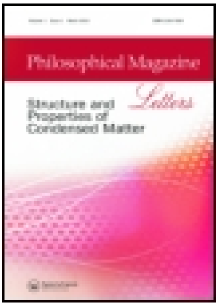

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Scaling relationships in indentation of power-law creep solids using self-similar indenters

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Scaling relationships in indentation of power-law creep solids using self-similar indenters

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ABSTRACT

We use dimensional analysis to derive scaling relationships for self-similar indenters indenting solids that exhibit power-law creep. We identify the parameter that represents the indentation strain rate. The scaling relationships are applied to several types of indentation creep experiment with constant displacement rate, constant loading rate or constant ratio of loading rate over load. The predictions compare favourably with experimental observations reported in the literature. Finally, a connection is found between creep and 'indentation-size effect' (i.e. changing hardness with indentation depth or load).

§ 1. INTRODUCTION

For more than 100 years, indentation experiments have been performed to obtain the hardness of materials. Recent years have seen a growing need to measure the mechanical properties of materials on small scales and significant improvements in indentation equipment. Today's equipment allows one to monitor continuously both the load F and the displacement h with accuracy in the nanometre and micro-newton range. In addition to hardness and elastic modulus, there is a renewed interest in using indentation with self-similar indenters (e.g. conical or pyramidal) to study the strain-rate-dependent properties of metals (Atkins *et al.* 1966, Mayo and Nix 1988, Syed and Pethica 1997, Lucas and Oliver 1999), ceramics (Han and Tomozawa 1990, Grau *et al.* 1998) and polymers (Briscoe *et al.* 1998, Tsukruk *et al.* 1998). A few theoretical and numerical studies of indentation in strain-rate-dependent solids have also appeared recently in the literature (Hill 1992, Bower *et al.* 1993).

Several types of indentation experiment have been proposed to gain insight into the strain-rate-dependent properties of materials using self similar indenters. Examples include the use of a constant loading rate $\dot{F}$, a constant displacement rate $\dot{h}$ or the keeping of parameters such as $\dot{h}/h$ or $\dot{F}/F$ constant. The 'indentation strain rate' is typically defined as $\dot{h}/h$ (Atkins *et al.* 1966, Mayo and Nix 1988, Syed

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and Pethica 1997). The strain-rate dependence of a measured property, such as hardness, is then expressed in terms of $\dot{h}/h$.

In this letter, we use dimensional analysis to derive scaling relationships for self-similar conical and pyramidal indenters indenting solids that exhibit power-law creep. This work is an extension of our previous work on indentation in elastic–plastic solids without strain-rate dependence (Cheng and Cheng 1998, 1999). We show that the parameter $\dot{h}/h$ can indeed be chosen to represent indentation strain rate. The scaling relationships are then applied to several types of indentation creep experiment in which $\dot{h}$, $\dot{F}$ or $\dot{F}/F$ is kept constant. The predictions are compared with experimental behaviour reported in the literature. We also show that an 'indentation-size effect' occurs in some but not all types of indentation creep experiment.

§ 2. Dimensional analysis

We consider a three-dimensional rigid self-similar indenter indenting normally into a homogeneous solid with power-law creep (Lubliner 1990, Dieter 1976):

$$\sigma = b\dot{\varepsilon}^{m}, \tag{1}$$

where σ is the stress, $\dot{\varepsilon}$ is the strain rate, and b and m are material constants. The geometry of the self-similar indenter (e.g. conical or pyramidal) can be described by a set of angles collectively denoted by θ. We assume that the friction coefficient at the contact surface between the indenter and the solid is zero. By assuming equation (1), the effects of elasticity, such as Young's modulus and Poisson's ratio, are precluded.

The quantities of interest include the force F and the contact area A_c under load from which the hardness H under load can be evaluated:

$$H = \frac{F}{A_c}. \tag{2}$$

For an isotropic solid obeying the creep rule given in equation (1), the two dependent variables F and A_c must be functions f and g of all the independent governing parameters b, m, indenter displacement h, rate $\dot{h}$ of indenter displacement and indenter angles θ:

$$F = f(b, m, h, \dot{h}, \theta), \tag{3a}$$

$$A_c = g(b, m, h, \dot{h}, \theta). \tag{3b}$$

Equations (3 *a*) and (3 *b*) are implicitly dependent on time t since h and $\dot{h}$ are dependent on time and $t = \int_{h(0)=0}^{h(t)} \mathrm{d}h/\dot{h}$.

Among the five governing parameters, three of them, namely b, h and $\dot{h}$, have independent dimensions. The dimensions of F and A_c are then given by $[F] = [b][\dot{h}]^m[h]^{2-m}$ and $[A_c] = [h]^2$ respectively. Applying the Π theorem in dimensional analysis (Barenblatt 1996), we obtain

$$F = b\left(\frac{\dot{h}}{h}\right)^{m} h^2 \Pi_\alpha(m, \theta), \tag{4}$$

$$A_c = h^2 \Pi_\beta(m, \theta), \tag{5}$$

where $\Pi_\alpha = F/b\dot{h}^m h^{2-m}$ and $\Pi_\beta = A_c/h^2$, m and θ are all dimensionless. Consequently, the hardness under load is

$$H = \frac{F}{A_c} = b\left(\frac{\dot{h}}{h}\right)^m \frac{\Pi_\alpha}{\Pi_\beta} \equiv b\Pi_\gamma\left(\frac{\dot{h}}{h}\right)^m, \tag{6}$$

where $\Pi_\gamma \equiv \Pi_\alpha/\Pi_\beta$. To simplify notation, $\Pi_i \equiv \Pi_i(m,\theta)$ for $i = \alpha$, β, γ in the following.

This equation shows that the strain-rate dependence of hardness is contained in the parameter $\dot{h}/h$. Comparing with equation (1), we observe that, aside from the pre-factor, the power-law dependence of hardness H on $\dot{h}/h$ in indentation experiments is the same as that of stress σ on strain rate $\dot{\varepsilon}$ in uniaxial creep tests. Thus, the parameter $\dot{h}/h$ may be chosen, aside from a time-independent pre-factor, to represent the indentation strain rate.

When the force, instead of the displacement, is the independent variable, equation (4) may be integrated to obtain

$$h(t) = \left(\frac{2}{m}\right)^{m/2} (b\Pi_\alpha)^{-1/2} \left(\int_0^t F^{1/m}(t)\,\mathrm{d}t\right)^{m/2}, \tag{7}$$

with the initial condition $h(0) = 0$.

An equation similar to equation (4) was first proposed by Grau *et al.* (1998) based on the assumptions that $H = b'\dot{\varepsilon}^m$ and $\dot{\varepsilon} = \dot{h}/h$. The present derivation is based on the self-similarity that exists in the problem of indentation in power-law creep solids using self-similar indenters. The assumptions in the derivation of Grau *et al.* are shown to be the consequence of this self-similarity. Furthermore, the parameter b' is not, in general, the same as b which was suggested by Grau *et al.* (1998).

§ 3. Discussion

In the following, the above equations are applied to several types of indentation experiment in which $\dot{h}$, $\dot{F}$ or $\dot{F}/F$ is kept constant.

3.1. *Constant-displacement-rate $\dot{h}_c$ experiments*

When $\dot{h}_c$ is constant, the force and hardness are, according to equations (4) and (6),

$$F = (b\Pi_\alpha)\left(\frac{\dot{h}_c}{h}\right)^m h^2, \tag{8}$$

$$H = (b\Pi_\gamma)\left(\frac{\dot{h}_c}{h}\right)^m. \tag{9}$$

These equations show that, during loading, the force is proportional to h^{2-m} and is no longer proportional to h^2. The square dependence is characteristic of indentation using self-similar indenters in elastic–plastic solids without strain-rate dependence (Cheng and Cheng 1998, 1999). The hardness decreases with increasing indentation depth. The creep exponent m can be obtained either from the indentation loading curve or from a graph of $\ln(H)$ versus $\ln(\dot{h}/h)$. Equations (8) and (9), scaled by their respective values at the maximum indenter displacement h_{max}, are illustrated in figure 1.

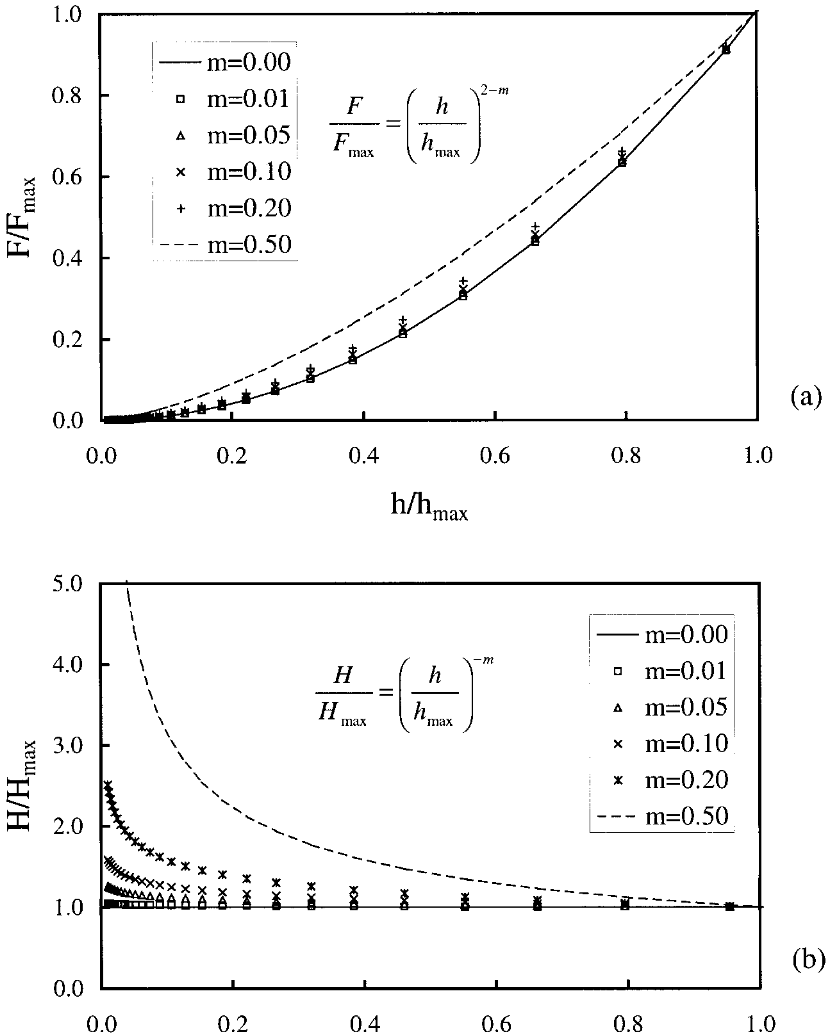

Figure 1. Scaling relationships (a) between the indenter load and displacement and (b) between the hardness and indenter displacement for constant-displacement-rate cases.

3.2. *Constant-loading-rate* $\dot{F}_c$ *experiments*

Substituting $F = \dot{F}_c t$ in equation (7), we obtain

$$h(t) = \left(\frac{\dot{F}_c}{b\Pi_\alpha}\right)^{1/2}\left(\frac{2}{m+1}\right)^{m/2} t^{(m+1)/2}. \qquad (10)$$

Consequently, the force, using equation (4), is

$$F = (b\Pi_\alpha)^{1/(m+1)}\left(\dot{F}_c\,\frac{m+1}{2}\right)^{m/(m+1)} h^{2/(m+1)}. \qquad (11)$$

The indentation strain rate can be expressed, using equation (10), as

$$\frac{\dot{h}}{h} = \frac{m+1}{2}\frac{1}{t} = \frac{m+1}{2}\frac{\dot{F}_c}{F}. \tag{12}$$

Using equation (6), the hardness becomes

$$H = (b\Pi_\gamma)\left(\frac{\dot{h}}{h}\right)^m = (b\Pi_\gamma)\left(\frac{m+1}{2}\right)^m\left(\frac{\dot{F}_c}{F}\right)^m. \tag{13}$$

These equations show that, during loading, the force is proportional to $h^{2/(m+1)}$ and is no longer proportional to h^2. The hardness decreases with increasing indentation load. The creep exponent m can be obtained from the indentation loading curve, from a graph of $\ln(H)$ versus $\ln(\dot{h}/h)$ or from a graph of $\ln(H)$ versus $\ln(\dot{F}/F)$. Equations (11) and (13), scaled by their respective values at the maximum indenter displacement $h_{\max}$, are illustrated in figure 2.

3.3. *Constant-loading-rate-over-load $\dot{F}/F$ experiments*

Since $\dot{F}/F = \lambda$ is a constant, the force is given by $F = F_0 \exp(\lambda t)$, where F_0 is the force at $t = 0$. Substituting into equation (7), we obtain a solution:

$$h(t) = \frac{1}{(b\Pi_\alpha)^{1/2}}\left(\frac{2}{\lambda}\right)^{m/2} F_0^{1/2}\left[\exp\left(\frac{\lambda t}{m}\right) - 1\right]^{m/2} \tag{14}$$

and, for large $t > m/\lambda$,

$$h(t) \approx \left(\frac{2^m F_0}{\lambda^m b\Pi_\alpha}\right)^{1/2} \exp\left(\frac{\lambda t}{2}\right). \tag{15}$$

Consequently, the indentation strain rate is given by

$$\frac{\dot{h}}{h} = \frac{\lambda}{2}\left[1 - \exp\left(\frac{-\lambda t}{m}\right)\right]^{-1} \approx \frac{\lambda}{2} = \frac{1}{2}\frac{\dot{F}}{F}. \tag{16}$$

Thus, the indentation strain rate $\dot{h}/h$ is half of $\dot{F}/F$ after a transient period of the order of m/λ. Using equations (4) and (7), the respective indentation loading curve and hardness may be written

$$F = F_0 \exp(\lambda t) \approx (b\Pi_\alpha)\left(\frac{\lambda}{2}\right)^m h^2, \tag{17}$$

$$H = (b\Pi_\gamma)\left(\frac{\dot{h}}{h}\right)^m \approx (b\Pi_\gamma)\left(\frac{\lambda}{2}\right)^m. \tag{18}$$

Equations (17) and (18), scaled by their respective values at $h_{\max}$, are shown in figure 3. Clearly, hardness reaches a steady-state value when $\dot{F}/F$ is kept constant. Correspondingly, the loading force is again proportional to h^2. The hardness increases with $(\dot{F}/F)^m$. The creep exponent m can be obtained either from a graph of $\ln(H)$ versus $\ln(\dot{F}/F)$ or from a graph of $\ln(H)$ versus $\ln(\dot{h}/h)$.

The results of the above analysis are consistent with the experimental behaviours reported in the literature. For example, numerous workers have shown a linear dependence between $\ln(H)$ and $\ln(\dot{h}/h)$ for all three loading conditions considered above (i.e. $\dot{h}$, $\dot{F}$ or $\dot{F}/F$ is kept constant). Furthermore, the creep exponent m has

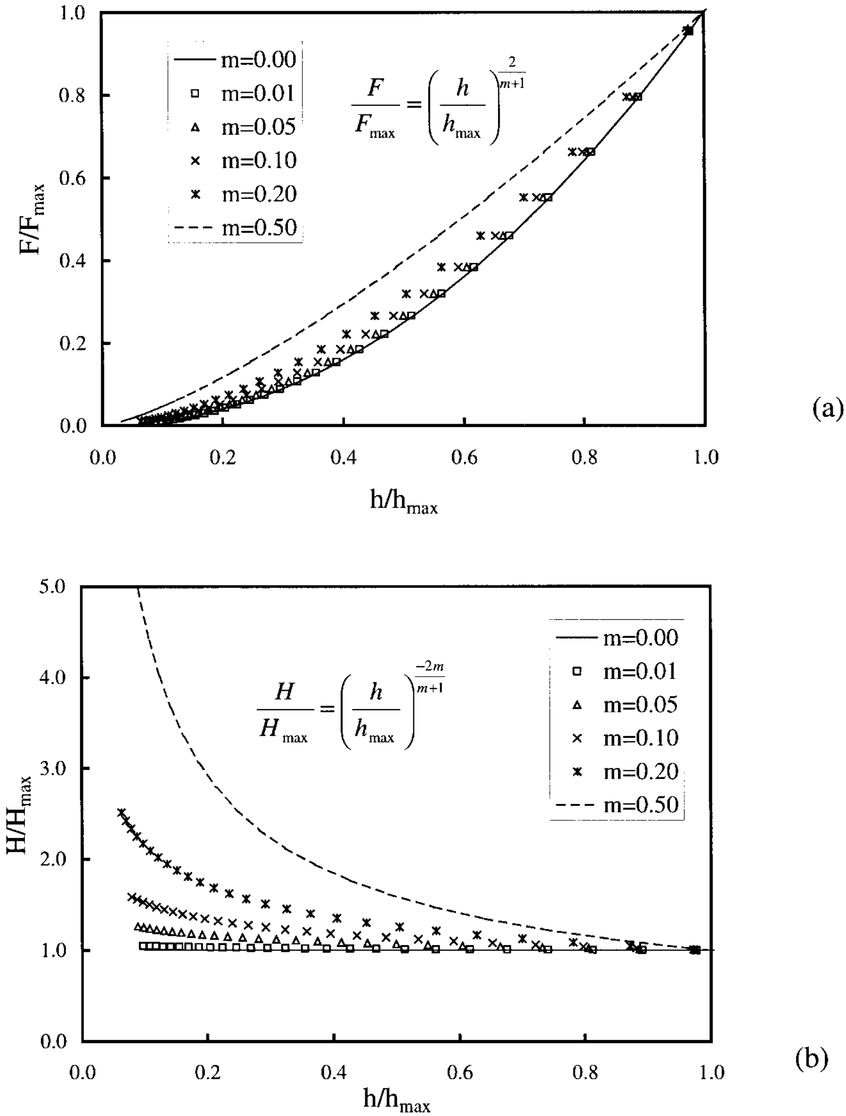

Figure 2. Scaling relationships (*a*) between the indenter load and displacement and (*b*) between the hardness and indenter displacement for constant-loading-rate cases.

been obtained from the slope of the straight lines in the graph of $\ln(H)$ versus $\ln(\dot{h}/h)$. The creep exponent m has also been obtained from indentation loading curves by Grau *et al.* (1998) using either constant-$\dot{h}$ or constant-$\dot{F}$ experiments and equations similar to equations (8) and (11).

Several researchers have reported an 'indentation-size effect' in constant-$\dot{h}$ or constant-$\dot{F}$ experiments as predicted by the above equations (equations (9) and (13)); the hardness decreases with increasing indentation depth or load (Stone and Yoder 1994, Lucas and Oliver 1999). It has also been demonstrated recently by Lucas and Oliver (1999) that, in constant-$\dot{F}/F$ experiments, the indentation strain rate reaches a 'steady state' and is given by $0.5\dot{F}/F$ (figure 4 of the paper by Lucas

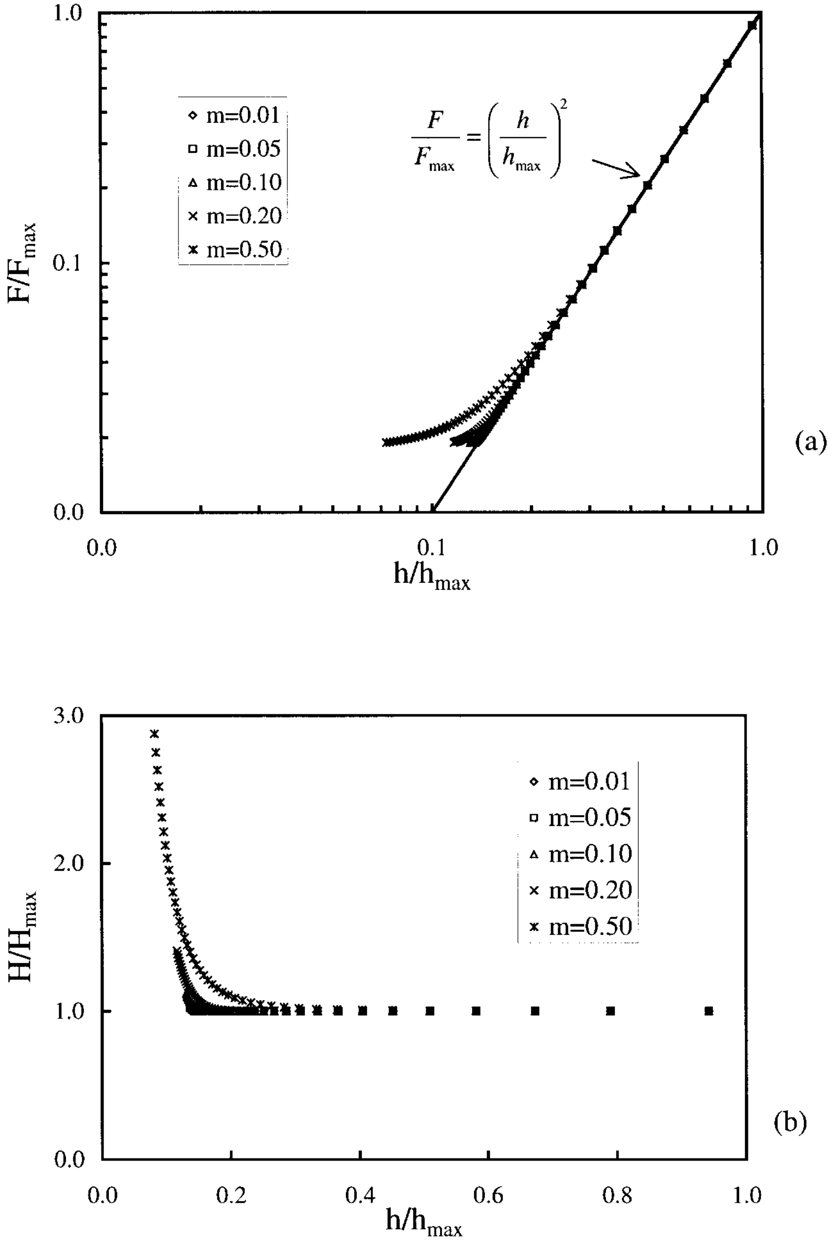

Figure 3. Scaling relationships (*a*) between the indenter load and displacement and (*b*) between the hardness and indenter displacement for constant-loading-rate-over-load cases where the parameter $\lambda t_{max} = 3$.

and Oliver (1999)). This observation agrees with the prediction of equation (16). Furthermore, these workers showed that the steady-state hardness is independent of the indentation depth and is proportional to $(\dot{F}/F)^m$ (figures 5 and 6 of the paper by Lucas and Oliver (1999)). These observations are also in agreement with equation (18).

§ 4. Conclusions

Scaling relationships in indentation of power-law creep solids using self-similar indenters have been established. Dimensional analysis shows that the parameter $\dot{h}/h$ can indeed represent the indentation strain rate. The relationships between the force and displacement and between the hardness and strain rate were obtained (equations (4)–(7)). These relationships were applied to several types of experiment in which $\dot{h}$, $\dot{F}$ or $\dot{F}/F$ was kept constant. The expressions for the loading curve and hardness were obtained. The predictions of the theory are consistent with experimental observations reported in the literature. In particular, it is shown that hardness decreases with increasing depth in both constant-$\dot{h}$ and constant-$\dot{F}$ experiments and reaches a steady-state value in constant-$\dot{F}/F$ experiments. Thus, a connection between power-law creep and 'indentation-size effects' is established.

In the future, several limitations of the present scaling theory will be explored. The scaling functions $\Pi_i(m, \theta)$ for $i = \alpha, \beta, \gamma$ cannot be obtained from dimensional analysis alone. They may be obtained from numerical analysis using finite-element methods. The parameter b in power-law creep (equation (1)) can then be determined from indentation experiments using equation (6), that is from the intercept on the graph of $\ln(H)$ versus $\ln(\dot{h}/h)$. Other factors that may contribute to 'indentation-size effect', including strain-gradient plasticity and imperfections in indenter or sample geometry, should also be explored in the future.

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