载流圆锥薄壳的磁弹性应力与变形分析*

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摘 要 在建立旋转壳体的非线性磁弹性运动方程的基础上,研究了电磁场和机械载荷联合作用下载流圆锥薄壳的磁弹性效应.通过算例,得到了载流圆锥薄壳的位移及应力与通电电流强度之间的关系,解决了圆锥薄壳顶点处的奇异性问题,给出了轴对称条件下的数值解.计算结果表明:改变通电电流强度,可以改变载流圆锥薄壳的应力与变形状态,达到控制圆锥薄壳的受力与变形的目的.

关键词 载流圆锥薄壳,磁弹性,应力,变形

0 引言

在许多现代工业领域中,常有一些结构元件处在电磁场和机械场联合作用的环境中,其变形和应力受电磁机械荷载的共同影响和控制.当该结构元件为载流薄壳,且电场又很强时,电磁场对应力及变形的影响就非常显著[1].

文中通过对载流圆锥薄壳在电磁场和机械荷载的联合作用下的磁弹性效应的分析,获得了耦合场中一些参量的变化规律.通过具体算例,给出了载流圆锥薄壳在电磁场和机械荷载作用下的位移及应力的关系.解决了圆锥薄壳顶点处的奇异性问题.给出了在轴对称条件下的数值解,计算结果表明:改变通电电流强度,可以改变载流圆锥薄壳的应力及变形状态,从而达到控制薄壳的受力和变形的目的.为改变在强电磁场环境下工程结构中的圆锥薄壳工作状态提供理论分析和数值计算方法.

1 基本方程

图1给出了在三维正交曲线坐标系 $P(s,\theta,n)$ 下圆锥薄壳及荷载、电流分布情况.由轴对称性的条件可将有关方程进行简化后得到对称荷载作用下的基本方程,其中有

圆锥薄壳的磁弹性运动方程[2]

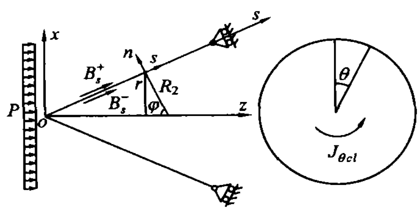

图1 滚支载流圆锥薄壳

$$\begin{cases} \dfrac{\partial(rN_s)}{\partial s} - \cos\phi N_\theta + r(P_s + \rho f_s^{\phi}) = r\rho h\dfrac{\partial^2 u}{\partial t^2} \\ \dfrac{\partial(rQ_s)}{\partial s} - \sin\phi N_\theta + r(P_n + \rho f_n^{\phi}) = r\rho h\dfrac{\partial^2 w}{\partial t^2} \\ \dfrac{\partial(rM_s)}{\partial s} - \cos\phi M_\theta - r\left(N_s - \dfrac{\sin\phi}{r}M_\theta\right)\theta_s - \\ \qquad rQ_s + r\rho f_s^{M} = \dfrac{r\rho h^3}{12}\dfrac{\partial^2\theta_s}{\partial t^2} \end{cases} \tag{1}$$

电动力学方程[2]

$$\begin{cases} -\dfrac{\partial B_n}{\partial t} = \dfrac{1}{r}\dfrac{\partial(rE_\theta)}{\partial s} \\ \sigma\mu\left[E_\theta + \dfrac{1}{2}\dfrac{\partial w}{\partial t}(B_s^+ + B_s^-) - \dfrac{\partial u}{\partial t}B_n\right] = \\ \qquad -\dfrac{\partial B_n}{\partial s} - \dfrac{B_s^+ - B_s^-}{rh} \end{cases} \tag{2}$$

洛仑兹力、洛仑兹力矩分量的表达式[3]

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$$\left\{\begin{aligned}
\rho f_s^{\phi} &= hJ_{\theta cl}B_n + \frac{1}{2}\sigma h\frac{\partial w}{\partial t}(B_s^+ + B_s^-)B_n - \sigma h\frac{\partial u}{\partial t}B_n^2 + \sigma hE_\theta B_n \\
\rho f_n^{\phi} &= -\frac{1}{2}hJ_{\theta cl}(B_s^+ + B_s^-) - \sigma h\frac{\partial w}{\partial t}\left[\frac{1}{4}(B_s^+ + B_s^-)^2 + \frac{1}{12}(B_s^+ - B_s^-)^2\right] + \frac{1}{2}\sigma h\frac{\partial u}{\partial t}B_n(B_s^+ + B_s^-) + \frac{1}{12}\sigma h^2\frac{\partial \theta_s}{\partial t}B_n(B_s^+ + B_s^-) - \frac{1}{2}\sigma hE_\theta(B_s^+ + B_s^-) \\
\rho f_s^{M} &= \frac{1}{12}\sigma h^2\frac{\partial w}{\partial t}B_n(B_s^+ - B_s^-) - \frac{1}{12}\sigma h^3\frac{\partial \theta_s}{\partial t}B_n^2
\end{aligned}\right. \tag{3}$$

位移与应变及弹性关系方程[2]

$$\left\{\begin{aligned}
&\varepsilon_s = \frac{\partial u}{\partial s} + \frac{1}{2}\theta_s^2;\quad \varepsilon_\theta = \frac{u}{r}\cos\phi + \frac{w}{r}\sin\phi \\
&\kappa_s = \frac{\partial \theta_s}{\partial s};\quad \kappa_\theta = \frac{\cos\phi}{r}\theta_s;\quad \theta_s = -\frac{\partial w}{\partial s} \\
&N_s = \frac{Eh}{1-\nu^2}(\varepsilon_s + \nu\varepsilon_\theta) \\
&N_\theta = \frac{Eh}{1-\nu^2}(\varepsilon_\theta + \nu\varepsilon_s) \\
&M_s = \frac{Eh^3}{12(1-\nu^2)}(\kappa_s + \nu\kappa_\theta) \\
&M_\theta = \frac{Eh^3}{12(1-\nu^2)}(\kappa_\theta + \nu\kappa_s)
\end{aligned}\right. \tag{4}$$

式(1)～(4)中,σ为介质的电导率;μ为磁导率;h为壳厚;ν为泊松比;E为材料弹性常数;B_s^+,B_s^-分别为壳上下表面的电磁感应强度;N_s为轴向内力;N_θ为环向内力;ε_θ,ε_s分别为环向与轴向应变;κ_s,κ_θ为弯曲应变;w为法向位移;u为轴向位移;θ_s为子午面内法向转角;E_θ为环向电场强度分量;B_n为法向磁感应分量;P_s,P_n分别为轴向与法向荷载分量;N_i,Q_i,M_i($i=s,\theta$)分别为载流圆锥薄壳的内力和力矩分量;ρf_s^{ϕ},ρf_n^{ϕ}为洛仑兹力分量,ρf_s^{M}为洛仑兹力矩分量.为获得Cauchy形式的可解方程组,选择u,w,θ_s,M_s,B_n,N_s,Q_s,E_θ作为基本可解函数,可得

$$\left\{\begin{aligned}
\frac{\partial u}{\partial s} &= \frac{(1-\nu^2)}{Eh}N_s - \frac{\nu\sin\phi}{r}w - \frac{\nu\cos\phi}{r}u - \frac{1}{2}\theta_s^2 \\
\frac{\partial w}{\partial s} &= -\theta_s \\
\frac{\partial \theta_s}{\partial s} &= \frac{12(1-\nu^2)}{Eh}M_s - \frac{\nu\cos\phi}{r}\theta_s \\
\frac{\partial N_s}{\partial s} &= \frac{\cos\phi}{r}\left[(\nu-1)N_s + Eh\left(\frac{\cos\phi}{r}u + \frac{\sin\phi}{r}w\right)\right] - P_s - hJ_{\theta cl}B_n + \rho h\frac{\partial^2 u}{\partial t^2} - \sigma h\left[E_\theta B_n + \frac{1}{2}\frac{\partial w}{\partial t}\cdot(B_s^+ + B_s^-)B_n - \frac{\partial u}{\partial t}B_n^2\right] \\
\frac{\partial Q_s}{\partial s} &= -\frac{\cos\phi}{r}Q_s + \frac{\nu\sin\phi}{r}N_s + \frac{\sin\phi}{r}Eh\left(\frac{\cos\phi}{r}u + \frac{\sin\phi}{r}w\right) - P_n + \rho h\frac{\partial^2 w}{\partial t^2} + \frac{1}{2}hJ_{\theta cl}(B_s^+ + B_s^-) + \sigma h\frac{\partial w}{\partial t}\left[\frac{1}{4}(B_s^+ + B_s^-)^2 + \frac{1}{12}(B_s^+ - B_s^-)^2\right] - \frac{1}{2}\sigma h\frac{\partial u}{\partial t}(B_s^+ + B_s^-)B_n - \frac{1}{12}\sigma h^2\frac{\partial \theta_s}{\partial t}(B_s^+ + B_s^-)B_n + \frac{1}{2}\sigma h(B_s^+ + B_s^-)E_\theta \\
\frac{\partial M_s}{\partial s} &= \frac{\cos\phi}{r}\left[(\nu-1)M_s + \frac{\cos\phi}{r}\frac{Eh^3}{12}\theta_s\right] + N_s\theta_s - \frac{\sin\phi}{r}\left(\nu M_s + \frac{\cos\phi}{r}\frac{Eh^3}{12}\theta_s\right)\theta_s + Q_s + \frac{\sigma h^3}{12}\frac{\partial \theta_s}{\partial t}B_n^2 - \frac{\sigma h^2}{12}\frac{\partial w}{\partial t}(B_s^+ - B_s^-)B_n + \frac{\rho h^3}{12}\frac{\partial^2 \theta_s}{\partial t^2} \\
\frac{\partial B_n}{\partial s} &= -\sigma\mu\left[E_s + \frac{1}{2}\frac{\partial w}{\partial t}(B_s^+ - B_s^-) - \frac{\partial u}{\partial t}B_n\right] - \frac{B_s^+ - B_s^-}{rh} \\
\frac{\partial E_\theta}{\partial s} &= -\frac{\partial B_n}{\partial t} - \frac{\cos\phi}{r}E_\theta
\end{aligned}\right. \tag{5}$$

2 消除奇异性及计算方法

2.1 计算方法

在不稳定的电磁场和机械荷载的作用下,对于薄壳的应力应变状态问题是针对指定瞬态求解的.采用Newmark稳定有限等差式[4]

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$$\begin{cases} \ddot{u}^{t+\Delta t} = \dfrac{u^{t+\Delta t} - u^{t}}{\beta(\Delta t)^2} - \left[\dfrac{\dot{u}^{t}}{\Delta t} + \ddot{u}^{t}\left(\dfrac{1}{2} - \beta\right)\right]\dfrac{1}{\beta} \\ \dot{u}^{t+\Delta t} = \dot{u}^{t} + \dfrac{\Delta t}{2}(\ddot{u}^{t} + \ddot{u}^{t+\Delta t}) \end{cases} \quad (6)$$

其中 β 为系统参数，此处取 $\beta = 0.25$，Δt 为时间增量，为进行离散化处理，将式(5)及相应的边界条件写成

$$\frac{\mathrm{d}\mathbf{N}}{\mathrm{d}s} = \mathbf{F}(s, \mathbf{N}),\ D_1\mathbf{N}|_{s=s_0} = \mathbf{d}_1,\ D_2\mathbf{N}|_{s=s_N} = \mathbf{d}_2 \quad (7)$$

式中：$\mathbf{N} = \{u, w, Q_s, N_s, \theta_s, M_s, E_\theta, B_n\}^{\mathrm{T}}$，$D_1$，$D_2$ 为给定的直角矩阵，$\mathbf{d}_1$，$\mathbf{d}_2$ 为给定的矢量。采用迭代法解此方程组。

令 $m = \rho s$ 由式(6)可得如下迭代方程

$$\frac{\mathrm{d}u^{(k+1)}}{\mathrm{d}m} = \frac{1-\nu^2}{\rho Eh}N_s^{(k+1)} - \frac{\nu\tan\phi}{m}w^{(k+1)} - \frac{\nu}{m}u^{(k+1)} + \frac{1}{2\rho}(\theta_s^{(k)})^2 - \frac{1}{\rho}\theta_s^{(k)}\theta_s^{(k+1)} \quad (8a)$$

$$\frac{\mathrm{d}w^{(k+1)}}{\mathrm{d}m} = -\frac{1}{\rho}\theta_s^{(k+1)} \quad (8b)$$

$$\frac{\mathrm{d}\theta_s^{(k+1)}}{\mathrm{d}m} = \frac{12(1-\nu^2)}{\rho Eh}M_s^{(k+1)} - \frac{\nu}{m}\theta_s^{(k+1)} \quad (8c)$$

$$\begin{aligned}\frac{\mathrm{d}N_s^{(k+1)}}{\mathrm{d}m} = &-\frac{1-\nu}{m}N_s^{(k+1)} + \frac{\rho Eh}{m^2}u^{(k+1)} + \frac{\rho Eh\tan\phi}{m^2}w^{(k+1)} - \frac{P_s}{\rho} - \frac{hJ_{\theta cl}}{\rho}B_n^{(k+1)} - \\ &\frac{\sigma h}{2\rho}(B_s^+ + B_s^-)[-(\dot{w}^{t+\Delta t})^{(k)}B_n^{(k)} + (\dot{w}^{t+\Delta t})^{(k+1)}B_n^{(k)} + (\dot{w}^{t+\Delta t})^{(k)}B_n^{(k+1)}] + \\ &\frac{\sigma h}{\rho}[-2(\dot{u}^{t+\Delta t})^{(k)}(B_n^{(k)})^2 + (\dot{u}^{t+\Delta t})^{(k+1)}(B_n^{(k)})^2 + 2(\dot{u}^{t+\Delta t})^{(k)} \cdot \\ &B_n^{(k)}B_n^{(k+1)}] - \frac{\sigma h}{\rho}(-B_n^{(k)}E_\theta^{(k)} + B_n^{(k+1)}E_\theta^{(k)} + B_n^{(k)}E_\theta^{(k+1)}) + h(\ddot{u}^{t+\Delta t})^{(k+1)}\end{aligned} \quad (8d)$$

$$\begin{aligned}\frac{\mathrm{d}Q_s^{(k+1)}}{\mathrm{d}m} = &-\frac{1}{m}Q_s^{(k+1)} + \frac{\nu\tan\phi}{m}N_s^{(k+1)} + \frac{\rho Eh\tan\phi}{m^2}u^{(k+1)} + \frac{\rho Eh\tan^2\phi}{m^2}w^{(k+1)} - \frac{P_n}{\rho} + \frac{\sigma h}{\rho}\Big[\frac{1}{4}(B_s^+ + B_s^-)^2 + \\ &\frac{1}{12}(B_s^+ - B_s^-)^2\Big](\dot{w}^{t+\Delta t})^{(k+1)} + \frac{hJ_{\theta cl}}{2\rho}(B_s^+ + B_s^-) + h(\ddot{w}^{t+\Delta t})^{(k+1)} - \frac{\sigma h}{2\rho}(B_s^+ + B_s^-) \cdot \\ &[-(\dot{u}^{t+\Delta t})^{(k)}B_n^{(k)} + (\dot{u}^{t+\Delta t})^{(k+1)}B_n^{(k)} + (\dot{u}_s^{t+\Delta t})^{(k)}B_n^{(k+1)} - E_\theta^{(k+1)}] - \frac{\sigma h^2}{12\rho} \cdot \\ &(B_s^+ + B_s^-)[-(\dot{\theta}_s^{t+\Delta t})^{(k)}B_n^{(k)} + (\dot{\theta}_s^{t+\Delta t})^{(k+1)}B_n^{(k)} + (\dot{\theta}_s^{t+\Delta t})^{(k)}B_n^{(k+1)}]\end{aligned} \quad (8e)$$

$$\begin{aligned}\frac{\mathrm{d}M_s^{(k+1)}}{\mathrm{d}m} = &-\frac{1-\nu}{m}M_s^{(k+1)} + \frac{\rho Eh^3}{12m^2}\theta_s^{(k+1)} + \frac{1}{\rho}(-N_s^{(k)}\theta_s^{(k)} + N_s^{(k+1)}\theta_s^{(k)} + N_s^{(k)}\theta_s^{(k+1)}) + \\ &\frac{1}{\rho}Q_s^{(k+1)} - \frac{\nu\tan\phi}{m}(-M_s^{(k)}\theta_s^{(k)} + M_s^{(k+1)}\theta_s^{(k)} + M_s^{(k)}\theta_s^{(k+1)}) - \frac{\rho Eh^3\tan\phi}{m^2}[2\theta_s^{(k+1)}\theta_s^{(k)} + (\theta_s^{(k)})^2] + \\ &\frac{\sigma h^3}{12\rho}[-2(\dot{\theta}_s^{t+\Delta t})^{(k)}(B_n^{(k)})^2 + (\dot{\theta}_s^{t+\Delta t})^{(k+1)}(B_n^{(k)})^2 + 2(\dot{\theta}_s^{t+\Delta t})^{(k)}B_n^{(k)}B_n^{(k+1)}] + \frac{h^3}{12}(\ddot{\theta}_s^{t+\Delta t})^{(k+1)} - \\ &\frac{\sigma h^2}{12\rho}(B_s^+ + B_s^-)[-(\dot{w}^{t+\Delta r})^{(k)}B_n^{(k)} + (\dot{w}^{t+\Delta r})^{(k+1)}B_n^{(k)} + (\dot{w}^{t+\Delta r})^{(k)}B_n^{(k+1)}]\end{aligned} \quad (8f)$$

$$\begin{aligned}\frac{\mathrm{d}B_n^{(k+1)}}{\mathrm{d}m} = &-\frac{\sigma\mu}{\rho}[E_s^{(k+1)} + 0.5(B_s^+ + B_s^-)(\dot{w}^{t+\Delta t})^{(k+1)} + (\dot{u}^{t+\Delta t})^{(k)}B_n^{(k)} - (\dot{u}^{t+\Delta t})^{(k+1)}B_n^{(k)} - \\ &(\dot{u}^{t+\Delta t})^{(k)}B_n^{(k+1)}] - \frac{B_s^+ - B_s^-}{mh\cos\phi}\end{aligned} \quad (8g)$$

$$\frac{\mathrm{d}E_\theta^{(k+1)}}{\mathrm{d}m} = -\frac{1}{\rho}(\dot{B}_n^{t+\Delta t})^{(k+1)} - \frac{1}{m}E_\theta^{(k+1)} \quad (8h)$$

2.2　消除圆锥顶点处的奇异性的方法

为了消除计算过程中在 $s = 0$ 的奇异性，可将微分方程组(8)作进一步处理。由参考文献[5]给出圆锥薄壳的轴力及位移方程

$$\begin{cases} N_S = -0.5Ps\tan\phi \\ u = \dfrac{P\tan\phi}{4Eh}(1 - 2\nu\cos\phi)(l^2 - s^2) \\ w = -\dfrac{P\tan^2\phi}{2Eh}[s^2(2\cos\phi - \nu) + 0.5(1 - 2\nu\cos\phi)(l^2 - s^2)] \end{cases} \quad (9)$$

其中 l 为圆锥薄壳的母线长。将上述理论经典解引入方程组(8)，作为方程组在 $s = 0$ 处迭代的初始值。同时，对形如 $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{y}{x} + g(x)$ 及 $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{b}{x^2} + f(x)$ (b 为常数)的方程作如下处理

将方程 $x\dfrac{\mathrm{d}y}{\mathrm{d}x} = -y + xg(x)$ 两边对 x 求导得 $\dfrac{\mathrm{d}y}{\mathrm{d}x} + x\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = -\dfrac{\mathrm{d}y}{\mathrm{d}x} + g(x) + x\dfrac{\mathrm{d}g(x)}{\mathrm{d}x}$ 则当 x 趋近 0 时有 $\left.\dfrac{\mathrm{d}y}{\mathrm{d}x}\right|_{x=0} = 0.5g(x)|_{x=0}$。同理，将方程 $x^2\dfrac{\mathrm{d}y}{\mathrm{d}x} = -b$

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$+ x^2 f(x)$ 两边对 x 求两次导后令 x 趋近 0 时可以得到 $\left.\frac{dy}{dx}\right|_{x=0} = f(x)|_{x=0}$. 例如,对式 $\frac{dE_\theta^{(k+1)}}{dm} = -\frac{1}{\rho}\cdot(\dot{B}_n^{t+\Delta t})^{(k+1)} - \frac{1}{m}E_\theta^{(k+1)}$,用前一种方法处理后可以得到式

$$\frac{dE_\theta^{(k+1)}}{dm} = -\frac{1}{2\rho}(\dot{B}_n^{t+\Delta t})^{(k+1)} \tag{10}$$

将式(9)代入到方程组(8)中的第三式并按上述后一种方法进行变换,得到

$$\begin{aligned}\frac{dN_s^{(k+1)}}{dm} = & -\frac{P\text{ctan}\varphi}{4\rho}(4\cos\varphi-2) - \frac{P_s}{\rho} - \frac{hJ_{\theta cl}}{\rho}B_n^{(k+1)} - \\ & \frac{\sigma h}{\rho}(-B_n^{(k)}E_\theta^{(k)} + B_n^{(k+1)}E_\theta^{(k)} + B_n^{(k)}E_\theta^{(k+1)}) - \\ & \frac{\sigma h}{2\rho}(B_s^+ + B_s^-)[-(\dot{w}^{t+\Delta t})^{(k)}B_n^{(k)} + \\ & (\dot{w}^{t+\Delta t})^{(k+1)}B_n^{(k)} + (\dot{w}^{t+\Delta t})^{(k)}B_n^{(k+1)}] + \\ & h(\ddot{u}^{t+\Delta t})^{(k+1)} + \frac{\sigma h}{\rho}[-2(\dot{u}^{t+\Delta t})^{(k)}(B_n^{(k)})^2 + \\ & (\dot{u}^{t+\Delta t})^{(k+1)}(B_n^{(k)})^2 + 2(\dot{u}^{t+\Delta t})^{(k)}B_n^{(k)}B_n^{(k+1)}]\end{aligned} \tag{11}$$

把所有含奇异项的式子作同样处理后,得到圆锥在 $S = 0$ 处的其他迭代方程为

$$\begin{aligned}\frac{du^{(k+1)}}{dm} = & \frac{1-\nu^2}{\rho Eh(1+\nu)}N_s^{(k+1)} + \frac{\nu\tan\varphi}{\rho(1+\nu)}\theta^{(k+1)} + \\ & \frac{1}{2\rho(1+\nu)}(\theta_s^{(k)})^2 - \frac{1}{\rho(1+\nu)}\theta_s^{(k)}\theta_s^{(k+1)}\end{aligned}$$

$$\frac{dw^{(k+1)}}{dm} = -\frac{1}{\rho}\theta_s^{(k+1)}$$

$$\frac{d\theta_s^{(k+1)}}{dm} = \frac{12(1-\nu^2)}{\rho Eh(1+\nu)}M_s^{(k+1)}$$

$$\begin{aligned}\frac{dQ_s^{(k+1)}}{dm} = & \frac{\sigma h}{\rho}\left[\frac{1}{4}(B_s^+ + B_s^-)^2 + \frac{1}{12}(B_s^+ - B_s^-)^2\right]\cdot \\ & (\dot{w}^{t+\Delta t})^{(k+1)} + \frac{hJ_{\theta cl}}{2\rho}(B_s^+ + B_s^-) + \\ & h(\dot{w}^{t+\Delta t})^{(k+1)} - \frac{\sigma h}{2\rho}(B_s^+ - B_s^-)\cdot \\ & [-(\dot{u}^{t+\Delta t})^{(k)}B_n^{(k)} + (\dot{u}^{t+\Delta t})^{(k+1)}B_n^{(k)} + \\ & (\dot{u}^{t+\Delta t})^{(k)}B_n^{(k+1)} - E_\theta^{(k+1)}] - \frac{\sigma h^2}{12\rho}(B_s^+ + B_s^-)\cdot \\ & [-(\dot{\theta}_s^{t+\Delta t})^{(k)}B_n^{(k)} + (\dot{\theta}_s^{t+\Delta t})^{(k+1)}B_n^{(k)} + \\ & (\dot{\theta}_s^{t+\Delta t})^{(k)}B_n^{(k+1)}] - \frac{P\cos\varphi}{\rho} - \frac{P_n}{\rho}\end{aligned}$$

$$\begin{aligned}\frac{dM_s^{(k+1)}}{dm} = & \frac{1}{\rho(2-\nu)}(-N_s^{(k)}\theta_s^{(k)} + N_s^{(k+1)}\theta_s^{(k)} + \\ & N_s^{(k)}\theta_s^{(k+1)}) + \frac{1}{\rho(2-\nu)}Q_s^{(k+1)} + \frac{h^3}{12(2-\nu)}\cdot \\ & (\ddot{\theta}_s^{t+\Delta t})^{(k+1)} - \frac{\sigma h^2}{12\rho(2-\nu)}(B_s^+ + B_s^-)\cdot \\ & [-(\dot{w}^{t+\Delta t})^{(k)}B_n^{(k)} + (\dot{w}^{t+\Delta t})^{(k+1)}B_n^{(k)}] + \\ & \frac{1}{(2-\nu)}(\dot{w}^{t+\Delta t})^{(k)}B_n^{(k+1)} + \frac{\sigma h^3}{12\rho(2-\nu)}\cdot \\ & [-2(\dot{\theta}_s^{t+\Delta t})^{(k)}(B_n^{(k)})^2 + (\dot{\theta}_s^{t+\Delta t})^{(k+1)}(B_n^{(k)})^2 + \\ & 2(\dot{\theta}_s^{t+\Delta t})^{(k)}B_n^{(k)}B_n^{(k+1)}]\end{aligned}$$

$$\begin{aligned}\frac{dB_n^{(k+1)}}{dm} = & -\frac{\sigma\mu}{\rho}[E_s^{(k+1)} + 0.5(B_s^+ + B_s^-)(\dot{w}^{t+\Delta t})^{(k+1)} + \\ & (\dot{u}^{t+\Delta t})^{(k)}B_n^{(k)} - (\dot{u}^{t+\Delta t})^{(k+1)}B_n^{(k)} - \\ & (\dot{u}^{t+\Delta t})^{(k)}B_n^{(k+1)}]\end{aligned} \tag{12}$$

在式(8)、(10)、(11)、(12)的基础上,利用数值解法进行求解,即可进一步求出应力、洛仑兹力、洛仑兹力矩及电磁场参数以及它们之间的变化关系.

3 算例

位于电磁场中且通有侧向电流 $J = \{0, J_{\theta cl}, 0\}$、同时作用机械荷载 $P = \{P_s, 0, P_n\}$ 的滑动支承圆锥薄壳如图 1 所示,圆锥薄壳处在磁场强度为 $B = \{B_s, 0, 0\}$ 中,已知 $l = 0.88$ m、$\varphi = \pi/2$、壳厚 $h = 5\times10^{-4}$ m、材料弹性常数 $E = 71$ GPa、质量密度 $\rho = 2670$ kg/m^3、泊松比 $\nu = 0.33$、载荷集度 $P_s = 680\times\sin\varphi$ Pa、$P_n = 680\times\cos\varphi$ Pa、电导率 $\sigma = 3.63\times10^7$ (Ω m)$^{-1}$、磁导率 $\mu = 1.256\times10^{-6}$ H/m、$\omega = \pi\times10^2$ s^{-1}、$J_{\theta cl} = J\sin\omega t$ A/m^2、$B_S = 0.001$ T.

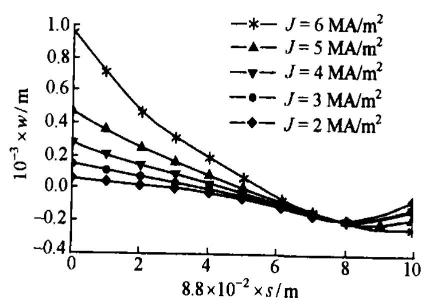

图 2 法向挠度与通电电流关系

壳体边界条件为:

在 $S = 0$ m 处,$N_s = 0$, $Q_S = 0$, $\theta_S = 0$, $B_n = 0$;

在 $S = 0.88$ m 处, $u = 0$, $Q_S = 0$, $M_S = 0$,

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$B_n = 0.1\sin\omega t$ T;

初始条件为： $N(m,t)|_{t=0} = 0, \dot{u}(m,t)|_{t=0} = 0,$
$\dot{w}(m,t)|_{t=0} = \dot{\theta}_S(m,t)|_{t=0} = 0.$

在式(8)、(10)～(12)的基础上,结合已知条件、边界条件及初始条件在微机上编程运算.可求解出八个基本函数 u、w、θ_s、M_s、B_n、$E\theta$、N_s、Q_s. 在此基础上改变电流强度,可确定机械量与电磁量之间的关系及变化规律:

(1) 如图 2 ($t = 5$ ms)、图 3 ($t = 5$ ms)所示,变换电流强度,壳体的轴向位移、法向挠度均随通电电流强度的增加而增加.同时电流强度越大,轴向位移及法向挠度的变化越明显.从图 2 中可见,在 $s = 0.65$ m 附近存在一个驻点,该点法向挠度不随电流强度的改变而改变.

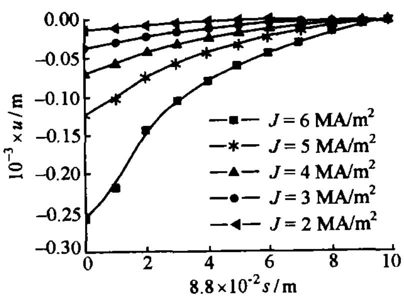

图 3　轴向位移与通电电流强度的关系

(2) 分析图 4 ($t = 5$ ms)可知:圆锥壳的轴向应力随电流强度的增加而增加;即电流强度越大,电磁力效应越大,已成为载流圆锥薄壳受力的主导原因.

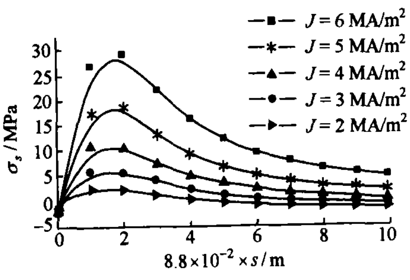

图 4　σ_s 与通电电流强度的关系

(3) 如图 5 ($t = 5$ ms)所示,圆锥壳中最大应力值随电流强度的增加而增加,同时改变通电电流方向应力值随之改变,如 $J = 6$ MA/m^2 与 $J = -6$ MA/m^2 应力峰值就相差约 10 MPa,这说明通过改变电流方向可以控制挠度,并且出现零应力点,这说明可以通过加入适当电流强度来实现该点不受力,从而达到对圆薄壳受力状态的控制.

(4) 由轴对称性可知圆锥薄壳的顶点应该是沿着 z 轴只发生水平位移.在 $J = 6$ MA/m^2 时,$w = 0.965$ mm, $u = 0.257$ mm,$u/w = 0.266$,$\tan(\pi/12) = 0.268$;误差为 7‰,这说明了该处理方法的有效性.

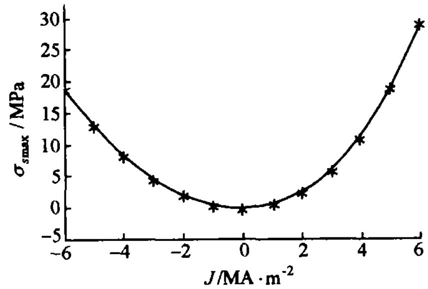

图 5　$\sigma_{s\max}$与通电电流强度的关系

4　结论

(1) 图 2 及图 3 说明了:改变通电电流强度的大小,可以控制圆锥薄壳的变形.因此,利用这一点,可以改变在电磁环境下工作的工程结构的工作状态.

(2) 图 4 及图 5 说明了:改变电流强度的大小,可以控制圆锥薄壳受力状况.因此可以通过对圆锥薄壳加适当强度的电流,来实现圆锥薄壳的某些部位的应力为零或是很小的控制.

(3) 通过对载流圆锥薄壳的磁弹性效应的分析结果表明:设计在强电磁场环境下工作的工程结构时,考虑电磁效应是必要的.

参　考　文　献

1　Амбарцумян С А, Багдасарян Г Е, Белубекян М В. Магнитоупругость тонких Оболочек и пластин. Изд Наука Мос,1977

2　Мольчено Л В. Математические Основы Нелинейной Магнитоупрогости Теории Оболочек. Киев,1988:10～25

3　徐耀铃等. 薄板薄壳非线性磁弹性问题基本方程. 燕山大学学报,1998, 22(4):358～362

4　Newmark N M. A method of computation for structural dynamics. J End Mech Div Proc ASCE, 1959, 85(7):67～97

5　黄克智. 板壳理论,清华大学出版社,1987,400～401

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ANALYSIS OF MAGNETO-ELASTIC STRESS AND DEFORMATION IN THIN CURRENT-CARRYING CONICAL SHELL

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Abstract The magneto - elastic effects in a thin current carrying conical shells under the combined action of electro - magnetic field and mechanical load are studied based on the nonlinear magneto-elastic kinetic equations of revolutionary shells. Through specific examples, the relationships between the electric current density and the displacement and the stress in a thin current carrying conical shells are obtained. The singularity in the apex of the conical shell is solved, and the numerical solution in an axisymmetric problem is obtained too. The computed results show that the stress and deformation in the thin current carrying conical shells can be controlled by changing the electric current density.

Key words thin current carrying conical shell, magneto-elastic, stress, deformation

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