Analytical Interaction of the Acoustic Wave and Turbulent Flame *

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A modified resonance model of a weakly turbulent flame in a high-frequency acoustic wave is derived analytically. Under the mechanism of Darrieus–Landau instability, the amplitude of flame wrinkles, which is as functions of the expansion coefficient and the perturbation wave number, increases greatly independent of the 'stationary' turbulence. The high perturbation wave number makes the resonance easier to be triggered but weakened with respect to the extra acoustic wave. In a closed burning chamber with the acoustic wave induced by the flame itself, the high perturbation wave number is to restrain the resonance for a realistic flame.

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The dynamics of the premixed turbulent flame is one of the most interesting and important problems in the combustion science. It has wide backgrounds in many fields, such as gas engines, future hypersonic propulsion systems and supernova explosion. The inherent hydrodynamic instability of a premixed flame front, known as Darrieus–Landau instability,[1] is vital in the flame dynamics and has been studied widely. It wrinkles, corrugates and accelerates the flame front, which will induce acoustic waves or pressure waves. Then the wave retroaction influences the combustion and instability, making the combustion flow field more complicated. Although the interaction between the laminar flame and acoustic waves have been investigated widely,[2,3] only recently the turbulent flame and acoustic wave interaction has been studied analytically, with the assumption of infinite thin flame.[4] In this Letter, previous analytical results on the acoustic wave and turbulent flame will be generalized into the regime of a thin but finite flamelets[5] and the perturbation effect with different wave numbers are discussed.

Following Bychkov's method,[4] the analysis of the flame response to the acoustic wave starts from the linear dynamic equation for a flame front with small perturbation amplitude. Assuming the corrugated flame is induced by a turbulent flow with velocity component u_z along the z axis, imposed at a planar flame front with an arbitrary gravitational field perpendicular to the planar flame front. Since the problem is linear, then the small perturbations may be presented in the form $f(x,t) = f(t)\cos(kx)$. With the assumption of the thin flame front and weak turbulence, the linear equation for the perturbation amplitude may be written as[6]

$$A\frac{d^2 f}{dt^2}+BU_f k\frac{df}{dt}-CU_f^2k^2 f+Dgkf = E\Big(U_f k+\frac{d}{dt}\Big)u_k, \tag{1}$$

where U_f is the laminar planar flame velocity, and g is the effective gravitational acceleration. The detailed dimensionless coefficients expressions are shown and discussed. In the simple case of the unit Lewis number and unit Prandtl number, and adopting the widely used hypothesis that thermal conductivity of an ideal gas varies as a square root of temperature, i.e. the simplified coefficients of Eq. (1) become

$$A = 1, \tag{2}$$

$$B = \frac{2\Theta}{\Theta+1}\Big[1+\frac{2\Theta}{\sqrt{\Theta}+1}kL_f\Big], \tag{3}$$

$$C = \Theta\frac{\Theta-1}{\Theta+1}\times\Big[1-\frac{2kL_f}{\Theta-1}\Big(\Theta^2(\Theta+1)\frac{\sqrt{\Theta}-1}{(\Theta-1)^2} + \frac{2}{3}\Theta^{3/2}-\sqrt{\Theta}+\frac{1}{3}\Big)\Big], \tag{4}$$

$$D = \frac{\Theta-1}{\Theta+1}, \tag{5}$$

$$E = \frac{2\Theta}{\Theta+1}, \tag{6}$$

where Θ is the expansion coefficient of the fuel, and L_f is the flame thickness.

To handle the right-hand side turbulence terms, Taylor hypothesis of "stationary" turbulence is adopted, which demonstrates that temporal pulsations of the turbulent flow are negligible in comparison with time variations caused by flame propagation.[7] When turbulence is isotropic, one turbulent mode velocity at the average flame position reads

$$u_k = U_T\cos(U_f kt), \tag{7}$$

where U_T is the mode amplitude. Following the previous results,[8] action of the sound wave on the flame is equivalent to an effective oscillating gravitational field:

$$g = \omega U_a\cos(\omega t+\varphi_a). \tag{8}$$

Substituting Eqs. (7) and (8) into Eq. (1), the dynamic equation may be written as

$$\frac{d^2 f}{dt^2}+BU_f k\frac{df}{dt}-CU_f^2k^2 f+D\omega U_a k\cos(\omega t+\varphi_a)f = \sqrt{2}EU_f kU_T\cos\Big(U_f kt+\frac{\pi}{4}\Big). \tag{9}$$

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Bychkov[4] has demonstrated that in the case of high-frequency acoustic waves with $\omega \gg U_f k$, there exits a solution in the form of superposition of a slow component $f_1(t)$ and a fast oscillating component $f_1 \cos(\omega t + \varphi)$, i.e.

$$f = f_1(t) + f_2 \cos(\omega t + \varphi), \tag{10}$$

where $f_1(t)$ changes with typical time $(U_f k)^{-1}$, and $f_1 \cos(\omega t + \varphi)$ changes with typical time ω^{-1}. Due to the high-frequency acoustic waves, we have

$$\frac{d^2 f}{dt^2} \propto \omega^2 f_2 \gg U_f k \frac{df_2}{dt} \gg U_f^2 k^2 f_2. \tag{11}$$

Substituting Eq. (10) into Eq. (9), the linear dynamic equation becomes

$$\begin{aligned}\frac{d^2 f_1}{dt^2} &+ BU_f k\frac{df_1}{dt} - CU_f^2 k^2 f_1 - (\omega^2 f_2 - D\omega U_a k f_1)\\ &\cdot \cos(\omega t + \varphi) + D\frac{\omega}{2}U_a k f_2[1 + \cos(2\omega t + 2\varphi)]\\ &= \sqrt{2}EU_f k U_t \cos(U_f k t).\end{aligned} \tag{12}$$

Separating the terms oscillating with frequency ω from Eq. (12), we can obtain the amplitudes f_1 and f_2 to read

$$f_2 = D\frac{U_a k}{\omega} f_1. \tag{13}$$

Then slowly changing terms of Eq. (12) constitute the following equation:

$$\begin{aligned}\frac{d^2 f_1}{dt^2} &+ BU_f k\frac{df_1}{dt} - CU_f^2 k^2 f_1 + \frac{1}{2}D^2(U_a k)^2 f_1\\ &= \sqrt{2}EU_f k U_t \cos(U_f k t).\end{aligned} \tag{14}$$

Solution of Eq. (14) for the slow component of the flame front perturbation has the form

$$f_1(t) = F\cos(U_f k t + \varphi_t). \tag{15}$$

Substituting Eq. (15) into Eq. (14), we have

$$F = \sqrt{2}E\frac{U_T}{U_f k}\Big\{\Big[\frac{D^2}{2}\Big(\frac{U_a}{U_f}\Big)^2 - C - 1\Big]^2 + B^2\Big\}^{-1/2}. \tag{16}$$

With F_0 representing the amplitude without acoustic wave $U_a = 0$, the scaled amplitude can be written as

$$\frac{F^2}{F_0^2} = \frac{(1+C)^2 + B^2}{\Big[\frac{D^2}{2}\Big(\frac{U_a}{U_f}\Big)^2 - C - 1\Big]^2 + B^2}. \tag{17}$$

Equation (17) and its coefficient Eqs. (2)–(6) give the amplitude increase F^2/F_0^2 as functions of the expansion coefficient Θ and the scaled perturbation wave number kL_f. If L_f is set to be zero, this relation can be reduced to the results with the infinite thin flame front.[4] The resonance amplitude F^2/F_0^2 characterizing influence of external turbulence on the flame velocity may increase by a large factor of 10–20 because of the resonance. The detailed discussion on this resonance's properties in the case of infinite thin front has been carried out.[4] It is deduced that the resonance may be one of the reasons for considerable scattering in experimental results on turbulent flame velocity. Also, the resonance effect may be responsible to sudden strong turbulization of a flame front observed in experiments. In the following context, we mainly focus on the influence of the perturbation.

To evaluate the perturbation effect, the critical wave number k_c must be decided first. It is the maximum value of the perturbation wave number, and DL instability will be prevented by diffusive effects when $k > k_c$. The previous results[6] have demonstrated that DL stability is limited when the coefficient C of Eq. (1) is zero. Therefore, from Eq. (4) the scaled perturbation wave number is derived:

$$\begin{aligned}k_c L_f = (\Theta - 1)\Big[&\Theta^2(\Theta+1)\frac{\sqrt{\Theta}-1}{(\Theta-1)^2}\\ &+ \frac{2}{3}\Theta^{3/2} - \sqrt{\Theta} + \frac{1}{3}\Big]^{-1}.\end{aligned} \tag{18}$$

Usually the realistic flame has the expansion coefficient $\Theta = 5$–7. Giving $\Theta = 6$ in Eq. (18), we can obtain $k_c L_f = 0.228$. Then the resonance amplitude versus the scaled oscillating velocity of an acoustic wave is shown in Fig. 1, for different perturbation scaled wave numbers $\alpha k_c L_f$ with $\alpha = 0$, 0.01, 0.1, 0.5. It is shown that perturbation wave may influence the resonance characteristics greatly. First the perturbation weakens the resonance amplitude. Increasing the wave number to the critical one, the amplitude decreases and the resonance gradually disappears. This can be attributed to Darrieus–Landau instability, which will be prevented for larger wave number due to diffusive effects. However, it can also be found out the acoustic velocity corresponding to the resonance point for each wave number decreases slowly meanwhile. That is to say, the resonance has a different characteristic acoustic velocity, which introduces the largest amplitude, for different perturbation wave numbers. Because the characteristic acoustic velocity decreases, the resonance is easier to be triggered. Interestingly, we find that the perturbation influence is similar to the influence by the various expansion ratio, which is discussed previously.[4] Because the expansion ratio indicates Darrieus–Landau instability effect, so it can be concluded that the perturbation weakens the instability effect. This resonance is from the instability and the turbulence interaction essentially, so the high perturbation wave number has the negative effect on the resonance. In a word, the high perturbation wave number can weaken the Darrieus–Landau instability effect, and the resonance is easier to be triggered with the cost of be weakened.

The above-discussed case is the resonance response to extra acoustic waves. However the acoustic wave can be generated by the flame itself in a closed burning chamber, in which the estimate characteristic acoustic velocity reads

$$U_a = (\Theta - 1)U_f. \tag{19}$$

Combining Eqs. (17) and (19), the resonance amplitude can be derived in the closed burning chamber case. Figure 2 shows the relation of the resonance amplitude and the expansion factor for different scaled wave numbers in a closed burning chamber. It is found out that there is a characteristic expansion ratio, which corresponds to the maximum amplitude. Figure 2 also shows that an extremum can be achieved in the case of $\alpha = 0$ and $\Theta = 5.6$, which is consistent with the previous results. This characteristic expansion ratio demonstrates that the resonance is not monotonously related to the Darrieus–Landau instability, which is indicated implicitly by the expansion ratio. Similarly to the resonance shown in Fig. 1, the perturbation wave may also weaken the resonance amplitude in the closed chamber, and the expansion coefficient corresponding to the resonance point also decreases gradually, from $\Theta = 5.6$, $\alpha = 0$ to $\Theta = 3.8$, $\alpha = 0.5$. In realistic burning chamber, the usual flame expansion coefficient usually ranges 5 to 7, so the high perturbation wave number makes the resonance difficult to be triggered. Therefore it can be concluded that the high perturbation wave number is against the resonance in the closed burning chamber.

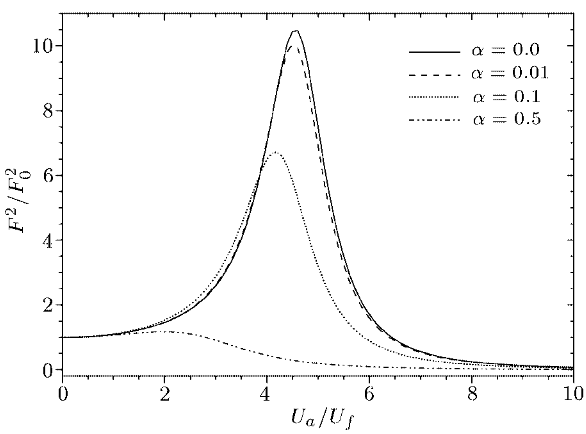

Fig. 1. Resonance amplitude versus the scaled oscillating velocity of an acoustic wave for different scaled wave numbers with the expansion coefficient 6.0.

Experimental validation of this analytical result is difficult to derive owing to two factors. First, the perturbation will weaken the resonance either in the open or closed chamber. Duo to the interaction between the turbulent and combustion expansion, the perturbation cannot be avoided, therefore the resonance is difficult to be observed. Second, the weak isotropic 'stationary' turbulence is hard to achieve. Although Taylor's hypothesis of 'stationary' turbulence is verified in the turbulent premixed flames in a time-dependent external flow,[9] the integral effects of various realistic turbulent flow on the resonance are still unknown, for they usually concerns the complicated multi-scale energy transfer.[10] Recent experiment[11] on acoustically forced lean premixed turbulent bluff-body stabilized flames found out that with higher inlet velocity amplitudes, the heat release response becomes nonlinear. Furthermore, the second nonlinearity was observed, whose origin is not yet clarified. It occurs at high amplitudes and at some narrow equivalence ratios, which induce a significant leakage of energy to higher harmonics. This phenomenon is similar to our model, and in our opinion the resonance is responsible to the second nonlinearity, although in experiments much more other interactions should be concerned.

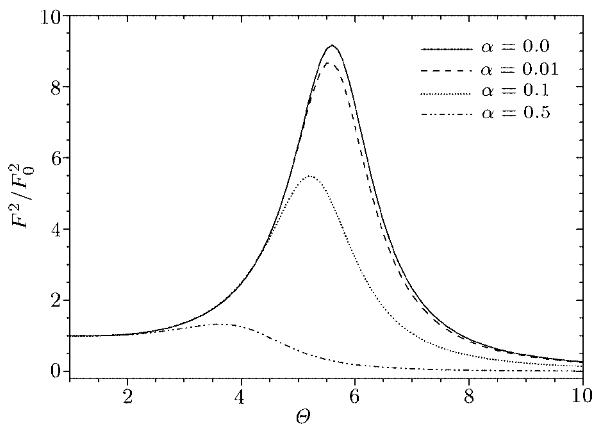

Fig. 2. Resonance amplitude versus the expansion factor for different scaled wave numbers in a closed burning chamber.

In conclusion, we have analytically derived a modified resonance model of a weakly turbulent flame in a high-frequency acoustic wave, in regime of the thin but finite flame front flamelets. This resonance increases the amplitude of flame wrinkles greatly independent of the turbulence, and the influence of perturbation is discussed. The analytical results show that the higher perturbation wave number makes the resonance easier to be triggered but weakened with respect to the extra acoustic wave, while it is to restrain the resonance for a realistic flame in a closed burning chamber.

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