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A NEW MODEL FOR THE ENERGY RELEASE RATE OF FIBRE/MATRIX INTERFACIAL FRACTURE

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Abstract

On the basis of the well-known shear-lag analysis of fibre/matrix interface stresses and the assumption of identical axial strains in the fibre and matrix, a new model for predicting the energy release rate of interfacial fracture of the fibre pull-out test model is attempted. The expressions for stresses in the fibre, matrix and interface are derived. The formula for interfacial debonding energy release rate is given. Numerical calculations are conducted and the results obtained are compared with those of the existing models. © 1998 Elsevier Science Ltd. All rights reserved

Keywords: B. Interfacial, B. debonding, B. fracture, fibre pull-out, energy release rate.

1 INTRODUCTION

The interface between fibres and matrix directly affects the off-axis properties in unidirectional composites and the shear strength of laminates. Interfacial debonding and fibre pull-out problems have attracted the interest of many investigators from mechanics[1] and materials science. The fibre pull-out test using the axisymmetric, cylindrical single-fibre/matrix composite model is one of the four commonly used methods for characterizing interfacial properties.[2] Chua and Piggott[3] and Piggott[4] have studied the fibre pull-out test and obtained the stress distribution and strain energy in the fibre, as well as an expression for the interfacial fracture energy release rate. On the basis of a fracture-mechanics approach, Zhou *et al.*[5] investigated the same problem and gave an elaborated analysis. Owing to the inherent difficulty of the problem, research into this topic is yet to be deepened.

In the present paper, a new shear-lag analysis model is proposed, formulae for stresses distributed in the fibre, matrix and interface are derived, and the expression approximating the interfacial fracture energy release rate is developed. A numerical calculation is conducted of the interfacial fracture energy release rate, G_i, and the result is compared with those of Refs 3 and 6. It is proved that the new model of this paper is valid and predicts a popular result.

2 THEORETICAL MODEL

2.1 The equilibrium relationship

The axisymmetric, cylindrical fibre/matrix composite fibre pull-out test model is shown in Fig. 1. r_f and r_m represent the radius of the fibre and matrix, respectively; L is the embedded fibre length, L' is the free length of the fibre; and $\bar{L} = L + L'$.

According to the simple equilibrium, the stresses in the fibre, σ_f, and in the matrix, σ_m, are related to pull-out stress, T, by the following formulas:

$$\sigma_f = T, \quad \text{for} - L' \leq x < 0^- \tag{1}$$

$$\sigma_f + \frac{2}{r_f^2} \times \int_{r_f}^{r_m} \sigma_m r \, dr = T, \quad \text{for} 0^+ \leq x \leq L \tag{2}$$

Because the distribution function of σ_m is not known, the integration in eqn (2) cannot be performed. However, to a first approximation, the assumption that the plane normal to the axial direction remains plane can be made, as is done in Ref. 5. This assumption leads to the 'identical strain assumption', which implies that the strains in the fibre and matrix are equal and both are uniformly distributed. Of course this assumption is not accurate, as revealed by the present author with a finite-element analysis.[7] It is expected that the variations of strain and stress in the matrix are substantial only near the fibre free end, and near the free matrix surface, where the stress should be zero. Furthermore, from a fracture mechanics viewpoint, at the interface point $x = 0$, the stress possesses a singularity. Nevertheless, for the whole body far from the matrix free surface, the assumption is valid and leads to an acceptable result for G_i (refer to Refs 2, 5 and 6). From the above reasoning and performing the integration of eqn (2), we obtained:

$$\sigma_f + \sigma_m \left(\frac{1-f}{f} \right) = T, \quad \text{for} 0^+ \leq x \leq L \tag{3}$$

where $f = r_f^2 / r_m^2$.

Referring to the analysis of the debonding mechanics in Ref. 6, the matrix stress, σ_m, and the fibre stress, σ_f, for the embedded fibre can also be related to the pull-out stress, T, as follows:

$$\sigma_f|_{x=0^+} = \frac{fT}{f+\lambda(1-f)} = \bar{f}T \quad (4)$$

$$\sigma_m|_{x=0^+} = \frac{fT\lambda}{f+\lambda(1-f)} = \bar{f}T\lambda \quad (5)$$

where $\lambda = E_m/E_f$, E_f and E_m being the Young's moduli of the fibre and matrix, respectively. It can be seen that eqns (4) and (5) imply the identical strain assumption and $\sigma_f/E_f = \sigma_m/E_m$.

2.2 Shear-lag analysis

Following the analysis method for the stress distribution in a fibre embedded in a matrix given by Ref. 8, the formula for the fibre stress for $0^+ \leq x \leq L$ was derived and obtained as:

$$\sigma_f = E_f e + (\sigma_f^+ - E_f e)\cosh(\beta x) - \frac{E_f e \sinh(\beta x)}{\sinh(\beta L)} - \frac{\cosh(\beta L)}{\sinh(\beta L)}(\sigma_f^+ - E_f e)\sinh(\beta x) \quad (6)$$

where

$$\sigma_f^+ = \sigma_f|_{x=0^+}$$

$$\beta^2 = \frac{2G_m}{E_f \ln(r_m/r_f)}$$

and e represents the strain in the matrix in the case of the fibre being absent or the strain at the location far from the fibre. Letting $e=0$, eqn (6) reduces to:

$$\sigma_f = \sigma_f^+\left(\cosh(\beta x) - \frac{\cosh(\beta L)}{\sinh(\beta L)}\sinh(\beta x)\right) \quad (7)$$

In Refs 3 and 4 the fibre stress is given as:

$$\sigma_f = T\left(\cosh(\beta x) - \frac{\cosh(\beta L)}{\sinh(\beta L)}\sinh(\beta x)\right) \quad (8)$$

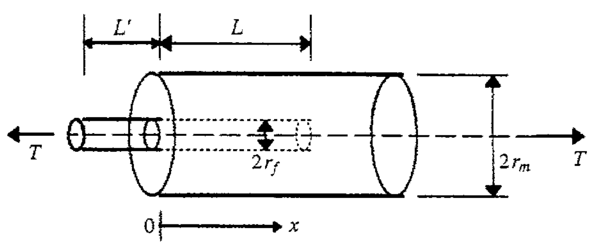

Fig. 1. Schematic drawing of the fibre pull-out model.

Here, $T = \sigma_f|_{x=0^-}$. So, eqn (8) is subjected to two objections: first, it implies that the assumption of $e=0$ is used, however in fact $e = \sigma_m^+/E_m$; second, at $x=0^+$, the fibre stress should be equal to σ_f^+, not T. Because the cross-section of the test sample at $x=0$ (see Fig. 1) changes abruptly, the stress distributions at the left and right sides of the cross-section at $x=0$ are obviously different. Since some portion of the axial force is shared by the matrix at $x=0^+$, the averaged fibre stresses exhibit a discontinuity at $x=0$.

Substituting $e = \sigma_m^+/E_m = T\lambda\bar{f}/E_m$ and $\sigma_f^+ = \bar{f}T$ into eqn (6), we obtained

$$\sigma_f = T\bar{f}\left(1 - \frac{\sinh(\beta x)}{\sinh(\beta L)}\right) \quad (9)$$

Although the two equations (8) and (9) depict a similar tendency of the variation of the axial fibre stress with coordinate x, an evident difference between them is that, at $x=0^+$, eqn (8) yields $\sigma_f^+ = T$; whereas eqn (9) gives $\sigma_f^+ = \bar{f}T$, which implies that the portion of force $T(1-\bar{f})$ is transferred to matrix. The difference between the two equations will become less and less as λ is gets smaller and f gets larger. It should be pointed out that both eqns (8) and (9) are approximate and cannot predict exact stress, especially for the region near the free fibre end (there should be a singularity of some order).

From eqn (3), the stress in matrix was obtained as:

$$\sigma_m = T\left(\frac{f}{1-f}\right)\left[1 - \bar{f}\left(1 - \frac{\sinh(\beta x)}{\sinh(\beta L)}\right)\right] \quad (10)$$

The interfacial shear stress, τ_i, can be derived from the equilibrium relationship

$$\tau_i = \frac{r_f}{2}\frac{d\sigma_f}{dx}$$

and is given as

$$\tau_i = -\frac{nT\bar{f}}{2}\frac{\cosh(\beta x)}{\sinh(\beta L)} \quad (11)$$

where $n = r_f\beta$.

By using the following equilibrium relationship

$$r\tau(r,x) = r_f\tau_i(x) \quad (12)$$

the expression for the shear stress in the matrix is obtained as:

$$\tau(r,x) = -\frac{r_f T\bar{f}n}{r}\left(\frac{\cosh(\beta x)}{\sinh(\beta L)}\right) \quad (13)$$

Thus eqns (9)–(11) and (13) constitute a complete set of stress expressions approximating the testing model of $0^+ \leq x \leq L$ (see Fig. 1).

2.3 Interfacial debonding energy release rate

When interfacial debonding occurs and (or) the interfacial crack grows, the length of free fibre, L', is increasing and the embedded fibre length, L, is decreasing, the strain energy stored in the whole sample changes. Therefore the expression for the interfacial fracture energy release rate can be derived as follows.

The total strain energy, $\bar{U}$, consists of four parts:

$$\bar{U} = U_{L'} + U_{\mathrm{f}} + U_{\mathrm{m}} + U_{\tau} \tag{14}$$

$U_{L'}$ is the strain energy in the free fibre length L',

$$U_{L'} = L' \frac{\pi T^2 r_{\mathrm{f}}^2}{2E_{\mathrm{f}}} = (\bar{L} - L)\frac{\pi T^2 r_{\mathrm{f}}^2}{2E_{\mathrm{f}}} \tag{15}$$

U_{f} and U_{m} are the normal strain energies in the embedded fibre and matrix, respectively, and U_{τ} is the shear strain energy of the matrix:

$$U_{\mathrm{f}} = \frac{\pi T^2 r_{\mathrm{f}}^2 \bar{f}^2}{2E_{\mathrm{f}}} \int_0^L \left(1 - \frac{\sinh(\beta x)}{\sinh(\beta L)}\right)^2 \mathrm{d}x \tag{16}$$

$$U_{\mathrm{m}} = \frac{\pi T^2 (r_{\mathrm{m}}^2 - r_{\mathrm{f}}^2)(f')^2}{2E_{\mathrm{f}}} \int_0^L \left[1 - \bar{f}\left(1 - \frac{\sinh(\beta x)}{\sinh(\beta L)}\right)\right]^2 \mathrm{d}x \tag{17}$$

where $f' = f/(1-f)$.

$$U_{\tau} = \frac{\pi n^2 T^2 r_{\mathrm{f}}^2 \bar{f}^2}{4G_{\mathrm{m}}} \int_{r_{\mathrm{f}}}^{r_{\mathrm{m}}} \frac{\mathrm{d}r}{r} \int_0^L \frac{\cosh^2(\beta x)}{\sinh^2(\beta L)} \mathrm{d}x \tag{18}$$

where G_{m} denotes the shear modulus of the matrix. Following fracture mechanics, the interfacial fracture energy release rate is given by:

$$G_{\mathrm{i}} = \frac{\mathrm{d}\bar{U}}{2\pi r r_{\mathrm{f}}\, \mathrm{d}L}$$

Thus we obtained

$$G_{\mathrm{i}} = G_{L'} + G_{\mathrm{f}} + G_{\mathrm{m}} + G_{\tau} \tag{19}$$

where

$$G_{L'} = -\frac{T^2 r_{\mathrm{f}}}{4E_{\mathrm{f}}} \tag{20}$$

$$G_{\mathrm{f}} = \frac{T^2 r_{\mathrm{f}} \bar{f}^2}{4E_{\mathrm{f}}} \left(\coth^2(\beta L) - \frac{2\cosh(\beta L)}{\sinh^2(\beta L)} + \frac{nL}{r_{\mathrm{f}}} \frac{\cosh(\beta L)}{\sinh^3(\beta L)}\right) \tag{21}$$

$$G_{\mathrm{m}} = \frac{T^2 r_{\mathrm{f}} f'}{4E_{\mathrm{f}}\, \lambda} \left\{1 - 2\bar{f}\left[\coth^2(\beta L)\left(1 - \frac{\bar{f}}{2}\right) + (\bar{f} - 1)\frac{\cosh(\beta L)}{\sinh^2(\beta L)} - \frac{\bar{f}\beta L \cosh(\beta L)}{2\sinh^3(\beta L)}\right]\right\} \tag{22}$$

and

$$G_{\tau} = \frac{T^2 r_{\mathrm{f}} \bar{f}^2}{4E_{\mathrm{f}}} \left(-\frac{\beta L \cosh(\beta L)}{\sinh^3(\beta L)}\right) \tag{23}$$

3 NUMERICAL EXAMPLE

Chua and Piggott[3] have presented the following formula for interfacial fracture energy release rate, G_{i}:

$$G_{\mathrm{i}} = \frac{T^2 r_{\mathrm{f}}}{4E_{\mathrm{f}}} \frac{1}{ns \tanh(ns)} \tag{24}$$

where $s = L/r_{\mathrm{f}}$ and $ns = \beta L$.

The expression for G_{i} presented by Zhou *et al.*[5] is the following:

$$G_{\mathrm{i}} = \frac{T^2 r_{\mathrm{f}}}{4E_{\mathrm{f}}} \frac{\lambda}{\lambda + f'} \tag{25}$$

In addition, Kim and Mai[9] give another equation predicting G_{i}:

$$G_{\mathrm{i}} = \frac{(1 - 2k\nu_{\mathrm{f}})^2}{\left[(1 - 2k\nu_{\mathrm{f}}) + \frac{\gamma}{\lambda}(1 - 2k\nu_{\mathrm{m}})\right]} \frac{T^2 r_{\mathrm{f}}}{4E_{\mathrm{f}}} \tag{26}$$

where

$$k = \frac{\lambda\nu_{\mathrm{f}} + \gamma\nu_{\mathrm{m}}}{\lambda(1 - \nu_{\mathrm{f}}) + 1 + \nu_{\mathrm{m}} + 2\gamma}$$

ν_{f} and ν_{m} are the Poisson's ratios of the fibre and matrix, and

$$\gamma = \frac{r_{\mathrm{f}}^2}{r_{\mathrm{m}}^2 - r_{\mathrm{f}}^2} = \frac{f}{1 - f}$$

Comparative calculations have been conducted for the material parameters pertinent to carbon/epoxy: $E_{\mathrm{f}} = 258{\cdot}6$ GPa, $E_{\mathrm{m}} = 3{\cdot}4$ GPa, $\nu_{\mathrm{m}} = 0{\cdot}35$ and $\nu_{\mathrm{f}} = 0{\cdot}25$.

As for the geometry of the testing model of Fig. 1, this is somewhat arbitrary. Different authors may design different sizes, say, of the radius of matrix, r_{m}. In the calculations of this example, we adopted: $r_{\mathrm{m}} = 16{\cdot}2$ mm, $r_{\mathrm{f}} = 4{\cdot}5$ mm and $L = 130$ mm. The calculated

Table 1. Results of the comparative calculations

	Model			
Equation	(19)	(24)	(25)	(26)
$\overline{G}_i$	0·188	0·403	0·135	0·134

results are listed in Table 1, where $\overline{G}_i$ is normalized energy release rate, i.e.

$$\overline{G}_i = \frac{G_i}{(T^2 r_f/4E_f)}$$

It can be seen from Table 1 that the new model of the present paper [eqn (19)] produces a reasonable result, which is similar to those of eqn (25) and (26), and has a larger difference respect to that of eqn (24).

4 CONCLUDING REMARKS

A new model of debonding and fibre pull-out interfacial fracture energy was attempted on the basis of the 'identical strain assumption' and shear-lag analysis. It is aimed at deepening the insight into the fracture mechanism of the fibre pull-out test. Needless to say, the present analysis is far from exact; to obtain an exact solution complicated elastic theory should be invoked.

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