

Source model for blasting vibration

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Abstract By analyzing and comparing the experimental data, the point source moment theory and the cavity theory, it is concluded that the vibrating signals away from the blasting explosive come mainly from the natural vibrations of the geological structures near the broken blasting area. The source impulses are not spread mainly by the inelastic properties (such as through media damping, as believed to be the case by many researchers) of the medium in the propagation pass, but by this structure. Then an equivalent source model for the blasting vibrations of a fragmenting blasting is proposed, which shows the important role of the impulse of the source's time function under certain conditions. For the purpose of numerical simulation, the model is realized in FEM. The finite element results are in good agreement with the experimental data.

Keywords: blasting vibration, equivalent source, finite element method

Explosion has found wide applications in various areas of rock and soil engineering such as water conservancy project, transport and communications, mining industry, petroleum industry, geological prospecting and safety protective engineering.

In a rock and soil blasting project, not all explosion energies can be effectively used. A part of the energy is propagated to the outside through the media around the blasting area, which is called the propagation of the blasting vibration and may have a detrimental effect on the surrounding buildings. Therefore, that part of the explosion energy should be made as small as possible. On the other hand, if the blasting vibration is to be used for strengthening the groundwork or for the geological prospecting, the respective energy should be made as large as possible. In both cases, it is necessary to have a good understanding of the characteristics of the blasting vibration sources.

The blasting processes in rock and soil are very complicated. Scientists have established several equivalent methods to analyze the blasting vibration sources.

The early attempts were focused on the equivalent cavity theories. Sharpe^[1] made a rather systematic theoretical and experimental study on the seismic waves induced by an enclosed explosion and established an equivalent cavity model for the seismic source for blasting waves in a homogeneous elastic medium. Then, Duvall^[2], Ricker^[3] and others studied spherical cavity explosion vibration in a visco-elastic medium, and obtained some results well consistent with the experimentally recorded waveforms. Blake^[4], Kisslinger^[5] and Favreau^[6] studied the impulse pressure, the spherical wave propagation and the effects of the medium structure and inhomogene-

ity of a point source explosion in an infinite medium and pointed out that the stress impulse produced during the explosion takes a form of a vibrating wave train with a large damping, whose characteristics are closely related to the physical properties of the media, the size of cavities and the nature of the charges. Heelan^[7] Gupta^[8] studied the line explosions and obtained results with similar characteristics as experimental results. Howell^[9] O'Brien^[10] studied the relationship between the explosion energy and the induced seismic energy. Kisslinger^[11] et al. studied the relationship between blasting-induced earthquake and the depth of buried explosives.

Recently, with the development of the measurement techniques and the computer technologies, it is possible to go beyond some simple analytical results and to obtain a more accurate description of the blasting vibrations under real conditions. Many scientists have devoted much effort in that direction. Ziolkowski^[12–14] et al. studied the characteristics of the explosion source in rock and soil using the similarity law for the seismic source for blasting waves and the wavelet theory for the source and obtained the time function of the source. Flynn^[15] studied the explosion source under different conditions and obtained the relationship between the depth of the explosion and the energy of the seismic wave. Yang^[16] analyzed the source mechanism of an explosively induced mine collapse. The above theoretical models are all based on the point source moment theory in seismology, where the characteristics of the blasting damage depend on the form of the moment and the related time function. Sun^[17] carried out an inverse transform for the time function of the blasting point source moment using a hereditary algorithm. Kennett^[18] made a detailed theoretical analysis of the point source moment for layered media and proposed related analytical methods.

With the help of rapidly developed computer technology, the numerical simulation has now become another powerful tool along with the experimental simulation in the scientific research work. Many attempts have been made to simulate the blasting processes and related wave propagations^[19]. Theoretically speaking, a numerical simulation may cover the whole explosion process, the damage process of rock and soil, the wave propagation (near field and far field). But due to the limitation of our understanding of the damage characteristics of media, the various time scales of different processes and the limit of current computer capacity, the calculation has hardly been carried out directly from the explosion of the explosive to the propagation of the blasting waves (especially the intermediate and far fields). One must make some simplifications of equivalence for the blasting source before the simulation in order to deal with blasting vibrations under some complicated conditions, which sees very bright prospect in a near future.

The mechanism and the characteristic properties of the blasting wave propagation are analyzed in this paper. A source model of blasting vibration is established for numerical purpose, and is implemented to simulate blasting wave propagation of fragmenting blasting in finite element method. The numerical results show a good accordance with the experimental data.

1 The main features of blasting waves

In all records of blasting experiments, the blasting seismic waves are always seen as complicated wave trains with different frequencies and amplitudes. These wave trains have characteristics of the explosion source, the physical properties of the media, propagation paths, observation environments and other effects. That is to say, the information of a great number of physical factors may be found in every complicated seismic wave train. This paper will focus on the information about the characteristics of the explosion source reflected by the seismic waves, which can be accessed through the observation signals. Therefore it is necessary to have an understanding of the properties of the media where the signals are propagating. Fig.1 shows a typical velocity waveform for the seismic waves on the free surface induced by half space shallowly buried segmenting blasting (elastic near field, in vertical direction).

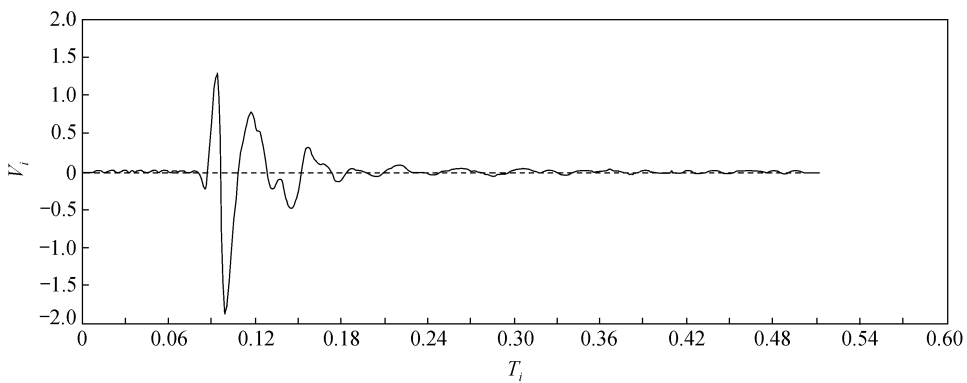


Fig. 1. A typical velocity waveform for the seismic waves on the free surfaces induced by half space shallowly buried fragmenting blasting

As is evident in fig. 1, the waveform of the blasting waves has a certain bandwidth and the characteristics of oscillation with a large damping, unlike a shock produced by explosive detonation wave. The results of most near field measurements of blasting waves show that the high frequency components of the vibrating velocity do not attenuate significantly with the distance. With an increase in the charge amount, the period of the waveform increases and the bandwidth decreases (fig.2).

It is known that in a process of fragmenting blasting, the charge may be regarded as an impulse load of a very short time duration acting on the surrounding media. The energy released by the charge is mainly transferred to the surrounding media through that impulse load, resulting in deformation, damage and movement of the media. Obviously, the im-

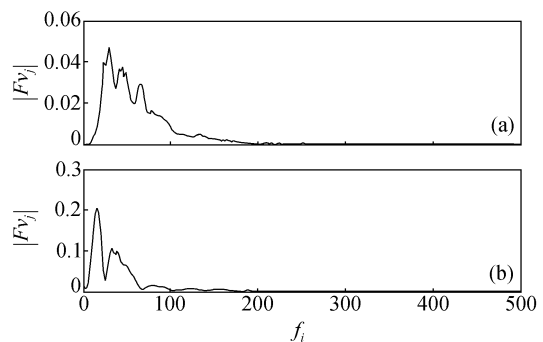


Fig. 2. The spectra of blasting waves with different charge amounts: (a) a charge of 2 kg; (b) a charge of 28 kg.

pulse load has not been transferred in a form of sharp impulse, but rather propagated to the outside through the filtering of the surrounding media. Many researchers believed that the filtering mainly depends on the inelastic properties of the media (such as damping)^[20]. From the analysis and comparison of waveforms recorded in experiments and the results obtained by the point source moment theory and the equivalent cavity theory, we conclude that the filtering mainly depends on the geological structures of the elastic media near the blasting area.

In this paper, we will see what physical mechanism underlies the filtering, what is the correct equivalent source model of blasting vibration and how it can be established. We will start with an analysis of conventional source theories of blasting waves.

2 Point source moment theory and equivalent cavity theory

In the seismic studies (on natural earthquakes and blasting quakes), the seismic source has long been assumed to be a point source moment, whose time function is determined by the far field waveform and its spectrum features. The inverse problem of the seismic source is turned into the determination of the form of the point moment and its time function. Of course, the equivalent cavity theory developed in the early times is also an efficient source model for blasting waves, but for lack of different kinds of load models and lack of knowledge of the time functions, it has not drawn enough attention recently.

In the source moment theory^[19], the equivalent source model is formed by the difference between the real stress τ_{ij} and the linear elastic model stress σ_{ij} ,

$$e_i = -\partial_j m_{ji}, \quad (1)$$

where

$$m_{ij} = \sigma_{ij} - \tau_{ij} \quad (2)$$

is the equivalent source moment. Generally speaking, m_{ij} is equal to zero everywhere except in a small (inelastic) region V_Q near the source. As the distance between the seismic signal observed and the seismic source is generally much greater than the size of the source region (inelastic region), the source may be regarded as a point source. The equivalent source moment takes the form

$$m_{ij}(t, X) = M_{ij}(t) \delta_Y(X), \quad (3)$$

where $\delta_Y(X)$ is the Dirac function at the spatial point Y , and

$$M_{ij}(t) = \int_{V_Q} m_{ij}(t, X) dV. \quad (4)$$

In a practical application, M_{ij} is usually decomposed into the multiplication of a second order tensor and a scalar time function

$$M_{ij}(t) = f(t) \cdot M = f(t) \cdot \{M_{ij}\}. \quad (5)$$

The point source moment tensor usually takes the following forms: ($\mathbf{n} = \{n_1, n_2, n_3\}$ is the surface normal vector; $\mathbf{v} = \{v_1, v_2, v_3\}$ is the sliding direction vector).

(i) For the spherically symmetrical explosion,

$$M_{ij} = A[\bar{u}] \lambda \delta_{ij}. \quad (6a)$$

(ii) For a crack surface:

$$M_{ij} = A[\bar{u}] \{ \lambda \delta_{ij} + 2\mu n_i n_j \}. \quad (6b)$$

(iii) For a sliding surface:

$$M_{ij} = A[\bar{u}] \{ \lambda n_k v_k \delta_{ij} + \mu (n_i v_j + n_j v_i) \}, \quad (6c)$$

where A is a scalar constant, $[\bar{u}]$ is the mean displacement or the difference in displacement, and λ and μ are Lamé constants of the medium.

As the point source moment takes the form of a spatial divergence in the equilibrium equations, and the smoothness of the derivative of Dirac function, in a best case, is in the space $H^{-2-\varepsilon}$ ($\varepsilon > 0$), the best smoothness of the solution of the linear elastic equations under the action of the point source moment is in the space $H^2(t; H^{-2-\varepsilon}) \cap L^2(t; H^{-\varepsilon})$. That is to say, the solution corresponding to the action of the point source moment is a generalized function, which cannot be defined in a point-to-point manner, but as a functional. We may explain it using the finite bandwidth solutions of the linear elastic equations under the action of the point source moment (which are corresponding to the smooth approximation of the generalized function). Fig.3 shows the solutions with different bandwidths as a near field solution of the linear elastic equations under the action of a point source moment with a rectangular time function. It can be known from the figure that, with the increase in the bandwidth, the waveform of the solution changes very little, but the duration time becomes shorter. That process of increasing the waveband and decreasing the time duration will continue infinitely, and the limiting state cannot be directly expressed, but can only be defined through similar processes or through a functional. When the width of the time function keeps a constant, the oscillation of the waveform will increase when the bandwidth is increased to a certain level; but the width of most single peaks will decrease, and its limiting state also cannot be directly expressed, but can only be defined through similar processes or through a functional. In the process of increasing the frequency bandwidth of the solution to increase the frequency limit, the high frequency components will be increased no matter what the loading process is. Therefore, it may be concluded that the analytical solution of the point source moment is singular, resulting in the appearance of components of infinitely high frequencies. In other words, these components of high frequencies come from the spatial singularities of the load. That is, if we do not limit the frequency range, we cannot obtain a waveform with a limited bandwidth. That is why people have to limit the bandwidth when using the point source moment.

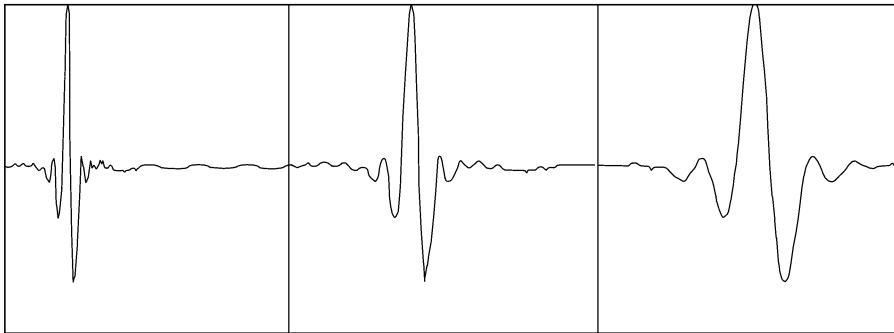


Fig. 3 The solution of the linear elastic equations under the action of point source moments with different bandwidths (which increase from right to left).

When the bandwidth of the solution is fixed and the loading time is less than the period of the main high frequency in the band, the configuration (waveform) of the solution will remain approximately unchanged and its amplitude will be proportional to the acting impulse of the load, almost independent of the time distribution of the load. At the same time, the waveform characteristics of the solution agree very well with experimental ones. But with the increase in the loading time, the waveform will change, the waveform characteristics cease to follow some regular manner, the state of the waveform will directly depend on the time distribution of the load and the waveform characteristics of the solution will start to deviate from experimental ones.

Then, what is the physical mechanism that limits the bandwidth? To explain that question, let us have a look at the cavity theory.

For a spherically symmetrical blasting in an infinite medium, according to the cavity theory, the seismic source may be considered as an equivalent cavity, on whose inner wall is acted a uniform pressure varying with time, and whose solution may be expressed as^[1, 18]

$$u(r, t) = \frac{1}{r^2} f\left(t - \frac{r}{c_p}\right) + \frac{1}{c_p r} f'\left(t - \frac{r}{c_p}\right), \quad (7)$$

where

$$f(t) = e^{-at} (A \cos(\omega t) + B \sin(\omega t)) + C. \quad (8)$$

The first term in eq. (8) is the homogeneous solution of the linear elastic equations for an infinite medium with a spherical cavity, C is a particular solution related with the load, A and B are parameters related to the initial value of the load, material properties of the medium, the initial state of the medium and the size of the cavity, and a and ω are constants related to the material properties and the size of the cavity. Actually, ω is the natural frequency of the cavity and can be expressed as

$$\omega = \frac{2c_s}{r_0} \sqrt{1 - \frac{c_s^2}{c_p^2}}, \quad (9)$$

where r_0 , c_s and c_p are, respectively, the cavity radius, the shear wave velocity and the longitudinal wave velocity of the medium. For a Poisson material, we have

$$\omega = \frac{2\sqrt{2}}{3r_0} c_p.$$

When the longitudinal wave velocity of the medium is around 3000m/s and the cavity radius is 1 m, the natural frequency of the cavity is 450 Hz. Generally speaking, in most loading conditions, the natural vibration mode of the cavity will be excited, and the waveform will take a similar form of the forced vibration superimposed with a natural vibration. When the wave signals are propagated, the excited natural vibration of the cavity will behave like a damped vibration ($a > 0$). When the loading time is much less than the period of the natural vibration, the propagated wave signals will take the waveform of the natural vibration. That kind of waveforms of the seismic signals, with the characteristic property of a large damping, agrees very well with the signals measured in experiments.

Thus, we have made an analysis of the point source moment solution and that of the cavity theory, with some of the experimental results compared. It may be concluded that for a fragmenting blasting, the seismic signals outside the neighborhood of the blasting area come mainly from the natural vibration of the geological structures near the blasting area, which may be considered as a cavity with some additional mass for a homogeneous medium. That is to say, the physical mechanism of limiting the bandwidth is the natural vibration of the geological structure after the blasting: The blasting process forms a weakened region (blasting area) in the medium like a cavity, making the medium have an evident structure feature. Part of the explosion energy is used to damage the medium, while the rest makes the surrounding medium move. Because the duration of the explosive load is very short, the surrounding medium could be regarded to have an initial velocity, so its movement afterwards depends mainly on the structure property of the medium and the energy release caused by the wave propagation, and is characterized by a damped vibration mode. This damped vibration characteristic could be seen clearly from the solution of the linear elastic equations for an infinite medium with a spherical cavity (eqs. (7) and (8)). The homogeneous solution of problem, i.e. the free vibration solution of the structure, is formed by a negative exponential factor and a harmonic function. This is the typical form of damped vibration solution. On the other hand, the structure of the medium surrounding the blasting area is by the explosive and the local properties of the material. The crucial parameter is the size of this region.

3 The equivalent source model

From the above analysis, we may make the following assumptions concerning the equivalent source model of the fragmenting blasting waves:

1. The equivalent load is a moment load uniformly distributed in a blast-weakened medium near the explosives. The size of the area depends on the nature of the medium, the arrangement of

the explosives, the amount of charges, etc. For a point explosion source, the equivalent source model may be regarded as a moment load uniformly distributed in a certain volume of the medium.

2. The moment load can be expressed by a moment tensor.

3. The intensity of the load depends on the acting impulse of the time function.

Thus, the equivalent source model for a fragmenting blasting by a point explosion can be expressed as follows.

For uniform load moment:

$$M(t) = f(t)((1-\alpha)M1 + \alpha M2), \quad (10)$$

where

$$M1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad M2 = \begin{bmatrix} \frac{\nu}{1-\nu} & 0 & 0 \\ 0 & \frac{\nu}{1-\nu} & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (11)$$

ν is the Poisson's ratio of the surrounding medium, α is a proportional factor for the cavity and cracking, and $f(t)$ is the time function of the load. With a dimension analysis, the size of the loading area can be estimated as

$$L = k_1 Q^{1/3}; \quad (12)$$

the impulse of the time function can approximately be expressed as

$$I = \int f(t)dt = \frac{1}{L^2} k_2 Q (1 - k_3 e^{-k_4 H / Q^{1/3}}), \quad (13)$$

where L, I, Q, H are, respectively, the size of the loading area, the magnitude of the impulse, the amount of the charge and the depth of the charge; k_1, k_2, k_3, k_4 are constants related to the properties of the explosive and medium. When the duration of the load action is significantly less than the period of the natural vibration of the medium after blasting, the shape of the time function has little influence on the wave propagation, which may be explained as follows.

Let v be the blasting wave response at some distance to the source, G the impulse response function, H the equivalent filtering function of the geological structure, and $f(t)$ the time function of the source. Then we have formally

$$v = G * H * f.$$

When the duration time of $f(t)$ is much less than the period of the main high frequency of filtering function H , we have approximately

$$H * f = I_f \cdot H, \quad I_f = \int f(t)dt.$$

Using the properties of the convolution, we have

$$v = I_f \cdot G * H.$$

That is, the response depends only on the magnitude of the acting impulse of the seismic source and the equivalent filtering characteristics of the geological structure.

4 Finite element simulation and comparison with experimental data

The equivalent source model of the blasting quake for a point source fragmenting blasting can be implemented in the finite element calculation as follows.

Let Ω be the acting area of the equivalent load. Then the corresponding variational equations can be obtained with the weighted residue method as

$$\int_V (\rho \ddot{u}_i - \sigma_{ij,j} + e_i) v_i dx = 0.$$

Letting E_k $k=1, \dots, K$ be the elements intersected by Ω , we can obtain the variation in nodal forces induced by the seismic source as

$$\sum_k \int_{\Omega \cap E_k} e_i v_i dx = f(t) \sum_k \int_{\Omega \cap E_k} v_{i,j} M_{ij} dx = f(t) \sum_k V_k^T P_k, \quad (14)$$

where

$$P_k = \int_{\Omega \cap E_k} DN^T M dx. \quad (15)$$

DN is the differentiation matrix of the shape functions in the finite element method, and M is the source moment matrix.

The numerical simulations are carried out using the above relations. First we will present the simulation results on different distribution of the time functions (for half space shallowly buried fragmenting blasting). Hexahedron elements are used in the simulation, and the loading area consists one or several elements exactly, whose modulus is less than that of the surrounding medium by two orders of magnitude, but with the same density.

Under the same conditions, three equal impulse time functions are used (fig. 4, where T is less than the period of the main high frequency in the seismic waveform).

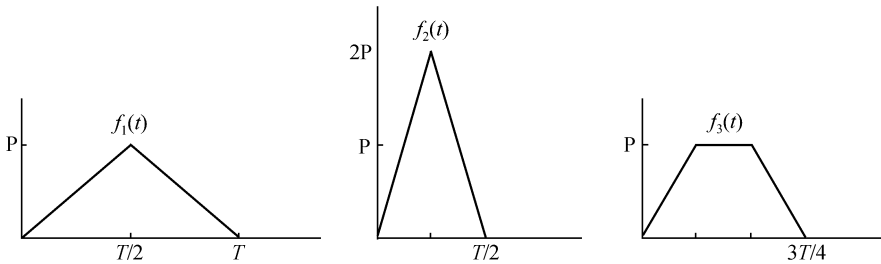


Fig. 4. Equal impulse time functions.

Figs. 5 and 6 show a comparison of the calculation results at the same measuring points using the three load time functions.

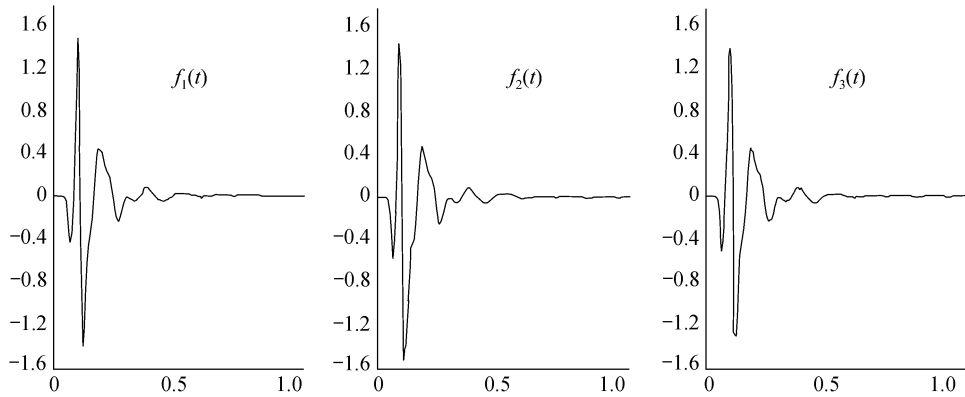


Fig. 5. The near field and near area response of the equal impulse time functions with different distributions.

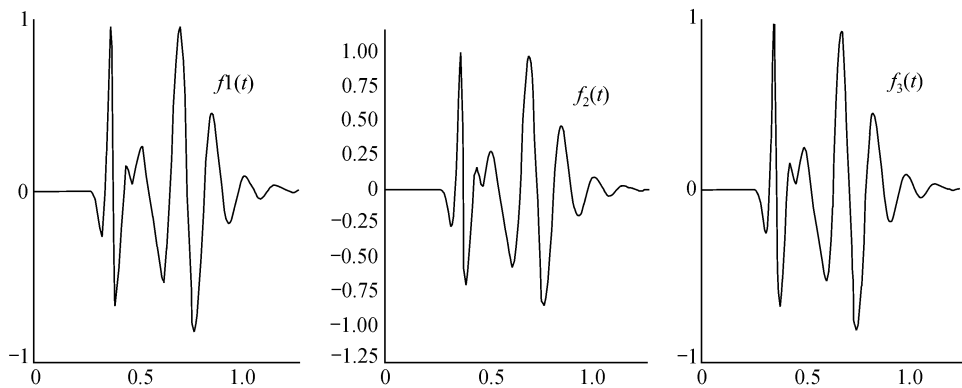


Fig. 6. The near field and far area response of the equal impulse time functions with different distributions.

It can be seen from the figures that the seismic waveforms obtained by the equal impulse time functions take nearly the same shape, with very little difference in their amplitudes as well. It shows that under that conditions the shape of the time functions has very limited effects on the wave propagation, and the main feature of the time function must depend on the magnitude of the impulse. This fact gives evidence from another angle that the features of a blasting wave mainly depend on the geological structure near the blasting area and the acting impulse of the source moment.

With the model of this paper, we have carried out numerical simulations of the experimental data from the blasting tests in Dahuichang, Beijing and Qingjiang Cofferdam, and obtained the following relations.

For tests in Dahuichang, Beijing:

$$\alpha = 0, \quad L = 3.4 \cdot \sqrt[3]{Q}, \quad I = 6.1 \cdot 10^5 \cdot (1 - e^{-0.2 \cdot H / \sqrt[3]{Q}}) \cdot \frac{Q}{L^2},$$

where the length is in unit of meters, the charge amount in units of kilograms. Fig.7 shows a comparison of the calculated and experimental waveforms at the corresponding points.

For tests in Qingjiang Cofferdam:

$$\alpha = 0.78, \quad L = 1.4 \cdot \sqrt[3]{Q}, \quad I = 1.5 \cdot 10^7 \frac{Q}{L^2}.$$

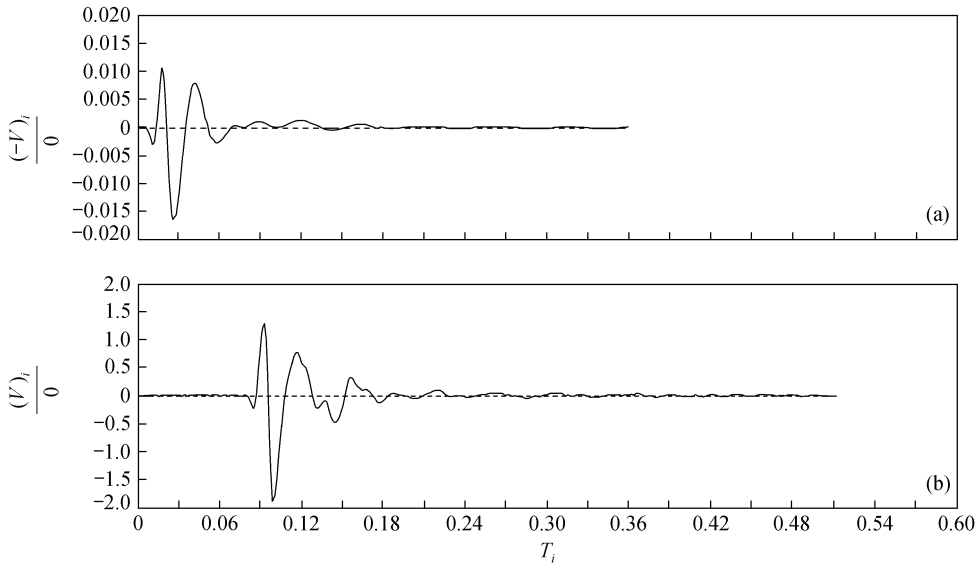


Fig. 7. The wave velocity waveform in the Dahuichang tests, (a) The calculated results; (b) the experimental results.

Fig.8 shows a comparison of the calculated and experimental waveforms at the corresponding points. It must be pointed out that the wave signals at the points near the blasting area show the formation of cracked surfaces and the movement of the layered blocks; the wave signals at the points far away from the blasting area show the influence of the blasting waves on the cofferdam. The disturbances induced by the formation of cracked surfaces and the movement of the layered blocks will not propagate a long distance and, therefore, will have little influence on the waveform

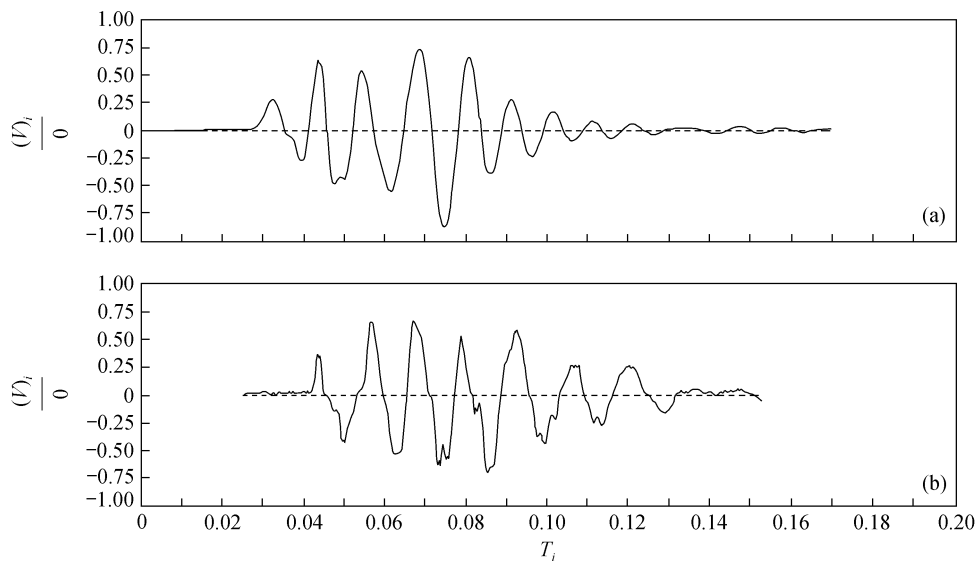


Fig. 8. The wave velocity waveform in the Qingjiang Cofferdam tests. (a) The calculated results; (b) the experimental results.

signals. That is to say, in the inverse fitting process and comparison, only signals far away from the blasting area are considered.

From the features of the waveforms, it can be seen that the numerical simulation results agree very well with the experimental ones, which shows that the model presented in this paper does reflect the actual processes.

5 Concluding remarks

Comparison with the experimental results, shows that the model proposed in this paper reflects the main physical characteristics of the blasting vibrating processes, and it is simple and easy to implement in applications. This paper also clarifies some mistakes in the simulations with equivalent seismic sources for the blasting waves. Further work in this direction will surely be helpful to evaluations and utilizations of blasting waves.

With different kinds of blasting, much work remains to be done on the simulation of the geological structures in this model. This paper gives some preliminary results. Further research is needed on the throwing blasting, the characteristics and physical mechanism of the wave propagation induced by the back throwing of the media.

As for the conclusion made in this paper about the impulse of the equivalent source moment of the blasting wave, its area of application also requires further exploration through experiments.

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