(t)}, \tag{34}$$

where

$$\alpha(t) = \frac{x(t)u(t) + \tilde{x}(t)\tilde{u}(t)}{x^2(t) + \tilde{x}^2(t)},$$

$$\beta(t) = \frac{x(t)\tilde{u}(t) - \tilde{x}(t)u(t)}{x^2(t) + \tilde{x}^2(t)},$$

$\tilde{u}(t)$ and $\tilde{x}(t)$ are the Hilbert transform of $u(t)$ and $x(t)$, respectively. It is the case of free vibration when α and β equals to zero.

Thus the skeleton curves can be plotted directly once the instantaneous response and instantaneous parameters of the GSLS are calculated.

5 Examples

Two numerical examples are considered to verify the validity of the technique developed in this paper.

Example **1** Consider the following system

$$\ddot{y} + c_1\dot{y} + c_2|\dot{y}|\dot{y} + k_1 y + k_2 y^3 + k_3 y^5 = u, \tag{35}$$

where

$$k_1 = 5\pi^2,\ k_2 = 3\times 10^6\pi^2,\ k_3 = 2\times 10^9\pi^2,\ c_1 = 0.05,\ c_2 = 0.2.$$

This is a polynomial system with hard spring, viscous damping and square damping. The skeleton curves are

$$\omega_0(a) = [k_1 + (3/4)k_2a^2 + (5/8)k_3a^4]^{1/2}, \tag{36}$$

$$h_0(a\omega) = c_1 + (4/3\pi)c_2 a\omega. \tag{37}$$

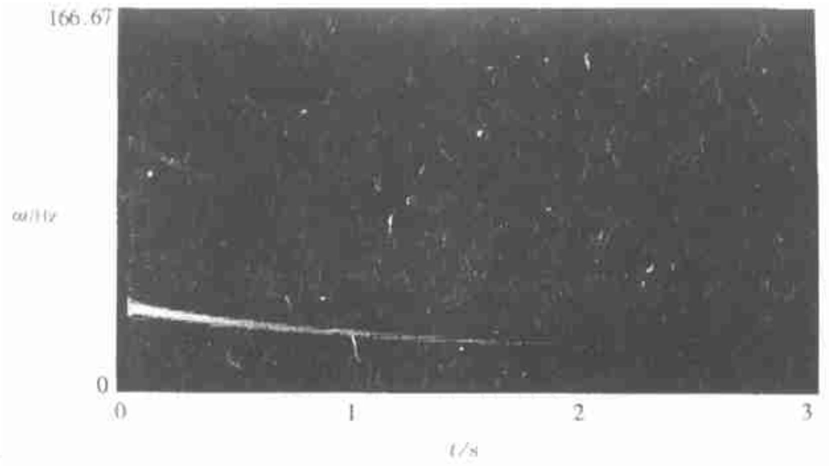

Fig. 1 Grayscale view of the quadratic time-frequency distribution of the acceleration in example 1

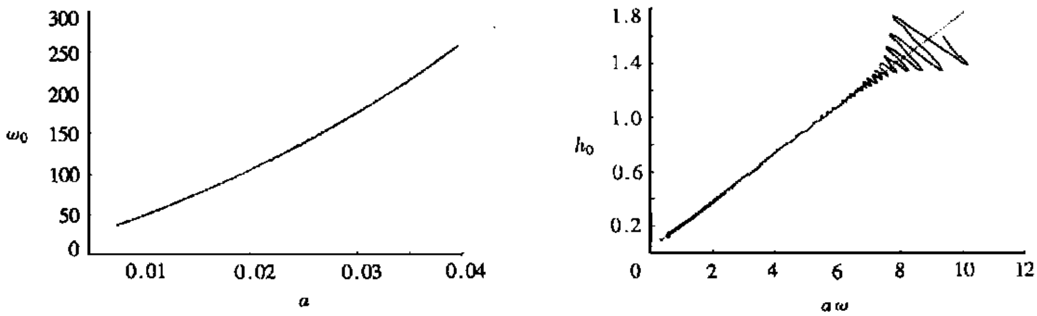

Fig. 2 The frequency and damping skeleton curves of example 1

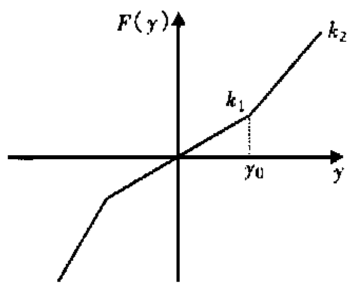

Fig. 3 The resilience versus displacement in a system with bilinear stiffness

The 4th-order Runge-Kutta method is used in the numerical calculation and a white noise is added to the response with consideration of the noise in actual measurements. The free vibration response is used with the following initial condition

$$y(0) = 0.02, \quad \dot{y}(0) = 0, \quad \ddot{y}(0) = 0.$$

The grayscale view of the quadratic time-frequency distribution of the acceleration within 0～3.07s is shown in Fig. 1. The abscissa represents the time (0～3.07s) while the ordinate denotes the frequency (0～166.67Hz). Skeleton curves identified (solid line) and theoretical predicted (dashed line) through Eqs. (36) and (37) at 0～20.48s are shown as Fig. 2. In this case, the instantaneous frequency of the principal component varies from 35.55Hz to 6.25Hz, and that of the third-harmonic component varies from 106.65Hz to 18.75Hz. So the principal component can not be extracted from the total response signal through the narrow-band filtering presented in Refs. [2 - 4] because of the overlapping.

Example **2** Consider the following system

$$\ddot{y} + c\dot{y} + M \cdot \text{sign}(\dot{y}) + F(y) = u, \tag{38}$$

where $F(y)$ is the resilience of a bilinear spring shown as Fig. 3. The system is with viscous damping and dry friction.

$$k_1 = 1\,600\pi^2,\ k_2 = 2\,500\pi^2,\ y_0 = 0.5,\ c = 0.8\pi,\ M = 60\pi.$$

Now we use the primary resonance for identification. Take the following excitation

$$u(t) = 500(1 + t)\cos\left[2\pi\left(20 + \frac{5}{10.24}t\right)t\right].$$

Although this system is with high level of nonlinearity in the common point of view, the response is an asymptotic signal approximately. And the instantaneous frequency of the response approaches that of the excitation. So the method developed in this paper is valid here

$$\omega_0(a) = \begin{cases} \left[k_2 + \dfrac{2(k_1 - k_2)}{\pi}\left(\arcsin\left(\dfrac{y_0}{a}\right) + \dfrac{y_0}{a}\sqrt{1 - \dfrac{y_0^2}{a^2}}\right)\right]^{1/2}, & a \geq y_0, \\ k_1, & a < y_0, \end{cases} \tag{39}$$

$$h_0(a\omega) = c/2 + 2\pi^{-1}M \cdot [a\omega]^{-1}. \tag{40}$$

Both the skeleton curves identified (solid line) and predicted (dashed line) are plotted in Fig. 4. It can be seen that the identified results are in good agreement with the theoretical predictions.

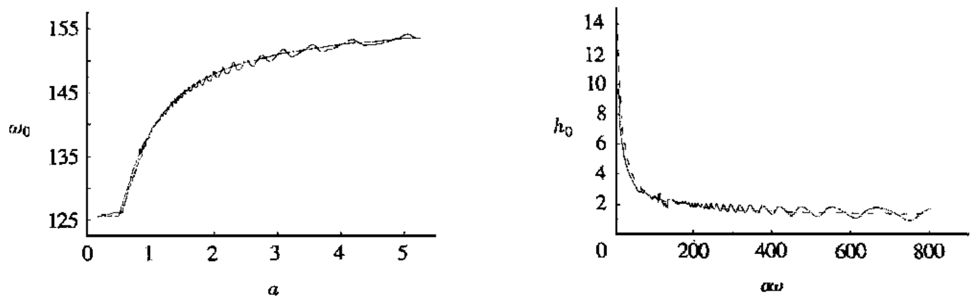

Fig. 4 The frequency and damping skeleton curves of example 2

6 Conclusions

In general, the behavior of a nonlinear system varies with the value of the instantaneous response. The nonlinearity is studied quantitatively based on the non-stationary response through the time-frequency filtering method is this paper.

The time-frequency masking operator together with the effective time-frequency region of an asymptotic signal are defined. The general skeleton linear system (GSLS) and skeleton curves of a class of nonlinear system are defined using the time-frequency filtering method. And the relationship between the two systems is clarified. The concepts and results are adaptable to non-stationary vibration of nonlinear systems within a large range.

The main nature of the nonlinear system is described quantitatively in visual forms through two skeleton curves. Compared with other methods, the approach developed here appears very interesting in regard to precision, formulation, testing time, and computational time. The numerical results are in good agreement with theoretical predictions. The work in this paper proposes a new approach for identification of nonlinear dynamic systems. Further research is expected in the future.

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