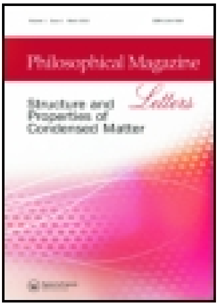

Philosophical Magazine Letters
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/tphl20

Effects of 'sinking in' and 'piling up' on estimating the contact area under load in indentation

Yang-Tse Cheng & Che-Min Cheng
Published online: 14 Nov 2010.

Philosophical Magazine Letters, 1998, Vol. 78, No. 2, 115–120

Effects of 'sinking in' and 'piling up' on estimating the contact area under load in indentation

By Yang-Tse Cheng† and Che-Min Cheng‡

† Physics and Physical Chemistry Department, General Motors Research and Development Center, Warren, Michigan 48090, USA
‡ Laboratory for Non-Linear Mechanics of Continuous Media, Institute of Mechanics, Chinese Academy of Sciences, Beijing 100080, PR China

[*Received 3 March 1998 and accepted 28 March 1998*]

Abstract

The phenomena of the 'piling up' and 'sinking-in' of surface profiles in conical indentation in elastic-plastic solids with work hardening are studied using dimensional and finite-element analysis. The degree of sinking in and piling up is shown to depend on the ratio of the initial yield strength Y to Young's modulus E and on the work-hardening exponent n. The widely used procedure proposed by Oliver and Pharr for estimating contact depth is then evaluated systematically. By comparing the contact depth obtained directly from finite-element calculations with that obtained from the initial unloading slope using the Oliver–Pharr procedure, the applicability of the procedure is discussed.

§ 1. Introduction

Indentation experiments have been performed to obtain the hardness of materials for nearly 100 years (Tabor 1996). Recently, there have been significant improvements in indentation equipment and a growing need for measuring the mechanical properties of materials on small scales. It is now possible to monitor, with high precision and accuracy, both the load and the displacement of an indenter during indentation experiments in the respective micro-Newton and nanometer range (Pethica *et al.* 1983, Bhushan *et al.* 1996). In addition to the hardness, the basic mechanical properties of materials, such as Young's modulus, the yield strength and the work-hardening exponent, may be deduced from curves of indentation load against displacement for loading and unloading. For example, the methods proposed by Oliver and Pharr (1992) and by Doerner and Nix (1986) have been broadly used to determine the hardness and Young's modulus from the peak load and the initial slope of unloading curves. However, the validity of their procedures for estimating the contact area under load has recently been questioned in the literature (for example, Laursen and Simo (1992) and Bolshakov and Pharr (1998)).

In this letter, we use a scaling approach to study the 'sinking in' and 'piling up' of the surface profiles caused by indentation in elastic–plastic solids with work hardening. The procedure proposed by Oliver and Pharr (1992) for estimating the contact depth is then evaluated systematically. By comparing the contact depth obtained directly from finite-element calculations with that obtained from the initial unloading slope using the Oliver–Pharr procedure, the applicability of the procedure is discussed.

0950–0839/98 $12.00

§ 2. Dimensional analysis

We consider a three-dimensional frictionless, ideally sharp rigid conical indenter of a given half-angle θ indenting normally into an isotropic elastic–plastic solid with work hardening. The stress–strain (σ– ε) curves of the solids under uniaxial tension are assumed to be given by

$$\sigma = \begin{cases} E\varepsilon, & \text{for } \varepsilon \le \dfrac{Y}{E}, \\ K\varepsilon^{n}, & \text{for } \varepsilon \ge \dfrac{Y}{E}, \end{cases} \tag{1}$$

where E is Young's modulus, Y is the initial yield stress, K is the strength coefficient and n is the work-hardening exponent (Lubliner 1990). To ensure continuity, we note that $K = Y(E/Y)^n$. Consequently, either E, Y and K, or E, Y and n, are sufficient to describe the stress–strain relationship. We use the latter set of parameters extensively in the following discussions. When n is zero, equation (1) becomes the model for elastic–perfectly plastic solids. For most metals, n has a value between 0.1 and 0.5 (Dieter 1976).

The contact depth h_c under load during loading (figure 1), is a function g, of all the independent governing parameters, namely Young's modulus E, Poisson's ratio υ, the initial yield strength Y, the work-hardening exponent n, the indenter displacement h and the indenter half-angle θ:

$$h_c = g(E, \upsilon, Y, n, h, \theta). \tag{2}$$

Dimensional analysis (Barenblatt 1996) shows that

$$h_c = h\Pi\left(\frac{Y}{E}, \upsilon, n, \theta\right), \tag{3}$$

where $\Pi = h_c/h$ is a dimensionless function of Y/E, υ, n and θ.

We note that, based on dimensional analysis, the contact depth h_c is proportional to the indenter displacement h, that is the ratio h_c/h is independent of the indenter displacement h. This ratio is a function of Y/E, υ and n, for a given θ. When this ratio is greater than one, 'piling up' of the surface profile occurs. When it is smaller than one, 'sinking in' occurs.

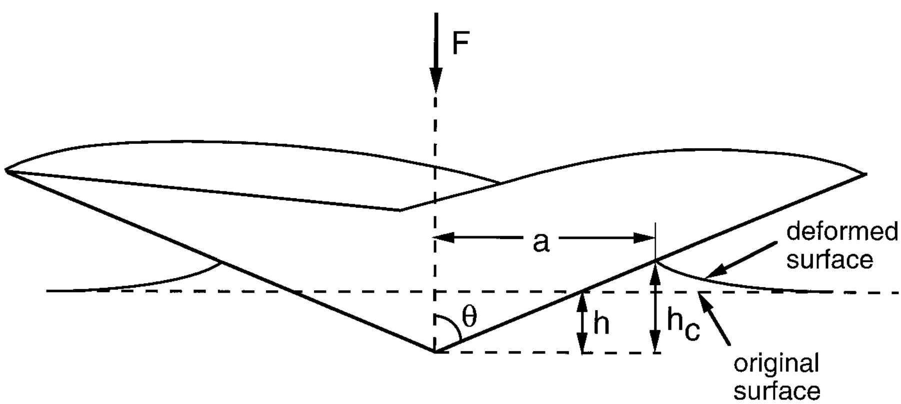

Figure 1. Illustration of conical indentation.

§ 3. Finite-element analysis

Finite-element calculations using ABAQUS (Hibbitt, Karlsson & Sorensen, Inc. 1996) have been carried out to evaluate the dimensionless function $\Pi(Y/E, \upsilon, n, \theta)$ in equation (3). The frequently used half-angle of 68° for the rigid indenter and a typical Poisson's ratio of 0.3 for the solid are chosen to illustrate the essential features of sinking-in and piling-up phenomena. This angle approximates the volume-to-depth relationship for the Vickers and the Berkovich indenters. Our previous results for conical indentation in elastic–perfectly plastic solids show that Poisson's ratio has little effect on the value of h_c/h when $0.2 < \upsilon < 0.4$ (Cheng and Cheng 1998a). To simplify the notation, $\Pi(Y/E, n)$ is used instead of $\Pi(Y/E, 0.3, n, 68°)$ in the following discussion.

The rate-independent incremental theory of plasticity in ABAQUS was used for the finite-element calculations. In particular, the plasticity theory uses the von Mises yield surface model with associated plastic flow rule. The hardening rule used was that of isotropic hardening and the hardening curves were given by equation (1). In each calculation using E, Y and n as input parameters, the loading and unloading curves were obtained together with the contact depth as a function of indenter displacement. The finite-element model has been discussed in detail previously (Cheng and Cheng 1998a, b).

Figure 2 displays the relationship between the calculated h_c/h and Y/E for several representative values of n. The value of h_c/h can be either greater or smaller than one, corresponding to the 'piling up' and 'sinking in' respectively of the displaced surface profiles. For large Y/E, sinking in occurs for all values of n. For small Y/E, both sinking in and piling up may occur depending on the degree of work hardening. In the case of severe work-hardening (i.e. $n = 0.5$), sinking in is expected even for very small values of Y/E, whereas piling up is expected for elastic–perfectly plastic solids and for solids with a small work-hardening exponent (e.g. $n = 0.1$).

These results are expected from analytical theories of conical indentation in elastic solids (Sneddon 1963), where 'sinking in' occurs, and in rigid–plastic solids (Lockett 1963), where 'piling up' occurs. They are also consistent with experimental observations of sinking-in and piling-up phenomena reported in the literature. For example in metals, such as Cu and mild steel where Y/E is small, sinking in is usually

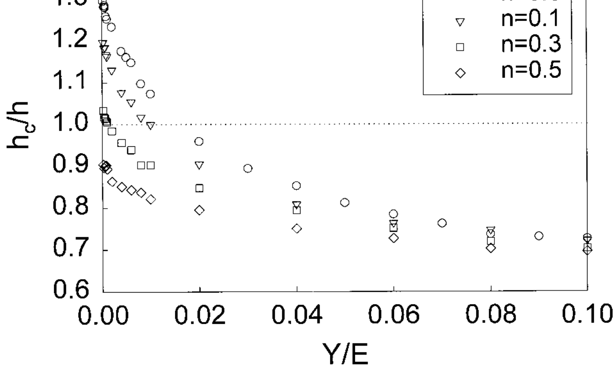

Figure 2. Scaling relationships for $h_c/h = \Pi(Y/E, n)$. (……), $h_c/h = 1.0$.

observed in fully annealed specimens, whereas piling up is seen in heavily work-hardened samples (Tabor 1970, Chaudhri and Winter 1988, Bec *et al.* 1996). In general, therefore, piling-up and sinking-in phenomena are determined by Y/E and the work-hardening exponent n.

§ 4. Discussion

We now consider the effects of piling up and sinking in on estimating the contact area under load in indentation. In general, hardness may be defined as the load divided by the contact area $A = \pi a^2$ under load (figure 1), that is

$$H = \frac{F}{A}, \tag{4}$$

and the elastic modulus can, in principle, be calculated from the initial unloading slope $\mathrm{d}F/\mathrm{d}h$, using

$$\frac{E}{1-\upsilon^2} = \frac{1}{2}\left(\frac{\pi}{A}\right)^{1/2}\frac{\mathrm{d}F}{\mathrm{d}h}. \tag{5}$$

Although equation (5) was known to hold for elastic solids, we have recently shown that it is also true for elastic–plastic solids with or without work hardening and internal stress (Cheng and Cheng 1997). In order to determine both the hardness and the modulus using equations (4) and (5), the contact area under load must be evaluated. However, the sinking in and piling up of the surface profiles can cause difficulties in estimating the contact depth or area. To overcome such difficulties, the procedure suggested by Oliver and Pharr (1992) for estimating the contact depth from the initial unloading slope is frequently used in the literature. The above dimensional and finite-element analysis provide an opportunity to evaluate systematically this procedure.

Based on the results of Sneddon (1963) on the shape of the surface outside the area of elastic contacts for an indenter of conical and paraboloid of revolution, Oliver and Pharr developed an expression for h_c at the indenter displacement h (same as h_{max} in the notation of Oliver and Pharr (1992)),

$$h_c = h - \frac{\xi F_m}{(\mathrm{d}F/\mathrm{d}h)_m}, \tag{6}$$

where F_m and $(\mathrm{d}F/\mathrm{d}h)_m$ are the respective load at the indenter displacement depth h and the initial slope of the unloading curve. The numerical value of ξ is 0.72 for a conical indenter, 0.75 for the paraboloid of revolution, and 1.0 for a flat punch.

Applying the Oliver–Pharr procedure to the loading-unloading curves obtained from finite-element calculations, we evaluate the contact depth using equation (6) and plot it in terms of h_c/h in figures 3 (*a*), (*b*), (*c*) and (*d*) for elastic–perfectly plastic solids ($n = 0.0$) and for elastic–plastic solids with increasing degree of work-hardening ($n = 0.1$, 0.3 and 0.5) respectively. For comparison, the values of h_c/h that were directly obtained from finite-element calculations (shown in figure 2) are also shown in figure 3.

It is apparent from figure 3 that the Oliver–Pharr procedure for estimating the contact depth under load is valid when the ratio of Y/E is large (e.g. greater than 0.05 for $0.0 < n < 0.5$). This is expected since this procedure is based on Sneddon's analysis of surface profiles for elastic contacts. Thus, the procedure may be used

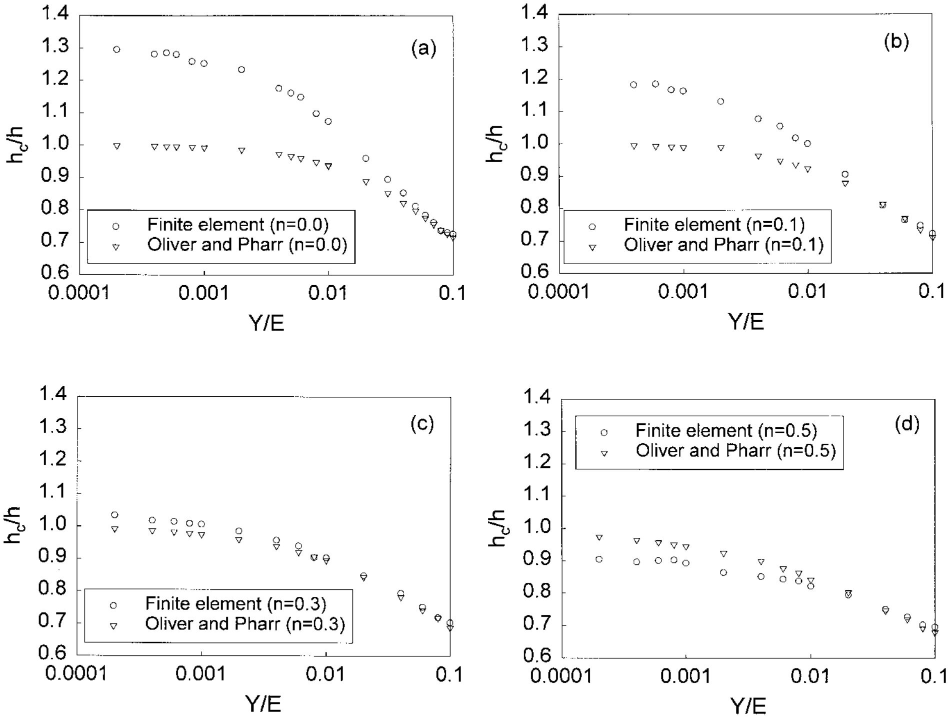

Figure 3. The values of h_c/h obtained using the Oliver–Pharr procedure and those directly from finite-element calculations for (*a*) elastic–perfectly plastic solids and (*b*)–(*d*) elastic–plastic solids with increasing work-hardening exponent (*b*) $n = 0.1$, (*c*) 0.3 and (*d*) 0.5.

with confidence for highly elastic materials, including hard coating materials such as diamond-like carbon and titanium nitride.

For materials with a wide range of Y/E (e.g. 10^{-4}–10^{-2}), such as metals, the Oliver–Pharr procedure should be used with caution. For example, the procedure is approximately correct for most Y/E values if the work-hardening exponent is approximately 0.3 (figure 3 (*c*)). However, the procedure underestimates the contact area for elastic–perfectly plastic solids over most Y/E values (figure 3 (*a*)). The error is most significant when piling up occurs, that is, $h_c/h > 1$. In fact, h_c/h estimated using the Oliver–Pharr procedure is always less than unity. It should also be noted that equation (6) could also overestimate the contact area for materials with a large work-hardening exponent, for example $n = 0.5$ (figure 3 (*d*)).

§ 5. Summary

Using dimensional and finite-element analysis, the phenomena of the piling up and sinking in of surface profiles in conical indentation in elastic-plastic solids with work-hardening has been studied in detail. The degree of sinking in and piling up is shown to depend on the ratio of initial yield strength Y to Young's modulus E and on the work-hardening exponent n. The widely used procedure proposed by Oliver and Pharr (1992) for estimating contact depth has been evaluated systematically. It is shown that the procedure may be used with confidence for highly elastic materials (e.g. $Y/E > 0.05$ for $0.0 < n < 0.5$). For materials with a wide range of Y/E (e.g. 10^{-4}–10^{-2}), however, the Oliver–Pharr procedure should be used with caution.

Acknowledgements

We should like to thank W. J. Meng, S. J. Harris, G. L. Eesley, J. R. Smith, L. C. Lev, B. Qiu, S. F. Yin, D. D. Snyder and K. C. Taylor for helpful discussions. We also thank R. J. Blint, M. S. Meyer, K. Ueno, J. Brown (Electronic Data Systems, Inc.), P. Lalor (Hibbitt, Karlsson and Sorensen, Inc.) and L. Hill (HKS) for maintaining the workstations and for helping with the use of ABAQUS. Communications with Professor G. M. Pharr and Professor D. Tabor have also been valuable.

References

Barenblatt, G. I., 1996, *Scaling, Self-similarity, and Intermediate Asymptotics* (Cambridge University Press).
Bec, S., Tonck, A., Georges, J.-M., Georges, E., and Loubet, J.-L., 1996, *Phil. Mag.* A, **74**, 1061.
Bhushan, B., Kulkarni, A., Bonin, W., and Wyrobek, J., 1996, *Phil. Mag.* A, **74**, 1117.
Bolshakov, A., and Pharr, G. M., 1998, *J. Mater. Res.* **13**, 1049.
Cheng, C.-M., and Cheng, Y.-T., 1997, *Appl. Phys. Lett.,* **71**, 2623.
Cheng, Y.-T., and Cheng, C.-M., 1998a, *Int. J. Solids Structures* (to be published); 1998b, *Phil. Mag. Lett.,* **77**, 39.
Chaudhri, M. M., and Winter, M., 1988, *J. Phys. D,* **21**, 370.
Dieter, G., 1976, *Mechanical Metallurgy* (New York: McGraw-Hill).
Doerner, M. F., and Nix, W. D., 1986, *J. Mater. Res.,* **1**, 601.
Hibbitt, Karlsson & Sorensen, Inc., 1996, ABAQUS, version 5.6 (Pawtucket, Rhode Island 02860, USA: Hibbitt, Karlsson & Sorensen, Inc).
Laursen, T. A., and Simo, J. C., 1992, *J. Mater. Res.,* **7**, 618.
Lockett, F. J., 1963, *J. Mech. Phys. Solids,* **11**, 345.
Lubliner, J., 1990, *Plasticity Theory* (London: Macmillan).
Oliver, W. C., and Pharr, G. M., 1992, *J. Mater. Res.,* **7**, 1564.
Pethica, J. B., Hutchings, R., and Oliver, C., 1983, *Phil. Mag.* A, **48**, 593.
Sneddon, I. N., 1963, *Int. J. Engng. Sci.,* **3**, 47.
Tabor, D., 1970, *Rev. Phys. Technol.,* **1**, 145; 1996, *Phil. Mag.* A, **74**, 1207.