Application of response number for dynamic plastic response of plates subjected to impulsive loading

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Abstract

A dimensionless number, termed response number, is applied to the dynamic plastic response of plates subjected to dynamic loading. Many theoretical and experimental results presented by different researchers are reformulated into new concise forms with the response number. The advantage of the new forms is twofold: (1) they are more physically meaningful, and (2) they are independent of the choice of units, thus, they have wider range of applications. © 2001 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Plates, beams, shells are basic structural elements for pressure vessels. Up to now, there have been a large number of theoretical and experimental studies on the dynamic plastic behaviour of structures, and in particular of plates subjected to various kinds of dynamic loading. These theoretical and experimental results usually have different target dimensions. In an attempt to compare results of deformed structures of similar geometries, boundary conditions and loading, it seems necessary to normalize all variables into dimensionless groups [1].

Johnson [2] defined a dimensionless number, damage number as

\[
D_n = \frac{\rho V_0^2}{\sigma_0},
\]

which is used for assessing the behaviour of metal structures under dynamic loading. Here \(\rho\) is the material density, \(V_0\) is the impact velocity and \(\sigma_0\) is the yield stress of the material. Johnson’s damage number is a basic dimensionless similarity parameter in material dynamics. This important dimensionless number can be derived by making the equation of motion dimensionless. For example, consider the equation of motion for a one-dimensional problem

\[
\frac{\partial \sigma}{\partial x} = \rho \frac{\partial V}{\partial t},
\]

where \(\sigma\) and \(V\) are stress and particle velocity, respectively. To make it dimensionless, introduce the dimensionless variables as follows,

\[
\Sigma = \frac{\sigma}{\sigma_0}, \quad \tau = \frac{t}{T}, \quad X = \frac{x}{V_0 T}, \quad \tilde{V} = \frac{V}{V_0},
\]

where \(T\) is a characteristic time. By substituting Eq. (3) into Eq. (2), the equation of motion Eq. (2) can be recast as

\[
\frac{\partial \Sigma}{\partial X} = \frac{\rho V_0^2}{\sigma_0} \frac{\partial \tilde{V}}{\partial \tau} = D_n \frac{\partial \tilde{V}}{\partial \tau}.
\]

The expression above demonstrates that the damage number is a dominant dimensionless parameter for the dynamic plastic response of the material. Johnson’s damage number can be presented in terms of the impulse as

\[
D_n = \frac{l^2}{A_0 \rho \sigma_0 d z^2} = \frac{l_0^2}{\rho \sigma_0 d z^2},
\]

where \(l\) is the total impulse, \(l_0\) is the impulse per unit area, \(A_0\) is the area of plate or beam over which the impulse is imparted, and \(t\) is the plate thickness.

When comparing results presented by different researchers with different plate dimensions and different plate materials, Johnson’s damage number does not provide a suitable means for comparison, and therefore an extension to Johnson’s damage number was needed. Recently, a new dimensionless response number has been suggested by Zhao for
dynamic plastic response of beams and plates [3]. This new number can be derived from making the governing equations of beams and plates dimensionless [4], that is,

$$ R_n = \frac{\rho V^2}{\sigma_0} \left( \frac{l}{t} \right)^2 = D_n \left( \frac{l}{t} \right)^2, \tag{6} $$

where \( l \) is the half length of beams or plates and \( t \) is the thickness of beams or plates. Similarly, in terms of the rectangular pressure pulse, the response number can be expressed as follows,

$$ R_n = \frac{l^2}{A^2 \rho \sigma_0 t^2} \left( \frac{l}{t} \right)^2 = \frac{l_0^2}{\rho \sigma_0 t^2} \left( \frac{l}{t} \right)^2. \tag{7} $$

It has been pointed out in Ref. [3] that the response number is an important independent dimensionless number for the dynamic plastic bending and membrane response of structural members. Zhao’s response number takes account of the geometrical influence of the structures on the dynamic response besides the inertia of the applied dynamic loading and the resistance ability of the material to the deformation due to the loading. In the following section, the present paper will recast some theoretical and experimental results of plates under impulsive loading into new forms with the response number.

2. Clamped circular plates subjected to uniform impulsive loading

For a circular plate with radius \( R \) and thickness \( t \) subjected to dynamic loading, the response number can be written as

$$ R_n = \frac{l_0^2}{\rho R \sigma_0 t^2} \left( \frac{R}{t} \right)^2 = D_n \left( \frac{R}{t} \right)^2 \tag{8} $$

Hudson [5] was one of the first to conduct theoretical studies into the influence of dynamic loads on the behaviour of thin disks or circular plates with clamped boundary but no strain rate consideration. Hudson [5] presented the midpoint deflected prediction \( w_m \) of a circular plate as

$$ w_m = \frac{0.318 I}{r R (\rho \sigma_0)^{1/2}}. \tag{9} $$

Combining Eqs. (8) and (9) and \( I = l_0 \pi R^2 \), then the dimensionless mid-point deflection \( w_m / t \) can be written as

$$ \frac{w_m}{t} = 0.318 \pi \frac{l_0}{\rho R (\sigma_0)^{1/2}} R, \tag{10} $$

It should be pointed out that Eq. (10) is not only more concise than Eq. (9), but also more physically meaningful by using Zhao’s response number.

Duffey [6] presented his prediction of \( w_m \) as

$$ w_m = \frac{0.242 l (1 - \nu + \nu^2)^{1/2}}{r R (\rho \sigma_0)^{1/2}}, \tag{11} $$

where \( \nu \) is Poisson’s ratio. By using Eq. (8), we can express Duffey’s \( w_m / t \) as

$$ \frac{w_m}{t} = \frac{0.242 l (1 - \nu + \nu^2)^{1/2}}{r^2 R (\rho \sigma_0)^{1/2}} = 0.760 (1 - \nu + \nu^2)^{1/2} \sqrt{R_n}. \tag{12} $$

Wierzbiicki and Florence [7] gave their result as

$$ w_m = \frac{0.027 l}{r^2 \rho R}. \tag{13} $$

Similarly, Wierzbiicki and Florence’s \( w_m / t \) can be rewritten with the aid of \( R_n \) as follows

$$ \frac{w_m}{t} = 0.266 R_n. \tag{14} $$

Batra and Dubey [8] presented their result for a circular plate as

$$ w_m = \frac{0.382 l}{r R (\rho \sigma_0)^{1/2}}. \tag{15} $$

Then Batra and Dubey’s \( w_m / t \) can be rewritten with \( R_n \) as

$$ \frac{w_m}{t} = 0.382 \pi \frac{l_0 R}{r^2 (\rho \sigma_0)^{1/2}} = 1.20 \sqrt{R_n}. \tag{16} $$

Lippman [9] presented his expression for a circular plate as

$$ w_m = \frac{0.132 l}{r R (\rho \sigma_0)^{1/2}}. \tag{17} $$

Then the dimensionless form will becomes

$$ \frac{w_m}{t} = 0.415 \sqrt{R_n}. \tag{18} $$

Ghosh and Weber [10] presented their expression for \( w_m \) as

$$ w_m = \frac{0.392 l}{r R (\rho \sigma_0)^{1/2}}. \tag{19} $$

Then \( w_m / t \) can be expressed as

$$ \frac{w_m}{t} = 0.392 \pi \frac{l_0 R}{r^2 (\rho \sigma_0)^{1/2}} = 1.232 \sqrt{R_n}. \tag{20} $$

Symonds and Wierzbiicki [11] gave their solution as

$$ w_m = \frac{0.212 l}{r R (\rho \sigma_0)^{1/2}}, \tag{21} $$

which leads to

$$ \frac{w_m}{t} = 0.666 \sqrt{R_n}. \tag{22} $$

Soares Guedes [12] presented his expression as

$$ w_m = \left( \frac{0.068 l^2}{r^2 R \rho \sigma_0 + l^2} \right)^{1/2}. \tag{23} $$
Then \( w_m/t \) can be expressed as

\[
\frac{w_m}{t} = r\left\{ 0.068\pi^2 \left( \frac{l_0^2}{\rho \sigma_t^2} \right) \left( \frac{R}{t} \right)^2 + 1 \right\}^{1/2} - 1
\]

\( = (0.671R_n + 1)^{1/2} - 1. \)  

(24)

Calladine’s result [1] was

\[
w_m = \frac{0.225l}{tR(\rho \sigma_0)^{1/2}},
\]

leading to

\[
\frac{w_m}{t} = 0.707\sqrt{R_n}.
\]

(26)

Perrone and Bhadra [13] presented their result as

\[
w_m = \frac{0.117l}{tR(\rho \sigma_0)^{1/2}}.
\]

(27)

Then \( w_m/t \) can be expressed as

\[
\frac{w_m}{t} = 0.368\sqrt{R_n}.
\]

(28)

Jones [14] presented the following result,

\[
w_m = \frac{0.260l}{tR(\rho \sigma_0)^{1/2}},
\]

and now \( w_m/t \) can be expressed as

\[
\frac{w_m}{t} = 0.817\sqrt{R_n}.
\]

(30)

Nurick [15] presented an experimental solution as

\[
w_m = \frac{0.135l}{tR(\rho \sigma_0)^{1/2}}.
\]

(31)

Then \( w_m/t \) can be expressed as

\[
\frac{w_m}{t} = 0.424\sqrt{R_n}.
\]

(32)

It has been shown in this section that the dynamic plastic response of circular plates subjected to impact loading is solely a function of Zhao’s response number.

3. Clamped quadrilateral plates subjected to uniform impulsive loading

In the case of quadrilateral plates, there should exist two dimensionless numbers. One is Zhao’s response number, which like the circular plate above, has the form

\[
R_n = \frac{l_0^2}{\rho \sigma_t^2} \left( \frac{l}{t} \right)^2 = D_n \left( \frac{l}{t} \right)^2,
\]

where \( l \) is the half length of the quadrilateral plate. The other dimensionless number is geometry number

\[
\beta = \frac{l}{b},
\]

(33)

where \( b \) is the half breadth of the plate.

Jones et al. [16] presented their theoretical prediction for a quadrilateral plate under dynamic loading as

\[
w_m = \frac{0.33l^2}{t^3L^2\rho \sigma_0} \left[ \left( 3 + \frac{B^2}{L^2} \right)^{1/2} - \frac{B}{L} \right] t,
\]

(35)

where \( B \) and \( L \) are the breadth and length of the quadrilateral plate, respectively. With Eqs. (33) and (34) and the half breadth of the plate \( b = B/2 \), \( w_m/t \) can be expressed as

\[
\frac{w_m}{t} = \frac{0.33l^2}{t^3L^2\rho \sigma_0} \left[ \left( 3 + \frac{B}{L} \beta \right)^{1/2} - \frac{1}{\beta} \right] t
\]

(36)

In Ref. [17], Jones presented the theoretical predictions for two cases. When \( \beta = 1 \), the result was

\[
w_m = t \left[ \left( 1 + \frac{l^2}{6\rho \sigma_t^4L^2} \right)^{1/2} - 1 \right].
\]

(37)

With the utility of response number \( R_n \) and \( \beta \), \( w_m/t \) can be written as

\[
\frac{w_m}{t} = t \left\{ 1 + \frac{2}{3} \left( \frac{l_0^2}{\rho \sigma_t^2} \right) \left( \frac{l}{t} \right)^2 \right\}^{1/2} - 1 \}
\]

(38)

\[
= \left( 1 + \frac{2}{3} R_n \right)^{1/2} - 1.
\]

When \( \beta = 1.616 \), the result was

\[
w_m = 0.776 \left( \left( 1 + \frac{1.98l^2}{6\rho \sigma_t^4L^2} \right)^{1/2} - 1 \right).
\]

(39)

and then the corresponding \( w_m/t \) can be written as

\[
\frac{w_m}{t} = 0.776t \left\{ 1 + 0.505 \left( \frac{l_0^2}{\rho \sigma_t^2} \right) \left( \frac{l}{t} \right)^2 \right\}^{1/2} - 1 \}
\]

(40)

\[
= 0.776 \left( 1 + 0.505 R_n \right)^{1/2} - 1.
\]

In Ref. [14], Jones presented the following two results when \( \beta = 1 \). One of the results was

\[
w_m = t \left( 1 + \frac{l^2}{6\rho \sigma_t^4L^2} \right)^{1/2} - 1.
\]

(41)

and then \( w_m/t \) can be rewritten as

\[
\frac{w_m}{t} = \left( 1 + \frac{2}{3} R_n \right)^{1/2} - 1.
\]

(42)

Another result was

\[
w_m = 0.857t \left( 1 + \frac{1.56l^2}{6\rho \sigma_t^4L^2} \right)^{1/2} - 1.
\]

(43)
and then \( \frac{w_m}{l} \) can be expressed as

\[
\frac{w_m}{l} = 0.857[(1 + 1.04R_a^{1/2}) - 1].
\]  

(44)

Baker [18] presented his two predictions for a quadrilateral plate. When \( \beta = 1 \), the result was

\[
w_m = \left[ \frac{0.077L^2}{t^2L^2 \rho \sigma_0} + 0.177r^2 \right]^{1/2} - 0.421l,
\]

(45)

then \( \frac{w_m}{l} \) can be expressed as

\[
\frac{w_m}{l} = (0.308R_a + 0.177)^{1/2} - 0.421.
\]

(46)

When \( \beta = 1.616 \), Baker’s prediction was

\[
w_m = \left[ \frac{0.121L^2}{t^2L^2 \rho \sigma_0} + 0.189r^2 \right]^{1/2} - 0.435l,
\]

(47)

then \( \frac{w_m}{l} \) can be expressed as

\[
\frac{w_m}{l} = (0.185R_a + 0.189)^{1/2} - 0.435.
\]

(48)

Nurick presented an experimental result for a quadrilateral plate in Ref. [15],

\[
w_m = \frac{0.235I}{t^2L^2 \rho \sigma_0}.
\]

(49)

then \( \frac{w_m}{l} \) can be expressed as

\[
\frac{w_m}{l} = 0.470 \left[ R_a \left( \frac{1}{\hat{\beta}} \right) \right]^{1/2}.
\]

(50)

4. Discussion and concluding remarks

It has been demonstrated that Zhao’s response number is the major influence on dynamic plastic response of structures subjected to dynamic loading. The theoretical and experimental results presented by many researchers for clamped plates subjected to impulsive loading have been reformulated into new concise forms with Zhao’s dimensionless response number \( R_a \).

For dynamic plastic response of circular plates, the mid point deflection of the plate is solely a function of the response number whereas for quadrilateral plates, a second dimensionless number, geometry number plays a role.

It should also be pointed out that the response number, suggested for the dynamic plastic response of beams and flat plates, has been generalized in Ref. [19] to study the elastic, plastic, dynamic elastic as well as dynamic plastic bulking problems of columns, plates as well as shells. Therefore, the response number will have a wider application for problems of structural dynamics.

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