ACTA MECHANICA SINICA (English Series), Vol.18, No.3, June 2002 ISSN 0567-7718
The Chinese Society of Theoretical and Applied Mechanics
Chinese Journal of Mechanics Press, Beijing, China
Allerton Press, INC., New York, U.S.A.

UNSTEADY MODEL OF DROP MARANGONI MIGRATION IN MICROGRAVITY*

Geng Ronghui (耿荣慧) Hu Wenrui (胡文瑞) Jin Youlan (金友兰)
(*Institute of Mechanics, Chinese Academy of Sciences, Beijing* 100080, *China*)
Ao Chao (敖 超)
(*Institute of Computational Mathematics, Chinese Academy of Sciences, Beijing* 100080, *China*)

ABSTRACT: The experiments of drop Marangoni migration have been performed by the drop shift facility of short period of 4.5 s, and the drop accelerates gradually to an asymptotic velocity during the free fall. The unsteady and axisymmetric model is developed to study the drop migration for the case of moderate Reynolds number $Re = O(1)$, and the results are compared with the experimental ones in the present paper. Both numerical and experimental results show that the migration velocity for moderate Reynolds number is several times smaller than that given by the linear YGB theory.

KEY WORDS: drop migration, thermocapillary effect, microgravity, fluid mechanics

1 INTRODUCTION

Drop Marangoni migration driven by a temperature gradient is a typical subject in the fluid mechanics and also in the microgravity science. Because the rate of change of the surface tension with temperature is negative, the particles in the interface move to the colder direction and so the uneven gradient of pressure makes the drop move from colder regions toward hotter regions. A linear analysis was suggested at first as the YGB model[1]. Then, the results were extended to include the weak non-linear terms by the analytical method[2,3], and the complete non-linear process in a theoretical model can be obtained only by the method of numerical simulation[4~6]. Most of these studies deal with the steady process of the drop migration, and the acceleration process should be studied by the unsteady model.

The drop Marangoni migration process in ground-based experiments is usually coupled with the drop buoyant migration, and the pure Marangoni migration process can only be performed in the microgravity environment. The space experimental results of the drop Marangoni migration agree with the YGB model in cases of small drops of $(11 \pm 1.5)\,\mu$m in diameter[7], however, the migration velocities of the microgravity sounding rocket experiments for the larger drops of 0.69~2.38 mm in diameter are smaller than those given by the YGB model[8].

Received 4 July 2000, revised 8 November 2001
* The project supported by the National Natural Science Foundation (19789201) and the Ministry of Science and Technology of China (95-Yu-34)

The drop Marangoni migration has been studied for the cases of middle and large Reynolds number in both experimental and numerical simulations. The numerical simulation of unsteady and two-dimensional model has been performed to study the non-linear feature of the drop Marangoni migration, and the vortex separations in the wake of a drop are obtained[9]. The drop migration experiment was performed by adopting immiscible vegetable oil and the 5 cst silicon oil as the experimental matrix liquid and drop liquid, respectively. The silicon oil drops of several millimeters in diameter are ejected into a tank of the vegetable oil with a vertical temperature gradient of several 10 °C/cm. Both the ground-based experiments[10] and the microgravity experiments of the drop shaft[11,12] give a smaller migration velocity in comparison with that of the YGB model. The drop shaft facility of the Japanese Microgravity Laboratory provides a microgravity period of only 4.5 s, so there raises a question whether the short period is enough for the drop acceleration. Therefore, a numerical simulation is desired to describe the drop acceleration process and to compare the numerical results with those of the drop shaft experiments.

In the present paper, the numerical simulation of the unsteady and 3-dimensional axisymmetric model is used to describe the acceleration process of a drop in immiscible matrix liquid with a temperature gradient for the case of middle Reynolds number $Re = O(1)$. The numerical results agree with the experiments in general, and both the numerical and the experimental results give an asymptotic migration velocity, both of which are several times smaller than the one of the YGB model in case of small Reynolds number $Re \ll 1$.

2 MODEL OF NUMERICAL SIMULATION

A spherical drop is surrounded by the matrix liquid in a cylindrical container, and the symmetric axis of the container is taken as the z-axis in a cylindrical coordinate system as shown in Fig.1. The conservation equations of the incompressible fluid for the unsteady and 3-dimensional axisymmetric model may be written as follows

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \tag{2.1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + 2\frac{\partial}{\partial r}\left(\nu\frac{\partial u}{\partial r}\right) + \frac{\partial}{\partial z}\left[\nu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right)\right] + \frac{\nu}{r}\left(2\frac{\partial u}{\partial r} - \frac{2u}{r}\right) + f_r \tag{2.2}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + 2\frac{\partial}{\partial z}\left(\nu\frac{\partial w}{\partial z}\right) + \frac{\partial}{\partial r}\left[\nu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right)\right] + \frac{\nu}{r}\left(\frac{\partial u}{\partial r} - \frac{\partial w}{\partial z}\right) + f_z \tag{2.3}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(\kappa\frac{\partial T}{\partial r}\right) + \frac{\partial}{\partial z}\left(\kappa\frac{\partial T}{\partial z}\right) \tag{2.4}$$

where $(u, 0, w)$ is the velocity vector, ρ, p and T are, respectively, the density, pressure and temperature, ν and κ are, respectively, the kinetic viscosity and thermal diffusion coefficient, t is the time, and $(f_r, 0, f_z)$ is the vector of the body force.

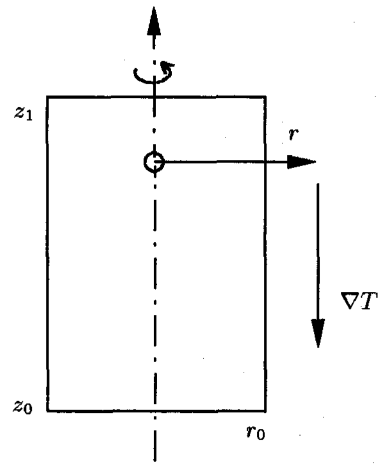


Fig.1 The schematic diagram of the axisymmetric model for drop Marangoni migration. A cylindrical coordinate system fixed at the position of the drop center of the initial time, the cylindrical radius r_0, upper cold boundary and lower hot boundary are, respectively, 14, 10 and 28 times of the drop radius in the calculation

The initial conditions may be given as

$$u(0,r,z)=0 \qquad w(0,r,z)=0 \qquad T(0,r,z)=T_0+A(z-z_0) \tag{2.5}$$

and the boundary conditions may be given as

$$\begin{array}{lll} u(t,r_0,z)=0 & \partial w(t,r_0,z)/\partial r=0 & T(t,r_0,z)=T_0+A(z-z_0) \\ u(t,0,z)=0 & \partial w(t,0,z)/\partial r=0 & \partial T(t,0,z)/\partial r=0 \\ \partial u(t,r,z_0)/\partial z=0 & w(t,r,z_0)=0 & \partial u(t,r,z_1)/\partial z=0 \\ w(t,r,z_1)=0 & T(t,r,z_0)=T_0 & T(t,r,z_1)=T_0+A(z-z_0) \end{array} \tag{2.6}$$

where $z=z_1, z=z_0$, and $r=r_0$ are, respectively, the upper colder boundary, the lower hotter boundary and the side wall boundary of a cylindrical container, and A is the constant temperature gradient. The drop is located at origin $(r,z)=(0,0)$ at the beginning, and then moves toward the lower and hotter boundary along the symmetric axis $r=0$.

The front-tracking method of the numerical simulation as discussed in [4, 9] is used in the present paper, and the essential is as follows:

(1) Tracking the surface of the drop;

(2) Determining the density ρ and the transport coefficients ν, κ and c_p across smoothly through the surface in a mesh and keeping the conservation relationship as

$$s=s_0+(s_1-s_0)\,\delta\,(r,z)$$

and

$$\delta\,(x,z)=\begin{cases} 0 & \text{outside drop} \\ V_1/V_0 & \text{near interface} \\ 1 & \text{inside drop} \end{cases}$$

where s denotes either ρ,ν,κ,c_p; s_0 and s_1 are values, respectively, outside and inside of the interface in a mesh; V_0 and V_1 are, respectively, the volume of the grid and the drop;

(3) Expressing the gradient of interface tension as a body force $(f_r, 0, f_z)$ in the meshes where the interface appears;

(4) Solving Eqs.(2.2)~(2.4) by the implicit finite differential method, and neglecting the pressure variation;

(5) Solving the pressure variation δp by the Poisson equation, and then modifying pressure p and velocity $(u, 0, w)$ to satisfy Eq.(2.1);

(6) Repeating procedure (1) to (5) until completing the calculation.

All quantities are non-dimensional in the numerical simulation. The radius of drop is taken as 1, and the container boundaries are taken as $z_1 = 10, z_0 = -28$ and $r_0 = 14$ in the calculations. The scale of computational grid is 1/16. The mediums used in the calculation are the same as those in the experiments, that is, a drop of 5 cst silicon oil and the mother liquid of vegetable oil, and the physical properties of these mediums are given elsewhere[11,12]. A temperature gradient of 32.0 °C/cm is applied vertically toward the lower hotter plane $z = z_0$. A Reynolds number is defined as $Re = v^* R/\nu$, where typical velocity $v^* = |\mathrm{d}\sigma/\mathrm{d}T||\partial T/\partial z|R/\rho v$, and σ is the interface tension. The experimental data of drop radii $R = 2.59, 3.26$ and $3.77\,\mathrm{mm}$[11,12] are adopted, respectively, in the numerical calculations.

3 RESULTS OF NUMERICAL SIMULATION

The drop center is located at $(r, z) = (0, 0)$ at the beginning, and then the drop accelerates toward the lower hotter plane along the symmetric axis $r = 0$ in the axisymmetric model. The migration velocity will increase from zero at the beginning to an asymptotic velocity w_a(mm/s), which corresponds to the migration velocity of the steady model. The asymptotic velocities for different Reynolds numbers are shown by the light curves in Fig.2. These distributions may be fitted by the following relationship

$$w_a = (0.967\,4\,Re + 0.123\,2)\mathrm{mm/s} \tag{3.1}$$

where the heavy lines in Fig.2 give the distributions of (3.1) for different Reynolds numbers.

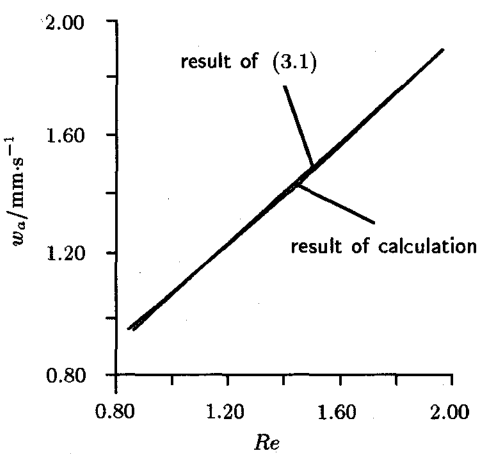


Fig.2 The distribution of asymptotic velocities. Comparison of the relationship (3.1) (heavy curve) and the calculation results (light curve)

The results of numerical simulations for the cases of different Reynolds numbers are reorganized by the non-dimensional migration velocity $w(t,0,z)/w_a$ and the non-dimensional time $\tau = t/t^*$, where the typical time $t^* = R/v^*$. The numerical simulation of all acceleration curves are nearly overlapped as shown by the light curves in Fig.2. These distributions may be fitted by the following relationship

$$w(t,0,z)/w_a = 1 - \exp[-(0.1544 - 0.1238\,Re + 0.04798Re^2)(t/t^*)] \qquad (3.2)$$

where the heavy lines in Fig.3 give the distributions of (3.2) for different Reynolds numbers.

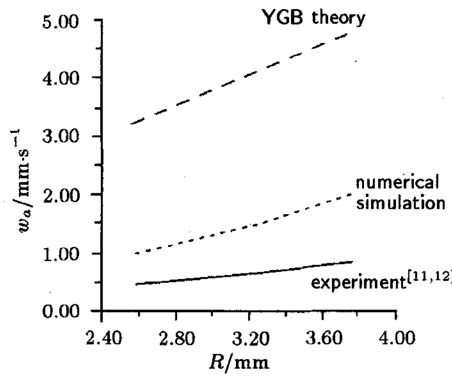


Fig.3 Acceleration processes of drop Marangoni migration. The results of relationship (3.2) (heavy curves) and the results of calculations (light curves) in cases of $Re = 1.8582, 1.3895, 0.8770$

The velocity and temperature distributions near the drop region at the time $t = 1.45, 5.017, 7.165$ and 10.22 s during the acceleration period are shown, respectively, in Figs.4 and 5. The velocity distributions in Fig.4 are given in a static frame of reference system, where the far field of velocity is zero. The thermocapillary flow is driven by the gradient of interface tension and the internal cell flow and the outside round flow are shown clearly in the static frame of reference system. The far field of temperature gradient is nearly uniform, but

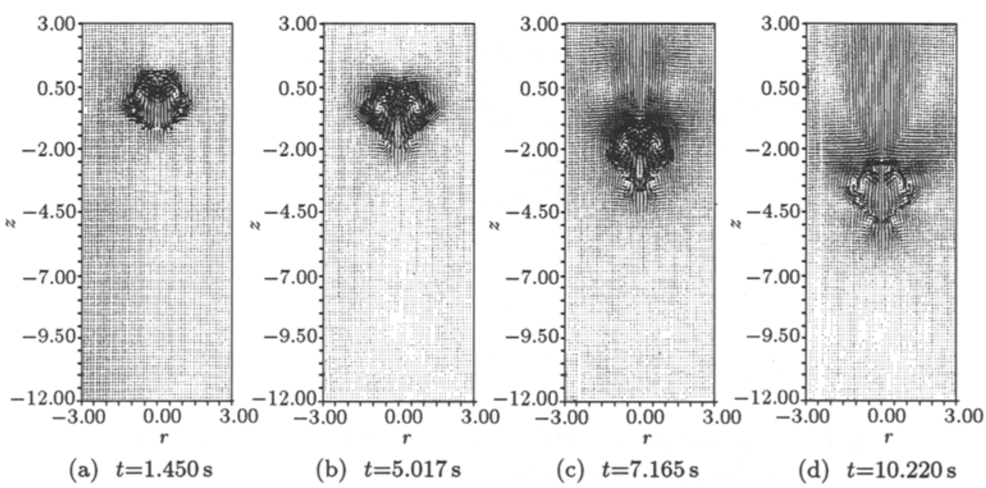
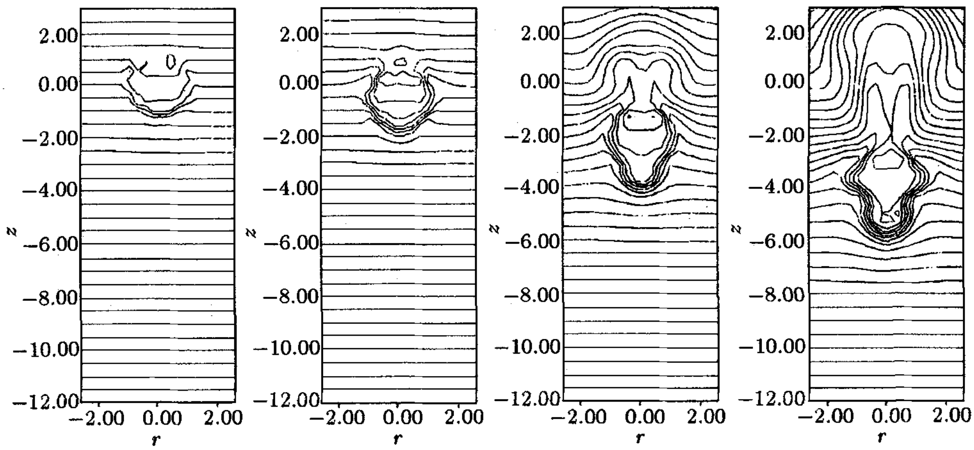


Fig.4 The evolution of velocity distributions. The velocity fields near the drop at different times $t = 1.450, 5.017, 7.165$ and 10.220 s for $Re = 1.8582$ in a reference system where the velocity is zero in the region far from the drop



(a) t=1.450 s (b) t=5.017 s (c) t=7.165 s (d) t=10.220 s

Fig.5 The evolution of temperature distributions. The ios-thermal counters of temperature distributions near the drop at different sequence $t = 1.450, 5.017, 7.165$ and 10.220 s for $Re = 1.858\,2$, and the temperature difference between the counters is $0.012\,5A(z_1 - z_0)R$. R is the drop radius

the temperature distributions near the drop change significantly due to the drop migration as shown in Fig.5. The typical time t^* is 0.365 s, and then, the acceleration period from zero velocity to an asymptotic velocity is nearly 10 s. This means that, the experimental period of 4.5 s by using the drop shaft facility of the experiments is a bit shorter than the required one. The migration velocity given by the asymptotic velocity of the present paper should be larger than that given by the drop shaft experiments because of the limited microgravity period of the experimental facility.

The relations between migration velocities and Reynolds numbers are given for the case of an applied temperature gradient 32.0 °C/cm. The results of the asymptotic drop velocity both from the numerical simulations of the present paper and the experiments[11,12] are compared with the linear YGB model as given in Fig.6. The results show that the

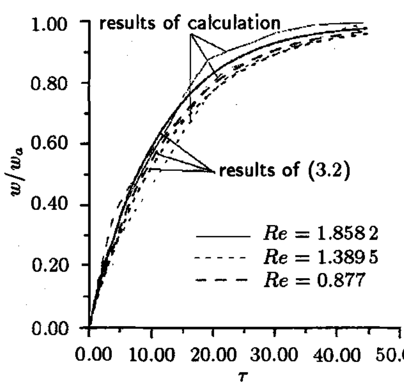


Fig.6 Comparison of the drop Marangoni migration velocities. The migration velocity distributions depending on the drop radii in case of an applied temperature difference $A = 32.0$ °C/cm for asymptotic velocity of experimental results, numerical simulation results of the present paper and YGB linear theory

migration velocities of numerical simulation are a bit larger than those of experiments. However, both results agree with each other in general, but are smaller by nearly one order of magnitude than those of the linear YGB model.

4 DISCUSSIONS

An unsteady and axisymmetric model of numerical simulation in the microgravity environment is given to study the acceleration process of the drop Marangoni migration for the case of large Reynolds number. The calculated results are compared with the experiments, which were performed by using the drop shaft facility in the Japan Microgravity Laboratory. The theoretical results are a bit larger than, but agree in general with the experimental ones. The differences between theoretical and experimental results may be explained by the limited microgravity period of the facility and the complexity of the experimental conditions due to the transition from the earth's gravity to the microgravity condition. The relationship between the migration velocity and Reynolds number is obtained in a non-dimensional general form.

For comparison with the experiments, the non-linear effects of both hydrodynamics and thermodynamics are considered by the numerical simulation in the present paper, and the Reynolds number of the drop Marangoni migration is in the range of $Re = O(10)$. The drop migration velocity increases with the increasing of the Reynolds number as shown in Fig.2, and the non-dimensional migration velocity depending on the time is similar in the calculated Reynolds range as shown in Fig.3. However the dimensional migration velocities depending on the time are different for different Reynolds number, and the larger the Reynolds number, the larger the migration velocity during the evolutionary process.

The numerical results show that the typical acceleration period is nearly 10 s in the Reynolds number range of $Re = 1.0 \sim 10.0$. Therefore, the experimental period of drop tower or drop shaft facility is a bit shorter than the requirement, but the results are still informative enough for the most part of the drop acceleration process in the parameter range. The drop tower facility is a useful tool in studying the drop Marangoni migration, especially for that with Reynolds number smaller than 1.0 in order of magnitude.

The acceleration process of the drop Marangoni migration for the case of large Reynolds number is studied in the present paper. The conclusion confirmed by the experiments is that the migration velocity for the case of large Reynolds number is nearly one order of magnitude smaller than that of the linear theory. It is expected that the drop or bubble migration may often appear in the microgravity environment. There have been limited studies on this subject until now, and more attentions should be paid to both experimental and theoretical researches.

REFERENCES

1 Young NO, Goldstein JS, Block MJ. The motion of bubbles in a vertical temperature gradient. *J Fluid Mechanics*, 1959, 6: 350

2 Balasubramaniam R, Chai AT. Thermocapillary migration of droplets: An exact solution for small Marangoni numbers. *J Colloid and Interface Science*, 1987, 119(2): 531～538

3 Haj-Hariri H, Nadim A, Borhan A. Effects of inertia on the thermocapillary velocity of a drop. *J Colloid and Interface Science*, 1990, 140(2): 277～286

4 Unverdi SO, Tryggvason G. A front-tracking method for viscous, in compressible, multifluid

flows. *J Computational Physics*, 1992, 100: 25~37

5 Golia C, Viviani A, Cioffi M. Thermocapillary bubble migration in confined medium. IAF-94-J.5.262, 1994

6 Balasubramaniam R, Lavery JE. Numerical simulation of thermocapillary bubble migration under microgravity for large Reynolds and Marangoni numbers. *Numerical Heat Transfer*, 1989, 16(2): 175~187

7 Braum B, Ikier C, Klein H. Thermocapillary migration of droplets in a binary mixture with miscibility gap during liquid/liquid phase separation under reduced gravity. *J Colloid and Interface Science*, 1993, 159(2): 515~516

8 Wozniak G. On the thermocapillary motion of droplets under reduced gravity. *J Colloid and Interface Science*, 1991, 141: 245~254

9 Geng RH, Ao C, Hu WR. Asymmetric drop Marangoni migration of larger Reynolds numbers. *Acta Astronautica*, 1998, 41: 757

10 Xie JC, Lin H, Han JH, et al. Drop migration of middle Reynolds numbers in a vertical temperature gradient. *Microgravity Science and Tech*, 1996, 9: 95

11 Xie JC, Lin H, Han JH, et al. Drop shaft experiments on Marangoni drop migrations for intermediate Reynolds numbers. In: Avduyevsky VS, Polezhaev VI, eds. Proc of Joint 9th European and 6th Russia Symposium on Physical Science in Microgravity. 1998. 189~195

12 Xie JC, Lin H, Han JH, et al. Experimental investigation on Marangoni drop migrations using drop shaft facility. *International J Heat & Mass Transfer*, 1998, 41(14): 2077~2081