

Strain gradient effects on deformation strengthening behavior of particle reinforced metal matrix composites

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Abstract

Dislocation models with considering the mismatch of elastic modulus between matrix and reinforcing particles are used to determine the effective strain gradient η for particle reinforced metal matrix composites (MMCp) in the present research. Based on Taylor relation and the kinetics of dislocation multiplication, glide and annihilation, a strain gradient dependent constitutive equation is developed. By using this strain gradient-dependent constitutive equation, size-dependent deformation strengthening behavior is characterized. The results demonstrate that the smaller the particle size, the more excellent in the reinforcing effect. Some comparisons with the available experimental results demonstrate that the present approach is satisfactory.

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1. Introduction

Due to potential applications of particle-reinforced metal matrix composites (MMCp), a considerable attention has been given to investigating the mechanical behavior of MMCp during the past decades. It is well known that the mechanical behavior of this class of materials is significantly influenced by their microstructure. During the past two decades, several attempts have been made to explore the relationship between microstructure and the deformation behavior in MMCp. Continuum models including the cell model [1,2], the modified shear lag theory [3] and homogenization models [4] provide a dependence of flow stress on volume fraction of reinforcing particles but not on particle size. Experimental results and preliminary theoretical investigations [5–12], however, have demonstrated that both particle size and volume fraction exert an influence. In order to explain the size effect in materials, several phenomenological strain gradient

plasticity theories were developed by Aifantis [13], Fleck et al. [14–16], Chen and Wang [17]. Quite dissimilar to the phenomenological strain gradient plasticity theories, Nix and Gao [18] proposed a strain gradient plasticity theory, the length scale is naturally introduced from the indentation test results and the Taylor relation. Recently, Gao et al. [19–21] developed a mechanism-based strain gradient plasticity theory (MSG) based on a multiscale framework linking the microscale concept of dislocations to mesoscale concept of plastic strain and strain gradient. Furthermore, Fleck and Hutchinson [15] suggested that an effective strain gradient can be characterized by $\eta = \sqrt{c_1 \eta_{iik} \eta_{ijk} + c_2 \eta_{ijk} \eta_{ijk} + c_3 \eta_{ijk} \eta_{kji}}$ and determined the three effective strain gradient constants c_1 , c_2 and c_3 from experimental results. Due to scarcity of the related experimental data, Gao et al. [19] determined the three effective gradient constants c_1 , c_2 and c_3 in their MSG theory from several dislocation models. However, these dislocation models are mainly focused on monolithic-phase materials. For particle-matrix two-phase MMCp, the introduction of reinforcing particles into the metal matrix will obviously influence the formation and activity of dislocations in the matrix. In this paper, this effect is considered for

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determining the constants c_1 , c_2 and c_3 , which characterize the effective strain gradient in MMCp.

In view of the aforementioned observations, dislocation models considering the effects of reinforcing particles are used to determine c_1 , c_2 and c_3 , related to effective strain gradient η , in the present study. Based on Taylor relation and the kinetics of dislocation multiplication, glide and annihilation, a strain gradient-dependent constitutive equation of MMCp is developed. By using this relationship, a size-dependent deformation strengthening behavior for MMCp is characterized. A comparison with the available experimental results demonstrates that the present approach is satisfactory.

2. Strain gradient dependent constitutive equation

For two-phase MMCp, current experimental investigations have demonstrated that the mechanical behavior is dependent strongly on the reinforcing particle size [5–7]. Obviously, classical constitutive equations containing no internal length parameters cannot capture size effect. In order to characterize the size-dependent inelastic behavior of MMCp, some strain gradient terms or length parameters should be incorporated into the classical constitutive equation.

To this end, a strain gradient strengthening law was developed by Dai et al. [8]. However, in this study, the kinetics of dislocation multiplication, glide and annihilation are neglected with the assumption that an incorporation of particles does not change the density of statistically stored dislocations. Apparently, introducing reinforcing particles into the metal matrix not only changes the geometrically necessary dislocation density but also changes the statistically stored dislocation density. With addition of reinforcing particles, the geometrically necessary dislocation will be generated to accommodate the mismatch of plastic deformation in the matrix. On the other hand, the incorporation of reinforcing particles will block the movement of dislocation and reduce the probability of trapping each other in a random way, such that the statistically stored dislocation density in the matrix will be decreased. Considering these effects, the total dislocation density of MMCp ρ_T can be written as:

$$\rho_T = \rho_G + (\rho_S + \rho_a) \quad (1)$$

where ρ_S is the statistically stored dislocation density in unreinforced matrix, ρ_G is the geometrically necessary dislocation density in MMCp, and ρ_a is the diminished part of the statistically stored dislocation due to the addition of reinforcing particles.

Assuming that the strengthening of MMCp is attributed to the deformation resistance induced by the reinforcing particles. According to Taylor relation, the

flow stress of MMCp can be given by:

$$\sigma_c = \sqrt{3}\alpha b\mu_m\sqrt{\rho_G + \rho_S - \rho_a} \quad (2)$$

where μ_m is the shear modulus of the matrix material, α a dimensionless parameter, and b is Burgers vector. The flow stress for the unreinforced matrix can be written as:

$$\sigma_m = \sigma_Y f(\varepsilon) = \sqrt{3}\alpha b\mu_m\sqrt{\rho_S} \quad (3)$$

For convenience, the state of plastic yield can be defined as:

$$\sigma = \sigma_Y, \quad \varepsilon = \varepsilon_Y, \quad f(\varepsilon_Y) = 1 \quad (4)$$

Combining Eq. (2) with Eq. (3) results in:

$$\left(\frac{\sigma_c}{\sigma_m}\right)^2 = 1 + \hat{l}\eta - f_{ad} \quad (5)$$

where:

$$\hat{l} = 3\alpha^2 \left(\frac{\mu_m}{\sigma_m}\right)^2 b \quad (6)$$

is identified as the material length, $f_{ad} = \rho_a/\rho_S$ is the percentage of ρ_a in ρ_S , and η is effective strain gradient and given by Fleck et al. [14] and Gao et al. [19]:

$$\eta = \rho_G b \quad (7)$$

Since b is a material parameter and is positive, it can be seen, from Eq. (7), that the effective strain gradient η is in direct proportion to the geometrically necessary dislocation density ρ_G . In our previous studies [8,9], we found that the geometrically necessary dislocation density ρ_G is controlled by the particle diameter d_p and the smaller the particle size the higher the geometrically necessary dislocation density. So, it can be deduced from Eq. (7) that the smaller the particle size the greater the effective strain gradient. This point will be demonstrated in details in next section.

According to the principle of the kinetics of dislocation, the value of f_{ad} is related to both microstructures and the extent of plastic deformation. There is not as yet an appropriate approach to identify it from experimental results. Because of the difficulty in identifying f_{ad} , we introduce another parameter to characterize it. From the available experimental data, we find that $\hat{l}\eta$ is usually less than one. Therefore, using Taylor expansion leads to:

$$1 + \hat{l}\eta - f_{md} \cong (1 + \hat{l}\eta)^\zeta \quad (8)$$

where $f_{md} = (1 - \zeta)[1 + 1/2\zeta\hat{l}\eta + 1/6\zeta(\zeta - 2)(\hat{l}\eta)^2]$ is the first order and second order terms in Taylor expansion. Here, the value of ζ ranges from 0 to 1. The benefit of Eq. (8) is that ζ can be determined from the stress–strain curves of MMCp with different size of reinforcing particles. Combining Eq. (5) with Eq. (8) leads to the following strain gradient-dependent constitutive equation for MMCp:

$$\left(\frac{\sigma_c}{\sigma_m}\right)^2 = (1 + \hat{l}\eta)^\zeta \quad (0 < \zeta \leq 1) \quad (9)$$

It is seen from this strain gradient dependent constitutive equation that strengthening (σ_c/σ_m) is controlled by both the effective strain gradient η and the material length \hat{l} . Furthermore, we find that the higher in the effective strain gradient the better in strengthening effect or the greater in the value of (σ_c/σ_m). According to Eq. (7) and the related discussions, we have known that the smaller the particle size the greater the effective strain gradient. So, from the strain gradient dependent constitutive equation, one can reach a conclusion that the smaller the particle size the better the strengthening effect for MMCp. Obviously, this qualitative analytical result is in accordance with the available experimental observations [5–7]. It is noted that the incorporation of strain gradient term into the conventional constitutive equation in the present approach is based on deformation mechanism and dislocation models, instead of adopting a phenomenological assumption [13–15].

3. Determination of effective strain gradient constants

To characterize size-dependent behavior of MMCp by making use of the strain gradient-dependent constitutive equation, the effective strain gradient should be first determined. According to Ashby's suggestion [22], the dislocations stored in metals during straining can be divided into two kinds: geometrically necessary dislocations and statistically stored dislocations. It has been demonstrated by Fleck et al. [14] and Gao et al. [19] that the effective strain gradient is controlled by the geometrically necessary dislocations that are required for compatible deformation of various parts in materials. Following the suggestion of Gao et al. [19], the effective strain gradient η is:

$$\eta = \sqrt{c_1 \eta_{iik} \eta_{jjk} + c_2 \eta_{ijk} \eta_{ijk} + c_3 \eta_{ijk} \eta_{kji}} \quad (10)$$

where the three effective constants (c_1, c_2, c_3) scale the three quadratic invariants for the incompressible third order strain gradient tensor η_{ijk} . Fleck and Hutchinson [15] attempted to determine c_1, c_2 and c_3 from experimental data. Due to a scarcity of experiments on strain gradient effects, Gao et al. [19] determined the three constants from three typical dislocation models. However, it is noted that Eq. (10) is initially developed for monolithic materials. To extend this equation to the case of discontinuous-reinforced MMCp, the effect of the reinforcing particles should be included.

In order to characterize the effective strain gradient in MMCp, the three constants c_1, c_2 and c_3 should be determined. To this end, three typical dislocation models: plane strain bending, pure torsion and 2-D axisymmetric cell, as did by Gao et al. [19] for

monolithic materials, are adopted. However, to determine c_1, c_2 and c_3 , the strain fields and density of geometrically necessary dislocation should be known. Since MMCp is a heterogeneous medium, even for simple cases, the strain fields in MMCp are very difficult to be obtained. To overcome this difficulty, MMCp is assumed to be a homogeneous effective medium, which is equal to the homogeneous matrix medium in a background of additional geometrically necessary dislocations generated by the presence of reinforcing particles. The total geometrically necessary dislocation density in MMCp can be written as:

$$\rho_G = \rho_G^0 + \rho_G' \quad (11)$$

where ρ_G^0 is the geometrically necessary dislocation density in the unreinforced matrix medium and ρ_G' is the additional geometrically necessary dislocation density due to the addition of reinforcing particles. Since ρ_G^0 and strain fields in the matrix medium are available, the current task is the determination of ρ_G' . This task will be completed by using the following three dislocation models, (a) plane strain bending; (b) simple torsion; and (c) 2-D axisymmetric tension of MMCp cell.

Consider a MMCp under plane bending. Due to a mismatch of elastic modulus between the matrix and the particles, a lot of geometrically necessary dislocations will be generated for accommodating the distortion deformation. This additional geometrically necessary dislocation configuration is shown in Fig. 1. The additional geometrically necessary dislocation density can be determined by the mismatch deformation volume in MMCp. According to the deformation-geometry condition under plane strain bending, the mismatched volume around a particle in the cell can be determined as:

$$\Delta V = \frac{1}{2} k f_p H^3 l \quad (12)$$

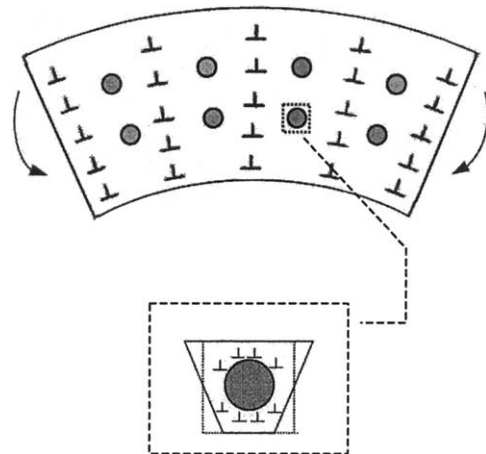


Fig. 1. Plane strain bending of composites and a cubical cell of composites including a particle plane strain bending.

where H is the length of each cubical cell, f_p the volume fraction of particles, k the curvature, and l is the unit length of arc. The number of the geometrically necessary dislocation loops imposed on the surface of a particle to accommodate the mismatch of plastic deformation n is given by:

$$n = \frac{4H^3}{\pi d_p^2 b} k f_p \quad (13)$$

where b is Burgers vector, d_p is the diameter of particles. In this cubical cell, the geometric condition gives:

$$H^3 = \frac{1}{6f_p} \pi d_p^3 \quad (14)$$

The total number of particles N_p is:

$$N_p = \frac{6f_p}{\pi d_p^3} \quad (15)$$

If the averaged length of each dislocation loop is taken as $w = \pi d_p/2$, then the additional geometrically necessary dislocation density ρ'_G is:

$$\rho'_G = N_p n w = \frac{2f_p k l}{b d_p} \quad (16)$$

For the case of unreinforced matrix material under plane bending, the geometrically necessary dislocation density ρ_G^0 is written as [19]:

$$\rho_G^0 = \frac{b}{k} \quad (17)$$

According Eq. (11), the total density of geometrically necessary dislocation in MMCp is given by:

$$\rho_G = \frac{k}{b} + \frac{2f_p k l}{b d_p} \quad (18)$$

According to Eq. (7), the effective strain gradient in MMCp is written as:

$$\eta = \frac{\rho_G}{b} = \left(1 + \frac{2f_p l}{d_p}\right) k \quad (19)$$

On the other hand, Gao et al. [19] have shown that the non-zero components of quadratic invariants of the third-order strain gradient tensor η_{ijk} are:

$$\eta_{iik}\eta_{jjk} = 4k^2, \quad \eta_{ijk}\eta_{ijk} = 4k^2, \quad \eta_{ijk}\eta_{kji} = 0 \quad (20)$$

Combining Eqs. (10), (19) and (20) leads to:

$$c_1 + c_2 = \frac{1}{4} \left(1 + \frac{2f_p l}{d_p}\right)^2 \quad (21)$$

For simple torsion of a cylinder of radius R shown in Fig. 2, the mismatched volume around a particle in a unit cell equal to:

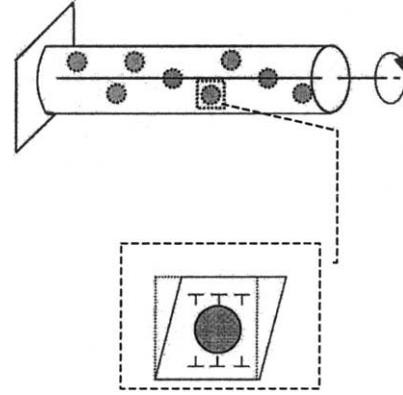


Fig. 2. Simple torsion of composites and a cubical cell of composites including a particle under simple torsion.

$$\Delta V = k l H^3 f_p \quad (22)$$

The number of the geometrically necessary dislocation loops imposed on the surface of a particle to accommodate the mismatch of simple torsion deformation is given by:

$$n = \frac{8kH^3 l f_p}{\pi d_p^2 b} \quad (23)$$

In the cubical cell, the geometric condition gives:

$$H^3 = \frac{1}{6f_p} \pi d_p^3 \quad (24)$$

If the averaged length of each dislocation loop is taken to be $w = \pi d_p/2$, then the additional geometrically necessary dislocation density ρ'_G is given by:

$$\rho'_G = \frac{4f_p k l}{b d_p} \quad (25)$$

Then the total density of geometrically necessary dislocation in MMCp is written as:

$$\rho_G = \frac{k}{b} + \frac{2f_p k l}{b d_p} \quad (26)$$

where k/b is the density of geometrically necessary dislocations in the unreinforcing matrix material. So, according to Eq. (7), the effective strain gradient expressed as:

$$\eta = \left(1 + \frac{4f_p l}{d_p}\right) k \quad (27)$$

From Eqs. (18), (19), (26) and (27), it is clearly seen that the smaller the particle size (d_p) the higher the geometrically necessary dislocation density (ρ_G) and the effective strain gradient (η). On the other hand, the non-zero components of quadratic invariants of the third-order strain gradient tensor η_{ijk} on simple torsion are given by [19]:

$$\eta_{iik}\eta_{jjk} = 0, \quad \eta_{ijk}\eta_{ijk} = 4k^2, \quad \eta_{ijk}\eta_{kji} = -2k^2 \quad (28)$$

Combining Eqs. (10), (27) and (28) leads to:

$$c_1 - \frac{1}{2}c_2 = \frac{1}{4} \left(1 + \frac{4f_p l}{d_p}\right)^2 \quad (29)$$

Now, consider the case of 2-D representative particle/matrix cell under axisymmetric loading, the additional geometrically necessary dislocation configuration is shown in Fig. 3. If particle is replaced by a void, the present representative cell is the same as the 2-D axisymmetric void growth model used by Gao et al. [19]. So, the displacements, strains and strain gradient tensor η_{ijk} in the present representative cell are the same as those of the 2-D axisymmetric void growth model, except that the displacement u_0 is now replaced by $\varepsilon_p a$. Here ε_p is the strain and a is the radius of particles. The dislocation model for MMCp under 2-D axisymmetric loading suggests:

$$c_2 + c_3 = \frac{1}{4} \quad (30)$$

Combining Eqs. (21), (29) and (30) leads to:

$$c_1 = \frac{1}{4} \left(1 + \frac{2lf_p}{d_p}\right)^2 - \frac{1}{6} \left(1 + \frac{4lf_p}{d_p}\right)^2 - \frac{1}{12}$$

$$c_2 = \frac{1}{6} \left(1 + \frac{4lf_p}{d_p}\right)^2 + \frac{1}{12}$$

$$c_3 = -\frac{1}{6} \left(1 + \frac{4lf_p}{d_p}\right)^2 + \frac{1}{6} \quad (31)$$

Obviously, the effective strain gradient in MMCp can be completely determined by making use of Eqs. (10) and (31). It is noted that the effects of volume fraction and size of particles on the three constants c_1 , c_2 and c_3 , which measure the effective strain gradient are included. If $f_p = 0$, the result given by Eq. (31) is the same as that of monolithic materials [19], i.e. $c_1 = c_2 = 0$, $c_3 = 1/4$.

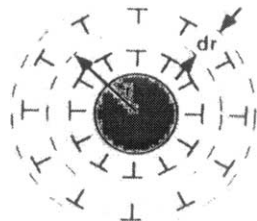


Fig. 3. The representative cell of MMCp under 2-D axisymmetric force.

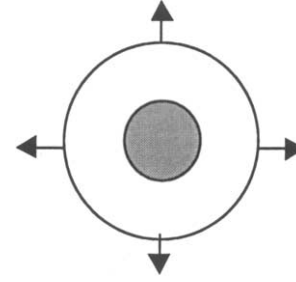


Fig. 4. The representative cell model of MMCp.

4. Comparisons and discussions

In order to identify the feasibility of the present approach, comparisons with available experimental data are made. Lloyd [7] and Ling et al. [5] examined, respectively, an aluminum alloy matrix reinforced with silicon carbide particles and observed increases in strength as the particle size was reduced from 16 to 7.5 μm and from 37 to 3 μm for a fixed particle volume fraction. Classical plasticity fails to explain the size dependence because its constitutive model does not possess an internal material length. In this section, we use the strain gradient dependent constitutive equation to model the size-dependent behavior of composite.

To characterize behavior of MMCp, a simple representative cell model is adopted. The model consists of a spherical particle of diameter d_p embedded in a concentric spherical matrix layer having an outer diameter d_c , as shown in Fig. 4. The radii represents the particle volume fraction f_p in the composite:

$$\frac{d_p^3}{d_c^3} = f_p \quad (32)$$

The particle is elastic, while the matrix is modeled as an incompressible solid. The system is subjected to symmetric tension, on the outer surface. In spherical coordinates, the non-vanishing displacement, strain and strain gradient are:

$$u_R = \frac{R_0^2}{R^2} u_0$$

$$\varepsilon_{RR} = -2\varepsilon_{\theta\theta} = -2\varepsilon_{\phi\phi} = -2\frac{R_0^2}{R^3} u_0$$

$$\eta_{RRR} = -2\eta_{R\theta\theta} = -2\eta_{\theta R\theta} = -2\eta_{R\phi\phi} = -2\eta_{\phi R\phi} = -2\eta_{\theta\theta R}$$

$$= -2\eta_{\phi\phi R} = 6\frac{R_0^2}{R^4} u_0 \quad (33)$$

For this case, the effective strain gradient is calculated using Eq. (10) and is given by:

$$\eta = \sqrt{(72c_1 + 90c_2 + 54c_3) \frac{R_0^2}{R^4} u_0 \frac{R_0^2}{R^4} u_0} \quad (34)$$

By volume integral averaging, the average effective strain gradient in a representative cell of MMCp can be written as:

$$\bar{\eta} = \sqrt{(72c_1 + 90c_2 + 54c_3)} \frac{\varepsilon_e \sqrt[3]{f_p}}{4d_p(1 - \sqrt[3]{f_p})} \ln \frac{1}{\sqrt[3]{f_p}} \quad (35)$$

where ε_e is the effective strain in the matrix. In order to mimic Lloyd's (1994) uniaxial tension experiments, Gao et al. [21] used the surface effective strain as the effective strain ε_e in the matrix. Here, we used the average strain as an effective strain. We find that on using the strain on the outer surface or on the inner surface as ε_e , the errors to the final results can be neglected. The variation of average effective strain gradient ($\bar{\eta}$) with d_p is shown in Fig. 5. In this figure, it is demonstrated that the smaller the particle size the larger the effective strain gradient.

Based on Eqs. (9) and (35), the effect of reinforcing particle size is shown in Fig. 6. The results demonstrate that the smaller the size of particles, the higher in the strain gradient and more in the contribution to strengthening effect.

To verify the aforementioned approach, we make comparisons with the available experimental results presented in [5–7]. It is observed, for an incompressible solid, the spherically symmetric strains are identical to those in uniaxial compression:

$$(\varepsilon_{RR}, \varepsilon_{\theta\theta}, \varepsilon_{\phi\phi}) = -\varepsilon_e \left(1, -\frac{1}{2}, -\frac{1}{2} \right) \quad (36)$$

It should be pointed out that the spherically symmetric stresses could be decomposed into a hydrostatic part and a uniaxial part:

$$(\sigma_{RR}, \sigma_{\theta\theta}, \sigma_{\phi\phi}) = (\sigma_{\theta\theta}, \sigma_{\theta\theta}, \sigma_{\theta\theta}) + (\sigma_{RR} - \sigma_{\theta\theta}, 0, 0) \quad (37)$$

where the 'uniaxial stress' $\sigma_{RR} - \sigma_{\theta\theta}$ is the same as the effective stress σ_e . The hydrostatic part $(\sigma_{\theta\theta}, \sigma_{\theta\theta}, \sigma_{\theta\theta})$ does not cause plastic deformation. Under the condition

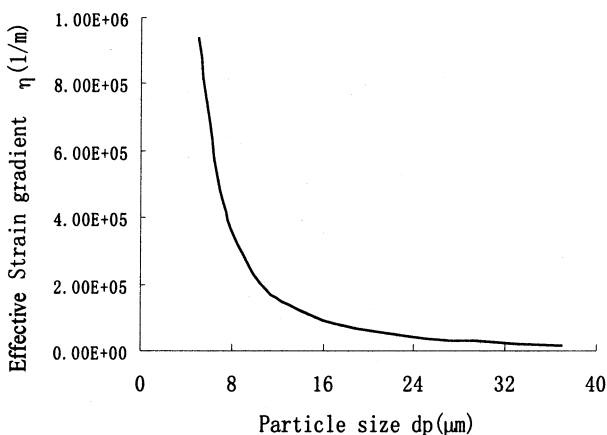


Fig. 5. The relation between η , and d_p .

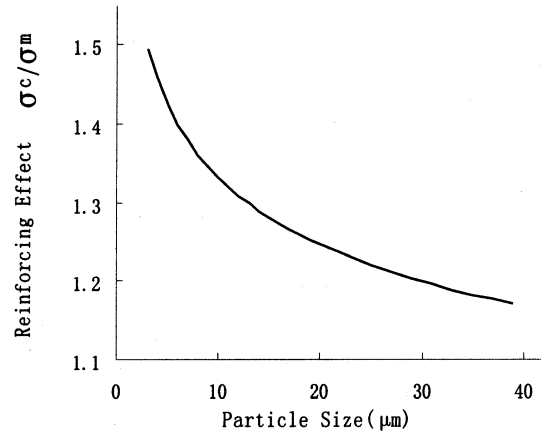


Fig. 6. The effect of reinforcing particle size in MMCp.

of neglecting elastic and shear strains, the effective strain can be written as:

$$\varepsilon_e = \varepsilon \quad (38)$$

Based on Eqs. (9), (35) and (38), the size-dependent behavior of MMCp can be determined if the constitutive behavior of the matrix σ_m is known. In the present calculations, σ_m is determined from the experimental data. The constitutive parameter ζ of MMCp can be determined by making use of the experimental data. Fig. 7 presents a comparison of the calculated flow stress-strain curves with the experimental results for SiCp/2124Al composites made by Ling et al. [5]. Figs. 8 and 9 provide a comparison of the calculated flow stress strain curves with the experimental results for SiCp/A356–T4 and SiCp/A356–T6 composites [7]. From these comparisons, we find that the calculated results based on the present approach agree well with the corresponding experimental results.

Actually, for a MMCp, the reinforcing particle deform elastically, the plastic deformation occurs only in the metal matrix. Obviously, the spacing between particles is an important geometrical parameter for

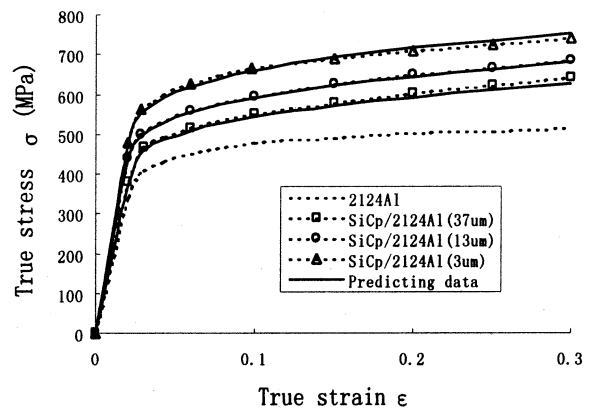


Fig. 7. A comparison of the calculated flow stress–strain curve with the related experimental stress–strain result of SiCp/2124Al composites.

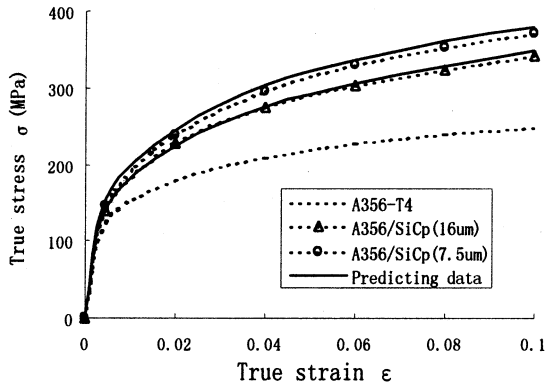


Fig. 8. A comparison of the calculated flow stress–strain curve with the related experimental stress–strain result of SiCp/A356–T4 composites.

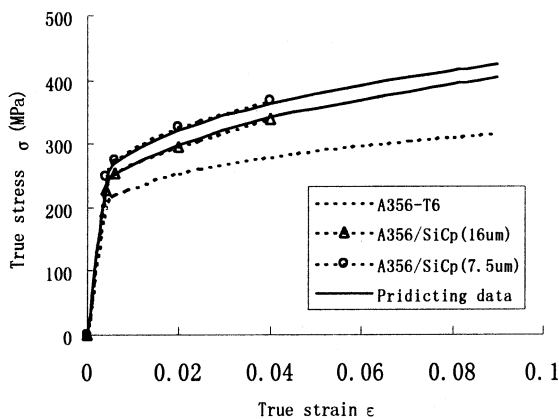


Fig. 9. A comparison of the calculated flow stress–strain curve with the related experimental stress–strain result of SiCp/A356–T6, composites.

controlling an inhomogeneous plastic deformation in the matrix. Such an inhomogeneous plastic deformation can be effectively characterized by strain gradient. According to [23], the average edge–edge spacing between particles in MMCp is given by:

$$\lambda_p = \frac{1}{2} d_p \left(\sqrt{\frac{2\pi}{3f_p} - \frac{4}{\pi}} \right) \quad (39)$$

It is seen from this equation that the smaller the particle size the smaller the particle spacing if the volume fraction of the reinforcing particle is kept fixed. Previous studies have demonstrated that the smaller the spacing the higher the strain gradient in MMCp [8]. The high strain gradient will result in a high work hardening and a high strengthening effect. So, the smaller in the particle size the better the strengthening effect. Obviously, this simple qualitative analysis and the aforementioned quantitative analysis based on strain gradient dependent constitutive equation draw an identical conclusion. This demonstrates that the present approach is satisfactory for characterizing the size-dependent behavior of MMCp.

5. Conclusions

In this paper, we have proposed three typical dislocation models for determining three effective strain gradient parameters. Based on these parameters, the effective strain gradient in two-phase MMCp is determined. According to Taylor strengthening relation and Ashby's geometrically necessary dislocation concept, a strain gradient dependent constitutive equation for MMCp is presented. By using this strain gradient dependent constitutive equation, the size-dependent behavior of MMCp is predicted. The results demonstrate that the smaller the reinforcing particle size, the higher is strain gradient and the better the strengthening effect for MMCp. A comparison with the available experimental results have shown that the present approach is satisfactory.

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