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# 二维抛物化稳定性方程的特征和次特征<sup>\*</sup>

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**摘要:**特征分析表明:对原始扰动量的抛物化稳定性方程组(PSE),它在亚、超音速区分别具有椭圆和抛物特性,给出PSE特征对马赫数的依赖关系,阐明PSE仅把信息对流-扩散传播特性抛物化,而保留了信息对流-扰动传播特性,因此PSE应称为扩散抛物化稳定性方程(DPSE)

**关键词:**扩散抛物化稳定性方程(DPSE);可压缩流动;特征;次特征

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## 1 引 言

Herbert<sup>[1,2]</sup>抛物化稳定性方程(PSE)理论在流体运动稳定性计算中得到越来越多的应用。该理论把扰动诸变量表示为快变波状分量和缓变形状函数的乘积,并对微分运算  $\frac{\partial^n}{\partial x^n} (i + \frac{\partial}{\partial x})^n$  作抛物化近似  $\frac{\partial^n}{\partial x^n} (i)^n + n(i)^{n-1} \frac{\partial}{\partial x}$ ,于是得到形状诸函数满足的抛物化稳定性方程组(PSE)。PSE 可用空间推进方法求解,计算维数减少一维,经济有效,然而,当推进步长小时,PSE 出现数值不稳定现象<sup>[3,5]</sup>,说明 PSE 并非真正的抛物形方程。文[4,5]关于 PS 的特征分析表明:当  $\frac{\partial u}{\partial x}$  和  $\frac{\partial p}{\partial x}$  的系数的实部(这里  $u, p$  为扰动量,  $x$  为主流方向)以及波状分量  $\exp(i_x)$  中 的实部不等于零时有复特征根,说明 PSE 具有椭圆特性,但文[4,5]没有给出 PSE 特征与马赫数的关系。近来文[6]从强粘性扰动流的分析出发,给出原始扰动量  $u, v, \dots$  和  $p$  等满足的扩散抛物化稳定性方程组(DPSE),若进一步把扰

动诸量表示为波状和形状函数的乘积,DSPE 简化为文[1,2]形状函数满足的 PSE。因此 PSE 特征分析应是 DSPE 特征分析的特例。

本文采用文[7]关于扩散抛物化 NS 方程组的特征次特征理论来分析扩散抛物化稳定性方程组(DPSE)的特征和次特征,表明 DPSE 在亚、超声速区分别具有椭圆和抛物特性。

## 2 稳定性方程组及其扩散抛物化方程组

### 2.1 二维可压线化稳定性方程组

考虑主流形式为  $(U(x, y, t), 0, T(x, y, t), \bar{U}(x, y, t))$  的二维可压缩边界层的稳定性问题,设线性化扰动为  $(u, v, \dots)$ ,总流场的速度、温度和密度记为  $\bar{U} = U + u, \bar{V} = v, \bar{T} = T + \tilde{t}, \bar{\rho} = \rho + \tilde{\rho}$

假设总流场满足流体力学基本方程(即文献[7]的(1)~(5)),利用状态方程消去压力  $p$ ,并减去主流所满足的流体力学基本方程组,再作线化处理,即可得到二维可压缩线化稳定性方程组

$$U \frac{\partial}{\partial x} + -\frac{\partial u}{\partial x} + -\frac{\partial v}{\partial y} = F_1 \quad (1)$$

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$$\begin{aligned} & \left( \frac{T}{\mu} + \frac{\mu}{T} \right) \frac{\partial}{\partial x} + U \frac{\partial u}{\partial x} - \frac{4\mu}{3(\frac{T}{\mu} + \frac{\mu}{T}) \text{Re}} \frac{\partial^2 u}{\partial x^2} + \\ & \frac{2\mu}{3(\frac{T}{\mu} + \frac{\mu}{T}) \text{Re}} \frac{\partial^2 v}{\partial x \partial y} + \frac{\partial}{\partial x} - \\ & \frac{\mu}{(\frac{T}{\mu} + \frac{\mu}{T}) \text{Re}} \frac{\partial^2 v}{\partial y \partial x} - \frac{\mu}{(\frac{T}{\mu} + \frac{\mu}{T}) \text{Re}} \frac{\partial^2 u}{\partial y^2} = F_2 \quad (2) \end{aligned}$$

$$\begin{aligned} & - \frac{\mu}{(\frac{T}{\mu} + \frac{\mu}{T}) \text{Re}} \frac{\partial^2 u}{\partial x \partial y} + U \frac{\partial v}{\partial x} - \frac{\mu}{(\frac{T}{\mu} + \frac{\mu}{T}) \text{Re}} \frac{\partial^2 v}{\partial x^2} + \\ & \frac{2\mu}{3(\frac{T}{\mu} + \frac{\mu}{T}) \text{Re}} \frac{\partial^2 u}{\partial y \partial x} + \frac{T}{(\frac{T}{\mu} + \frac{\mu}{T})} \frac{\partial}{\partial y} - \\ & + \frac{\partial}{\partial y} = F_3 \frac{4\mu}{3(\frac{T}{\mu} + \frac{\mu}{T}) \text{Re}} \frac{\partial^2 v}{\partial y^2} \quad (3) \end{aligned}$$

$$\begin{aligned} & - U \frac{\partial}{\partial x} + (C_p - 1) - U \frac{\partial}{\partial x} - \frac{C_p \mu}{P_r \text{Re}} \frac{\partial^2}{\partial x^2} - \\ & - \frac{C_p \mu}{P_r \text{Re}} \frac{\partial^2}{\partial y^2} = F_4 \quad (4) \end{aligned}$$

其中  $U, u, v$  用  $U_e$  归一化,  $x, y$  用边界长度  $L$  归一化,  $T, \mu$  和  $C_p$  用气体常数  $R$  归一化,  $\mu_e$  和  $\mu_e$  归一化,  $\text{Re} = \frac{e U_e L}{\mu_e}$  为雷数数。  $F_1, F_2, F_3$  和  $F_4$  表示关于  $\frac{\partial}{\partial x}, \frac{\partial}{\partial y}$  以外的其它项。

## 2.2 扩散抛物化稳定性方程组(DPSE)

根据文[6]对线化稳定性方程组的扩散抛物化处理,或把文[1,2]对形状函数稳定性方程的抛物化处理返回到原始扰动量形式,即在线化稳定性方程组(1)~(4)中略去对  $x$  求偏导数的粘性项后,得到扩散抛物化稳定性方程为

$$U \frac{\partial}{\partial x} + - \frac{\partial u}{\partial x} + - \frac{\partial v}{\partial y} = F_1 \quad (5)$$

$$\begin{aligned} & \left( \frac{T}{\mu} + \frac{\mu}{T} \right) \frac{\partial}{\partial x} + U \frac{\partial u}{\partial x} + \frac{\partial}{\partial x} - \frac{\mu}{(\frac{T}{\mu} + \frac{\mu}{T}) \text{Re}} \frac{\partial^2 u}{\partial y^2} = F_2 \quad (6) \end{aligned}$$

$$U \frac{\partial v}{\partial x} + \frac{T}{(\frac{T}{\mu} + \frac{\mu}{T})} \frac{\partial}{\partial y} - \frac{4\mu}{3(\frac{T}{\mu} + \frac{\mu}{T}) \text{Re}} \frac{\partial^2 v}{\partial y^2} + \frac{\partial}{\partial y} = F_3 \quad (7)$$

$$- U \frac{\partial}{\partial x} + (C_p - 1) - U \frac{\partial}{\partial x} - \frac{C_p \mu}{P_r \text{Re}} \frac{\partial^2}{\partial x^2} = F_4 \quad (8)$$

## 3 DPSE 的特征分析

### 3.1 扩散抛物化稳定性方程组

根据文[7]的理论分析,信息在流场中的传播主要有两个途径,一是对流-扩散传播,一是对流-扰动传播,前一个传播特性由流体运动方程(包括二阶粘性项)或其简化方程组的特征(称为主特征)所确定,后一个传播特性由流体运动方程组或其简化方程组丢掉所有粘性项得到的微分方程组的特征(称为次特征)所确定。为了求得扩散抛物化稳定性方程组(5)~(8)的主特征,我们把它们转换为关于

$$Z = ( , u, \frac{\partial u}{\partial y}, v, \frac{\partial v}{\partial y}, , \frac{\partial}{\partial y}) \quad (9)$$

的联立一阶拟线性偏微分方程

$$A \frac{\partial Z}{\partial x} + B \frac{\partial Z}{\partial y} = F \quad (10)$$

其中  $Z$  和  $F$  是 7 维列向量,  $A$  和  $B$  为  $7 \times 7$  阶矩阵, 特征方程为

$$\det(-_1 a_{ij} + _2 b_{ij}) = 0 \quad (11)$$

其中

$$\begin{aligned} \det(-_1 a_{ij} + _2 b_{ij}) &= \begin{vmatrix} U_1 & -_1 & 0 & -_2 & 0 & 0 & 0 \\ \frac{T}{\mu} + \frac{\mu}{T} & U_1 & \frac{-\mu}{(\frac{T}{\mu} + \frac{\mu}{T}) \text{Re}} & 0 & 0 & 1 & 0 \\ (\frac{T}{\mu} + \frac{\mu}{T})^2 & 0 & 0 & U_1 & \frac{-4\mu}{3(\frac{T}{\mu} + \frac{\mu}{T}) \text{Re}} & 2 & 0 \\ -U_1 & 0 & 0 & 0 & 0 & -U(C_p - 1)_1 & \frac{-C_p \mu}{P_r \text{Re}} & 2 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{vmatrix} \\ &= (U^2 - \frac{-T}{(\frac{T}{\mu} + \frac{\mu}{T}) \text{Re}}) \frac{4C_p \mu^2}{3P_r (\frac{T}{\mu} + \frac{\mu}{T}) \text{Re}^2} {}^{1,2}_{5,2} \end{aligned}$$

求得主特征值为

$$_1^5 = 0, _2^2 = 0 \quad (12)$$

主特征全为零,说明扩散抛物化稳定性方程组(5)~(8)为抛物型。

通过类似的数学处理,不难证明线化稳定性方程组(1)~(4)的主特征为

$$_1^4 = 0, _5, _6 = \pm i, _7, _8 = \pm i, _9, _{10} = \pm i \quad (13)$$

这里  $\lambda = -\frac{1}{2}$ , 六个虚特征表明线化稳定方程组(1)~(4)为椭圆型。

### 3.2 稳定性方程组(5)~(8)的次特征

将二维可压稳定性方程组(5)~(8)的粘性项全部去掉, 得到如下次特征方程组

$$U \frac{\partial u}{\partial x} + -\frac{\partial u}{\partial x} + -\frac{\partial v}{\partial y} = F_1 \quad (14)$$

$$\frac{T}{(\gamma + 1)} \frac{\partial}{\partial x} + U \frac{\partial u}{\partial x} + \frac{\partial}{\partial x} = F_2 \quad (15)$$

$$U \frac{\partial v}{\partial x} + \frac{T}{(\gamma + 1)} \frac{\partial}{\partial y} + \frac{\partial}{\partial y} = F_3 \quad (16)$$

$$-U \frac{\partial}{\partial x} + (C_p - 1) \frac{\partial}{\partial x} = F_4 \quad (17)$$

为了求得方程组(14)~(17)的特征, 采用与上面相同的处理, 把它们转换为关于  $Z = (u, v, \frac{\partial v}{\partial y}, \dots)$  的联立一阶拟线性偏微分方程组

$$A \frac{\partial Z}{\partial x} + B \frac{\partial Z}{\partial y} = F \quad (18)$$

该方程组的特征方程为

$$\det \begin{pmatrix} 1 & a_{ij} & 2 & b_{ij} \\ U_1 & -1 & -2 & 0 \\ \frac{T}{\gamma + 1} & U_1 & 0 & 1 \\ \frac{T}{\gamma + 1} & 0 & U_1 & 2 \\ -TU_1 & 0 & 0 & -U(C_p - 1)_1 \\ -U^2(C_p - 1)_2 & \frac{TC_p}{C_p - 1}_1 & \frac{TC_p}{C_p - 1}_2 & 0 \end{pmatrix} = 0 \quad (19)$$

其中  $\frac{TC_p}{C_p - 1} = a^2$ ,  $a$  为声速, 由(19)求得

$$a_1^2 = 0, \quad a_3 = \frac{1}{\sqrt{M_U^2 - 1}}, \quad a_4 = -\frac{1}{\sqrt{M_U^2 - 1}} \quad (20)$$

其中  $\lambda = -\frac{1}{2}$ ,  $M_U = \frac{U}{a}$ , 二维可压扩散抛物化稳定性方程组的次特征与主流方向的马赫数  $M_U$  有关。当  $M_U > 1$  时, 次特征为零和实特征, 当  $M_U < 1$  时, 次特征为虚特征。

## 4 结 论

1) 二维可压线化扩散抛物化稳定性方程组的特征分析表明,  $M_U > 1$  时, 它为抛物-双曲型, 即“抛物化”的简化处理只把信息对流-扩散传播抛物化了, 而信息对流-扰动传播特性仍得到保留, 因此抛物化的称呼并不能正确反映线化稳定性方程组的特性, 正确的称呼应是扩散抛物化稳定性方程组(DPSE)。

2) 线化 DPSE 特征与扰动马赫数  $u/a$  和  $v/a$  无关, 而仅与未扰流马赫数  $M_U$  有关, 这是线化小扰动  $u, v \ll U$  造成的结果。

3) 文[4,5]关于稳定性方程组特征分析的椭圆型结论, 相当于本文  $M_U < 1$  时的部分结论。

4) 由于扩散抛物化稳定性方程组在  $M_U > 1$  时为抛物-双曲型, 空间推进求解适定, 计算维数减少一维, 经济有效;  $M_U < 1$  时为椭圆型, 然而由次特征方程(14)~(17)知道只要合理处理  $\frac{\partial u}{\partial x}$  即可排除椭圆特性, 使空间推进求解适定。

5) 求解原始扰动量形式的稳定性方程组是流动稳定性计算的一个主要方向。求解扩散抛物化稳定性方程组勿需规定下游边界条件, 因此解决了在下游边界设置扰动量边界条件的难题, 对流动稳定性计算十分有用。

## 参 考 文 献

- Herbert Th., Bertolotti F P., Stability analysis of non-parallel boundary layers, Bull American Phys Soc, 1987, 32: 2097
- Herbert Th., Nonlinear stability of parallel flows by high-order amplitude expansion, AIAA J, 1980, 18(3): 243~248
- Chang C-L, Malik M R, Erleracher G, et al., Compressible stability of growing boundary layers using parabolized stability equation, AIAA 91-1636, New York: AAIA, 1991
- Haj-Hariri H., Characteristics analysis of the parabolized stability equations, Stud Appl Math, 1994, 92(1): 41~53
- 郭乃龙, 三维不可压边界层抛物化稳定性方程的椭圆特性研究, 航空学报, 1999, 20(2): 104~106
- 高智, 流体运动诸方程计算的尺度效应和它们的简化方程, 北京计算流体力学讨论会文集, 2000
- 高智, 流体力学基本方程组(BEFM)的层次结构理论和简化 Navier-Stokes 方程组(SNHSE), 力学学报, 1988, 20(2): 107~116

## Characteristics and Sub-characteristic of Two Dimensional Parabolized Stability Equation

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**Abstract :** In this paper, characteristics analysis shows that there exist ellipticity and paraboloid for parabolized stability equations (PSE) of original disturbance variables respectively in the subsonic and supersonic fields. The dependence of PSE characteristics on Mach number is given also. It is implied that the characteristics of convection-diffusion of information is parabolized, otherwise convection-disturbances of information is remained. For this reason, PSE should be called diffusion parabolized stability equations (DPSE).

**Keywords :** *diffusion parabolized stability equations (DPSE), compressible flows, characteristic, sub-characteristic.*

## Seismic Behavior of Multiple Tuned Mass Dampers based on Control of the Acceleration Response

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**Abstract :** Seismic behavior of multiple tuned mass dampers (MTMD) consisting of many tuned mass dampers (TMDs) is studied in the present paper. Each damper keeps their stiffness and damping constant and the system has a linear distribution of natural frequencies. Based on both the pseudo-excitation approach and the Kanai-Tajimi and Clough-Penzien Spectrums, the acceleration transfer function for the structure with MTMD are formulated. The explicit expression for the acceleration dynamic magnification factor of the structure with MTMD, denoted by ADMF, is then derived. The criterion for the optimum searching is the minimization of the minimum values of the maximum acceleration dynamic magnification factors [i.e. Min. Min. Max. ADMF]. Through the optimum searching, the optimum frequency spacing, average damping ratio, tuning frequency ratio and corresponding index representing the control effectiveness may be found. Take different ratio between the structural frequency in the mode to be mitigated and the dominant ground frequencies, research is carried out regarding the influence of the dominant ground frequencies on the MTMD optimum parameters and its effectiveness.

**Keywords :** *vibration control, multiple tuned mass dampers (MTMD), Min. Min. Max. ADMF, optimum parameters, control effectiveness index, dominant ground frequencies.*

## An Experiment Study on Dynamic Fracture Toughness and Damage Expansion for Composite Laminates

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**Abstract :** The combined effect of temperature and strain rate of the mechanical properties for composite lami-