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SPALLATION ANALYSIS WITH A CLOSED TRANS-SCALE FORMULATION OF DAMAGE EVOLUTION*

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ABSTRACT: A closed, trans-scale formulation of damage evolution based on the statistical microdamage mechanics is summarized in this paper. The dynamic function of damage bridges the mesoscopic and macroscopic evolution of damage. The spallation in an aluminium plate is studied with this formulation. It is found that the damage evolution is governed by several dimensionless parameters, i.e., imposed Deborah numbers De^* and De, Mach number M and damage number S. In particular, the most critical mode of the macroscopic damage evolution, i.e., the damage localization, is determined by Deborah number De^*. Deborah number De^* reflects the coupling and competition between the macroscopic loading and the microdamage growth. Therefore, our results reveal the multi-scale nature of spallation. In fact, the damage localization results from the nonlinearity of the microdamage growth. In addition, the dependence of the damage rate on imposed Deborah numbers De^* and De, Mach number M and damage number S is discussed.

KEY WORDS: spallation, statistical microdamage mechanics, damage evolution, damage localization, closed trans-scale formulation

1 INTRODUCTION

Spallation is a dynamic process. Therefore, to understand spallation one must understand both the threshold conditions that trigger the process and the kinetics of the process. However, some distinct features of spallation complicate and hinder the understanding of it. Firstly, the spallation process is of multiple scales, including microscopic, mesoscopic and macroscopic scales. Spallation usually results from an ccumulation of microdamages and is governed by the collective effects of numerous microdamages[1,2]. Roughly speaking, the nucleation of microdamages usually is in a size of particulates or grains of micrometers, and the typical density of such microdamages on the surface of metals is in the range of $100\sim10\,000/\mathrm{mm}^2$[3,4]. However, the specimen under spallation generally has a size of millimeters or even larger. Secondly, the spallation process is in a state far from equilibrium. The phenomena at various scales can be excited and their physics may be different. In addition, there may be strong or sensitive coupling between different scales. Therefore, it is impossible to deal with the phenomena at each scale separately. It means that a trans-scale (from meso- to macro-scopic) understanding of the damage evolution in spallation is badly needed.

There have been various efforts to formulate this multi-scale process, such as an integral criterion, a continuum measure of spallation, microstatistical fracture mechanics, etc.[1,5,6]. These studies provide progressively helpful means to reveal the essence of spalling. For instance, Davison and Stevens[6] indicated the importance of the compound damage in spallation, though no microscopic physical basis for the compound damage was indicated. In recent ten years, some new and informative studies relevant to spallation were carried out for deeper understanding of the process[7~11]. All these studies intended

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to link the macroscopic spallation and the microdamage evolution inside materials. However, as reviewed by Meyers[2], there is still an acute lack of quantitative/predictive models based on the dynamics of nucleation-and-growth of microdamages and their continuum measure in the study of spallation.

In this paper, a closed trans-scale formulation of damage evolution based on the statistical microdamage mechanics is presented. The trans-scale formulation consists of equations of continuum, momentum, and microdamage evolution, with several time scales and length scales on meso- and macroscopic levels. With this formulation, the damage evolution process in an Al alloy subjected to normal plate impact is analyzed numerically. Effects of several dimensionless parameters on the damage evolution are discussed.

2 EXPERIMENTAL OBSERVATIONS OF MICRODAMAGE EVOLUTION

For experimental observation of microdamage evolution in spallation, we use one stage light gas gun to establish a uniaxial strain state in the specimen. The testing material used is an aluminum alloy similar to 2219-T6 Al (with 6% Cu, etc.), for more details see Refs.[9,12]. A short stress pulse of about 0.1 μs duration was used to investigate the nucleation[3] and a multi-stress pulse technique to study the successive nucleation, growth and coalescence of microdamages[4]. In all these experiments, the transient recording of stresses and the counting of microdamages in specimens after recovery are combined.

In order to investigate the evolution of microdamages, the proper counting of microdamages in tested specimens is a key step. After impact, specimens should be carefully recovered by a specially designed catcher in the gun to prevent a secondary damage. Then the tested specimens were sectioned and polished carefully. Microscopic observations and counting of microdamages were conducted with an S-570 scanning electron microscope and a Q-520 image analysis system with Polyvar-Met Microscope[3,4]. Figure 1 shows a typical microscopic picture of microdamges on a section of an impacted specimen. Based on the experimental observations of microcracks and stereology principles, the nucleation rate and growth rate of microdamages in the impacted Al alloy were obtained[3,4].

The nucleation rate of microdamages n_{N} can be approximately fitted to a function as

$$n_{\mathrm{N}}(c_0,\sigma) = n_{\mathrm{N}}^* g_1(\bar{\sigma}) h(\bar{c}_0) \tag{1}$$

$$\bar{\sigma} = \frac{\sigma}{\sigma^*} \qquad g_1(\bar{\sigma}) = \begin{cases} \bar{\sigma} - 1 & \text{if } \bar{\sigma} \geq 1 \\ 0 & \text{if } \bar{\sigma} < 1 \end{cases} \tag{2}$$

$$\bar{c}_0 = \frac{c_0}{c^*} \qquad h(\bar{c}_0) = \bar{c}_0^{k-1} \cdot \exp\big(-\bar{c}_0^k\big) \tag{3}$$

where c_0 is the size of microdamages at nucleation and $c^* = 4.27\,\mu$m is a characteristic size of the material, roughly the size of the second-phase in the alloy; k is the fitted Weibull modulus and $k = 2.33$; σ is the stress and $\sigma^* = 450\,$MPa is the threshold of nucleation of microdamages. The characteristic nucleation rate of microdamage density n_{N}^* is $5.22 \times 10^4/(\mathrm{mm}^3 \cdot \mu\mathrm{m} \cdot \mu\mathrm{s})$. n_{N}^* is one of the most important parameters of the microdamage evolution in the material.

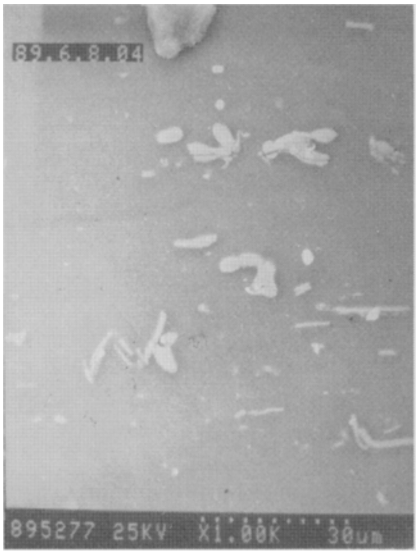


Fig.1 Microcracks on a section of an impacted aluminium alloy specimen

Apart from the nucleation rate, all other mesoscopic dynamic information involved in the experimental data (such as the growth and coalescence of microcracks) are included in the growth rate of microdamages $V = \dot{c}$, when data fitting is carried out by means of the statistical equation of microdamage evolution. The fitting gives the following formula

$$V(c, c_0; \sigma) = V^* \cdot g_2(\bar{\sigma})(\bar{c} - \bar{c}_0)^{\nu} \tag{4}$$

where $\bar{c} = c/c^*$, $\nu = 0.775$ and $V^* \cdot g_2(\bar{\sigma}) = V_0 = 8.1\,$m/s at $1\,470\,$MPa.

3 TRANS-SCALE FORMULATION OF DAMAGE EVOLUTION

Based on the observations of the microcrack evolution outlined in the last section, we developed a theory of the statistical microdamage mechanics. Since the framework of the theory has been well documented in our previous papers[12~15], in this paper, a brief summary of the theory will be given.

According to the statistical microdamage mechanics, the evolution of number density of microdamages n is governed by the equation[12,13]

$$\frac{\partial n}{\partial t}+\frac{\partial(n\cdot A)}{\partial c}+\frac{\partial(n\cdot \boldsymbol{v})}{\partial \boldsymbol{x}}=n_{\rm N} \tag{5}$$

where t is time, $\boldsymbol{x}$ the macroscopic coordinate, c the current size of microdamages, A the average growth rate of microdamages with size c, $A=\int_0^c V(c,c_0)n_0(c,c_0,t)\mathrm{d}c_0\Big/\int_0^c n_0(c,c_0,t)\mathrm{d}c_0$ (n_0 is the microdamage number density in the phase space of $\{c,\ c_0\}$), $\boldsymbol{v}$ is the particle velocity of the macroscopic element, $n_{\rm N}$ the nucleation rate of the microdamage density.

The continuum damage D is defined as an integral

$$D(t,\boldsymbol{x})=\int_0^\infty n(t,\boldsymbol{x},c)\tau\mathrm{d}c \tag{6}$$

where τ is the failure volume of an individual microdamage with size c. For a spherical microdamage, $\tau\sim\pi c^3/6$.

Integrating Eq.(5) under proper boundary conditions converts the statistical evolution equation of microdamage number density to the continuum damage field equation[14,15]

$$\frac{\partial D}{\partial t}+\nabla\cdot(D\boldsymbol{v})=f \tag{7}$$

$$f=\int_0^\infty n_{\rm N}(c_0;\sigma)\cdot\tau\cdot\mathrm{d}c_0+\int_0^\infty n(t,\boldsymbol{x},c)\cdot A(c;\sigma)\cdot\tau'\cdot\mathrm{d}c \tag{8}$$

f is the dynamic function of damage (DFD), which represents the statistical average effects of nucleation and growth of microdamages and $\tau'=\mathrm{d}\tau/\mathrm{d}c$. Obviously, the DFD defined in Eq.(8) is an agent bridging microdamages n and continuum damage D.

Consider cases when the stress keeps constant or varies much slower than the damage. In either case, the basic solution of $n(t,c;\sigma)$ is obtained as

$$n(t,c;\sigma)=\begin{cases}\displaystyle\int_0^c\frac{n_{\rm N}(c_0;\sigma)}{V(c,c_0;\sigma)}\mathrm{d}c_0 & c<c_{\rm f,0}\\[2ex] \displaystyle\int_{c_{0\rm f}}^c\frac{n_{\rm N}(c_0;\sigma)}{V(c,c_0;\sigma)}\mathrm{d}c_0 & c\geq c_{\rm f,0}\end{cases} \tag{9}$$

where $c_{\rm f,0}=c_{\rm f}(t,c_0=0)$ is the front of microdamages with infinitesimal initial size (Fig.2). Substitution of Eq.(9) into the integral Eq.(8) leads to a trans-scale DFD without microdamage number density but still with mesoscopic kinetics. The DFD is directly expressed by two mesoscopic kinetic laws of nucleation and growth rates of microdamages[14,15]

$$f=f_1+f_2=\int_0^\infty n_{\rm N}(c_0;\sigma)\cdot\tau\cdot\mathrm{d}c_0+\int_0^\infty(\tau_{\rm f}-\tau_0)n_{\rm N}(c_0;\sigma)\mathrm{d}c_0 \tag{10}$$

where $f_1=\int_0^\infty n_{\rm N}(c_0;\sigma)\cdot\tau\cdot\mathrm{d}c_0$, $f_2=\int_0^\infty(\tau_{\rm f}-\tau_0)n_{\rm N}(c_0;\sigma)\mathrm{d}c_0$, $\tau_{\rm f}=\pi c_{\rm f}^3/6$, and $c_{\rm f}$ is defined as $t=\int_{c_0}^{c_{\rm f}}\frac{\mathrm{d}c'}{V(c',c_0;\sigma)}$.

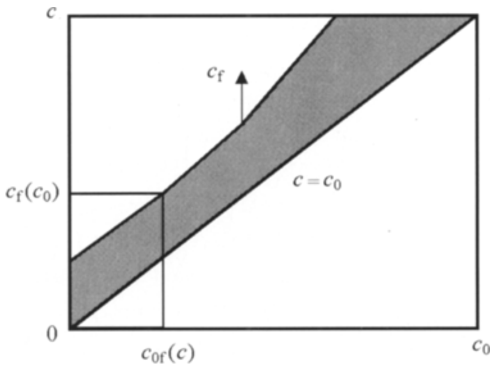


Fig.2 The solution region of microdamage number density $n_0(t,c,c_0)$. The shaded area indicates where a non-zero solution locates. $c=c_{\rm f}(c_0,t)$ or $c_0=c_{0\rm f}(c,t)$ are the microdamage fronts moving upward

4 NUMERICAL ANALYSIS OF SPALLATION

Consider a problem of damage evolution owing to the normal impact of a flying plate of thickness L with velocity v_f striking on a target plate, i.e. the spallation. For the time-dependent damage process, associated equations of continuum, momentum and damage evolution should be used. In one-dimensional strain state, they are

$$\frac{\partial\varepsilon}{\partial T}=\frac{\partial v}{\partial X} \tag{11}$$

$$\rho_0\frac{\partial v}{\partial T}=\frac{\partial\sigma}{\partial X} \tag{12}$$

$$\frac{\partial D}{\partial T}+\frac{D}{1+\varepsilon}\frac{\partial v}{\partial X}=f \tag{13}$$

where ρ_0 is the density of the intact material and f is the dynamic function of damage (DFD). It is noticeable that the damage evolution equation (7) is written in the Eulerian coordinate system (t, x), while Eq.(13) in the Lagrangian coordinate system (T, X). The two systems are related by $\dfrac{\partial}{\partial t} + v\dfrac{\partial v}{\partial X} = \dfrac{\partial}{\partial T}$ and $\dfrac{\partial}{\partial x} = \dfrac{1}{1+\varepsilon}\dfrac{\partial}{\partial X}$. For simplicity, the Al alloy is assumed to be an elastic material in the simulation. The constitutive equation is

$$\mathrm{d}\sigma = E(1-D)\mathrm{d}\varepsilon - E\varepsilon\mathrm{d}D \tag{14}$$

Since the magnitude of the stress pulse is almost constant, we employ the DFD in Eq.(10) for constant stress loading in the simulation.

In section 2, the nucleation and growth laws of microdamages are expressed as Eq.(1) and Eq.(4). Substituting these equations into Eq.(10), and assuming $\tau = \pi c^3/6$, we have

$$f_1 = \frac{\pi n_{\mathrm{N}}^* c^{*4}}{6} h_3 g_1 \tag{15}$$

$$f_2 = \frac{\pi n_{\mathrm{N}}^* c^{*4}}{6} g_1 \int_0^\infty \left(\left(1 + \left(\frac{(1-\nu)V^* t}{c^*} g_2 \right)^{\frac{1}{1-\nu}} \frac{1}{\bar{c}_0} \right)^3 - 1 \right) \bar{c}_0^3 h \mathrm{d}\bar{c}_0 \tag{16}$$

$$f = \frac{\pi n_{\mathrm{N}}^* c^{*4}}{6} h_3 \left(1 + \frac{\displaystyle\int_0^\infty \left(\left(1 + \left(\frac{(1-\nu)V^* t}{c^*} g_2 \right)^{\frac{1}{1-\nu}} \frac{1}{\bar{c}_0} \right)^3 - 1 \right) \bar{c}_0^3 h \mathrm{d}\bar{c}_0}{h_3} \right) g_1 \tag{17}$$

where h_3 is the third order moment of $h(\bar{c}_0)$, $h_3 = \displaystyle\int_0^\infty h(\bar{c}_0) \cdot \bar{c}_0^3 \mathrm{d}\bar{c}_0$. Equation (17) is the dynamic function of damage for spallation analysis.

With the trans-scale nature of spallation in mind the variables in Eq.(11) to Eq.(13) may be non-dimensionalized. Then the dimensionless equations are

$$\frac{\partial \varepsilon}{\partial \bar{T}} = M \frac{\partial \bar{v}}{\partial \bar{X}} \tag{18}$$

$$\frac{\partial \bar{v}}{\partial \bar{T}} = S \frac{\partial \bar{\sigma}}{\partial \bar{X}} \tag{19}$$

$$\mathrm{d}\bar{\sigma} = \frac{E}{\sigma^*}[(1-D)\mathrm{d}\bar{\sigma} - \mathrm{d}D] \tag{20}$$

$$\frac{\partial D}{\partial \bar{T}} + M \frac{D}{1+\varepsilon} \frac{\partial \bar{v}}{\partial \bar{X}} = \frac{\pi h_3 g_1}{6} \frac{1}{De} \left(1 + \frac{\displaystyle\int_0^\infty \left(\left(1 + \left((1-\nu) g_2 \frac{\bar{T}}{De^*} \right)^{\frac{1}{1-\nu}} \frac{1}{\bar{c}_0} \right)^3 - 1 \right) \bar{c}_0^3 h \mathrm{d}\bar{c}_0}{h_3} \right) \tag{21}$$

where the dimensionless variables are:

independent variables

$$\bar{T} = \frac{aT}{L} \qquad \bar{X} = \frac{X}{L} \tag{22}$$

where a is the acoustic speed in the target plate, and dependent variables

$$\bar{v} = \frac{v}{v_{\mathrm{f}}} \qquad \bar{\sigma} = \frac{\sigma}{\sigma^*} \tag{23}$$

The dimensionless numbers are defined as follows:

Mach number

$$M = \frac{v_{\mathrm{f}}}{a} \tag{24}$$

Damage number

$$S = \frac{\sigma^*}{\rho_0 a v_f} \tag{25}$$

and imposed Deborah numbers

$$De = \frac{a}{L n_N^* c^{*4}} \tag{26}$$

$$De^* = \frac{a c^*}{L V^*} \tag{27}$$

The four dimensionless numbers (M, S, De and De^*) govern the damage evolution process in the target plate. Any critical measures should be functions of these dimensionless numbers.

5 RESULTS

In order to validate the presented trans-scale damage evolution model and the numerical program, we compare the numerical results from the Eq.(17) and the phenomenological model proposed by Davison and Stevens[6]. The results of damage evolution obtained from these two models agree very well with each other[16]. Hence, both the trans-scale model and the numerical program are verified. In addition, as most successful models, this model clearly shows the variations of pull-back signals on rear free surface with the residual strength of the spallation area, as shown in Fig.3. Besides, this model can also show that mesoscopic kinetics governs the most critical mode of damage evolution, i.e. the damage localization.

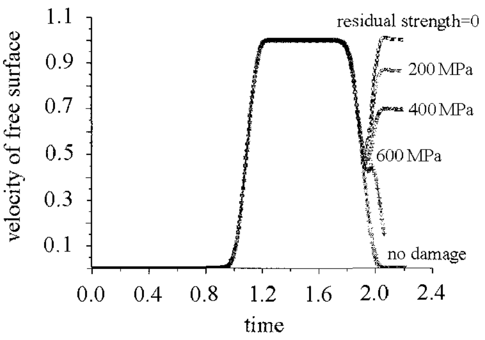


Fig.3 Variation of the free surface velocity with the residual strength of spallation area ($M = 0.030\,5$, $S = 0.153$, $De = 6.59$, $De^* = 0.073\,2$)

Effects of Mach number M and damage number S are studied by varying the velocity of flying plate v_f. Figure 4 demonstrates the effects of v_f on the maximum damage evolution in the target plate. It is evident that with v_f increasing (or, with M increasing and S decreasing), the maximum damage in the target plate increases.

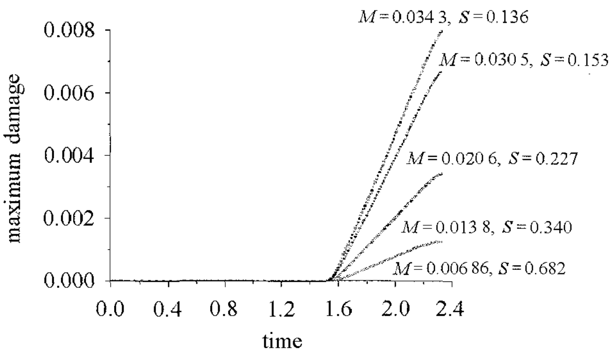


Fig.4 Effect of v_f on the maximum damage evolution in the target plate ($De = 65.9, De^* = 0.415$)

The effect of imposed Deborah number De on the evolution of the maximum damage in the target plate is shown in Fig.5. In these calculations, Mach number M, Damage number S and the imposed Deborah number De^* remain unchanged for all curves in Fig.5. Obviously, the maximum damage in the target plate increases with decreasing De . Therefore, the decrease of De speeds up the process of microdamage growing.

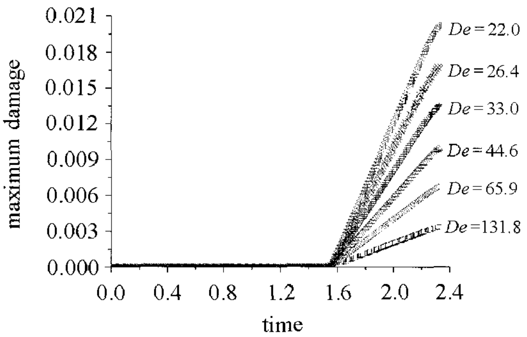


Fig.5 Effects of De on the evolution of maximum damage in target plate ($M = 0.030\,5$, $S = 0.153$, $De^* = 0.415$)

Figure 6 illustrates the effects of imposed Deborah number De^* on the evolution of the maximum damage in the target plate. Similar to the effect of De, the maximum damage in the target plate increases with De^* decreasing. Therefore, decreasing De^* accelerates the damage evolution process too.

Then, what is the difference between the effects of the two imposed Deborah numbers De and De^*? Further simulations show that De^* affects the damage localization behaviour significantly. Figure 7 shows the normalized accumulated damage distribution at a fixed time. The damage is normalized by the maximum damage in the target plate. It is evident that

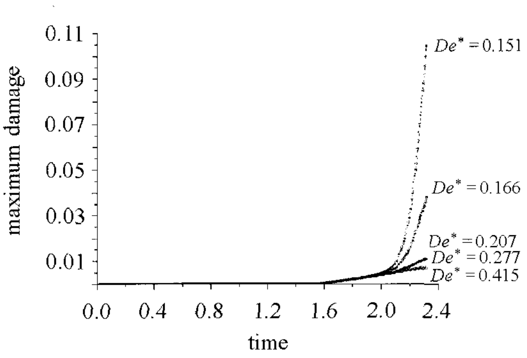


Fig.6 Effect of De^* on the maximum damage evolution in the target plate ($M = 0.030\,5$, $S = 0.167$, $De = 65.9$)

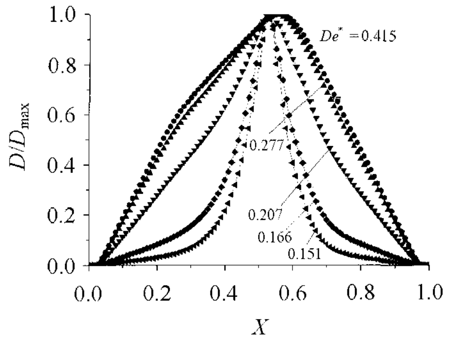


Fig.7 Effects of De^* on damage localization ($M = 0.030\,5$, $S = 0.153$, $De = 65.9$, $\bar{T} = 2.33$)

the damage distributes not uniformly in the plate. In particular, when the imposed Deborah number De^* decreases, the damage gets more localized in the plate.

6 DISCUSSIONS

Physically speaking, there are three kinetic processes involved in spallation. They are: the macroscopic impact loading, the nucleation of microdamages, and the growth of microdamages. The characteristic time scales of these processes are characteristic impact time

$$t_{\mathrm{i}} = \frac{a}{L} \tag{28}$$

characteristic nucleation time of microdamages

$$t_{\mathrm{N}} = \frac{1}{n_{\mathrm{N}}^* c^{*4}} \tag{29}$$

and characteristic growth time of a typical microdamage

$$t_{\mathrm{V}} = \frac{c^*}{V^*} \tag{30}$$

Hence, t_{i} characterizes the macroscopic process, while both t_{N} and t_{V} characterize mesoscopic processes. The ratios between these characteristic times denote the competition and coupling between the three processes. Two independent ratios are imposed Deborah numbers

$$De = \frac{a}{n_{\mathrm{N}}^* c^{*4} L} = \frac{t_{\mathrm{N}}}{t_{\mathrm{i}}} \tag{31}$$

$$De^* = \frac{ac^*}{LV^*} = \frac{t_{\mathrm{V}}}{t_{\mathrm{i}}} \tag{32}$$

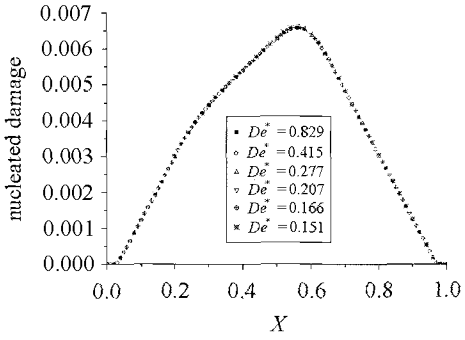
Therefore, we may emphasize that imposed Deborah numbers De and De^* essentially are at the bottom of the multi-scale nature of spallation. Moreover, they are related to the competition and coupling between damage evolution processes at different scales.

As shown in Eq.(31), De is related to the competition and coupling between the process of macroscopic loading and the process of microdamage nucleation. For a given impact loading time, a smaller De means a higher microdamage nucleation rate. Therefore, a smaller De results in more damage in the plate owning to nucleation. This is qualitatively consistent with the numerical results (Fig.5).

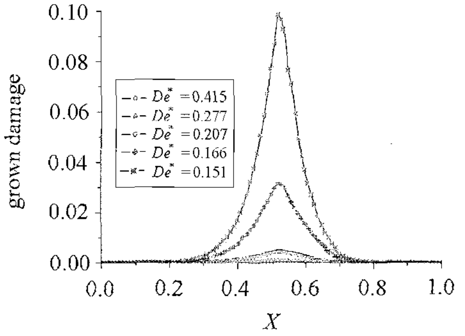
Similarly, Eq.(32) directly relates the imposed Deborah number De^* to the competition and coupling between the process of macroscopic loading and the process of microdamage growth. A smaller De^* physically means faster microdamage growth. Therefore, a smaller De^* causes more damage in the plate too, but owing to the microdamage growth. This is consistent with the numerical results (Fig.6).

However, the damage localization behaviour should be attributed to the imposed Deborah number De^* only. For a given real stress, the two terms in the right hand in the damage evolution Eq.(21) are controlled by De and De^*, respectively. Based on the above discussion on De and De^*, those two terms correspond to the contribution of microdamage nucleation and growth, respectively. We name the accumulated damage contributed by nucleation and growth of microdamage as the nucleated damage and the grown damage. This is in accord with the concept of simple and compound damage proposed by Davison and Stevens[6]. Therefore, it shows that the microscopic basis of simple and compound damage is the nucleated damage and the grown damage, respectively.

Figures 8(a) and 8(b) illustrate the nucleated damage and the grown damage in the target plate, respectively. Figure 8(a) shows that the microdamage nucleation results in an inhomogeneous distribution of damage, which is caused by the non-uniform distribution of stress in the target plate. Since $D \ll 1$, the history of the stress distribution is almost the same for all cases with the same Mach number M and damage number S. Therefore, the nucleated damage is the same for cases with the same De and different De^* (Fig.8(a)). However, the grown damage varies significantly for cases with different De^*, as shown in Fig.8(b). Generally, when De^* decreases, the grown damage increases and becomes more localized in the target plate. Comparing Fig.8(a) and Fig.8(b), we can see that for cases with smaller De^*, the grown damage is much more localized than the nucleated damage. Actually, the term of the grown damage in Eq.(21) serves as a positive feedback. The non-uniformity of the nucleated damage will be amplified by the microdamage growing. The amplification gets stronger with De^* decreasing. Therefore, it can be seen that the damage localization is caused by the growth of microdamage. Since the growth of microdamage is controlled by De^*, De^* is closely related to the damage localization and the damage is prone to localize in materials with smaller De^*.



(a) Nucleated damage v.s. De^*



(b) Grown damage v.s. De^*

Fig.8 Variation of nucleated damage, grown damage and flowed damage with De^* ($M = 0.0305$, $S = 0.153$, $De = 65.9$, $\bar{T} = 2.33$)

7 SUMMARY

A trans-scale formulation of damage evolution is presented based on the statistical microdamage mechanics. With the presented model, the spallation in an Al plate is studied numerically. The study shows that the model provides realistic simulation results of damage evolution in materials under spallation.

Mach number M, damage number S, imposed Deborah numbers De^* and De govern the damage evolution process in the target plate. With increasing M or decreasing S, the maximum damage in the target plate increases. The decrease of De^* or De accelerates the damage evolution in the target plate.

Both imposed Deborah numbers, De and De^*, are at the bottom of the multi-scale nature of spallation. More importantly, De^*, characterizing the microdamage growth, is closely related to the damage localization and the accelerated damage. The damage is prone to localize in materials with smaller De^*.

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