Structure of Vortex Dislocations in a Wake-Type Flow *

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Structure and dynamical processes of vortex dislocations in a kind of wake-type flow are described clearly by vortex lines, which are directly constructed from data of three-dimensional direct numerical simulations of the flow evolution.

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It has been confirmed that in the cylinder wake,[1] mixing layers[2] and some nonlinear wave flows, the flow transition involves the appearance of large-scale vortex structures with a highly three-dimensional configuration, referred to as "*vortex dislocations*" or wave pattern "*defect*." Vortex dislocations are generated between spanwise vortex shedding cells out of phase. They play an important role in the flow transition and are considered as a new kind of the mechanisms for the transition. Experimental visualization and measurement have provided much information about the generation, development and dynamics of vortex dislocation, but only a little knowledge about the characteristics and the dynamical process of vortex dislocations. There are few numerical studies on vortex dislocation. Recently, Ling and Xiong[3] and Braza *et al.*[4] have made numerical exploration on the vortex dislocations in wake-type flow and cylinder wake. The generation of forced vortex dislocations and natural vortex dislocations and their influences on the flow field are reported. However, the understanding of the generation mechanism, the structure of the vortex dislocations and the real information of the vortex reconnections in vortex dislocations are still far from the complete. The influences of vortex dislocations on flow need be studied deeply. In order to study the physical cause of dislocation phenomena in details, we study the vortex dislocation occurring in a wake-type flow instead of in a real cylinder wake. Removing the real object offers a considerable reduction of the required computation and avoids the complexity of various vortex shedding modes induced by different cylinder aspect ratios.

In view of the above discussion, a more sufficiency direct numerical simulation has been carried out in the previous works based on Ling and Xiong[3] The computation is performed for a long-time flow in a larger flow domain, and the spanwise resolution is higher. A clear identification of scenario of the formation of vortex dislocations in wake-type flow with a local spanwise non-uniformity is presented, and the basic features of the complex vortex linkages in vortex dislocations are described by analysing the substantial modification of vortex line tracks. To our best knowledge, there has been no report using vortex lines to describe vortex dislocations. By these investigations, we can understand how the wake flow behaviour accommodates the differences in phase and strength between shedding cells via vortex dislocations.

The governing equations for the wake-type flow are the continuity and the Navier–Stokes equations for incompressible fluid. To obtain high numerical accuracy and wavenumber resolution in simulation of temporal–spatial evolution of the flow the compact finite difference-Fourier Spectral hybrid method suggested by Xiong and Ling[5] is used. The method procedure is summarized as follows. In the spanwise direction of the flow, the periodic boundary conditions are applied. From the Navier–Stokes equations, the $m^{\rm th}$ harmonic component equations is

$$\frac{\partial \boldsymbol{u}_m}{\partial t} + F_m[(\boldsymbol{u}\cdot\nabla)\boldsymbol{u}] = -\nabla_m p_m + \frac{1}{Re}\nabla_m^2 \boldsymbol{u}_m,$$

where the flow variables are normalized by characteristic length D and velocity U_0, respectively. For time discretization to the equation, a third-order mixed explicit-implicit schemes is used. The solution procedure at every time step is split into three substeps. The pseudo-spectral method is adopted to evaluate the nonlinear terms in the split equations in which the fifth-order upwind compact scheme is used in physical space. For solving Helmholtz equations for pressure and velocity, a nine-point compact scheme of fourth-order accuracy is derived and fourth-order centre compact scheme is used for nonhomogeneous term calculations. In this method, a semi-discretized pressure boundary conditions and a generalized nonreflecting-type outflow boundary conditions are used. The wake-type inflow velocity profile is taken as $U(y,z) = 1.0 - a(z)(2.0-\cosh(by)^2){\rm e}^{-(cy)^2}$, which represents the time average stream-wise velocity profile in the cylinder near wake where the flow is in most unstable, determined by referring to direct numerical simulations[6] as well as the experimental measurement.[7] The parameters $a = 1.1 + 0.4{\rm e}^{-z^2}$, $b = 1.1$, $c = 1.2$, where $a(z)$ is imposed to simulate a local span-wise variation in momentum defect, which is exponentially decay with the spanwise distance. The Reynolds number is defined as $R_e = U_0 D/\nu = 200$. The computation domain is $100, 30, 30(D)$ in stream-wise, transverse and spanwise directions, respectively. The cutoff of the truncated Fourier series is $N = 64$, and the corresponding number of the grid points in the

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$x-y$ plane are 202×62. The numerical code used in this paper has been verified first. It shows that evolution of the wake-type flow without the local span-wise non-uniformity results in a normal Kármán vortex street with the Strouhal number of 0.189, which is well compared with both the numerical simulation of wake-type flow evolution ($St=0.195$)[6] and the direct numerical simulation (DNS) results of flow around cylinder ($St=0.179$).[8]

Figure 1 shows an iso-surface diagram of $|\boldsymbol{\omega}|=0.12$ in the full calculation domain at $t=300.0$. It exhibits the whole process of the vortex dislocations, from generation to decay. The instability waves are first developed at two sides of the span and lead to rolling-up of vortices, forming Kármán vortex streets, while the vortices near the central are still not rolling-up. Hence the vortices begin to roll up and to form Kármán vortex in a very narrow area near centre. In a little downstream area, the Kármán vortex street has developed and a rather regular coherent structure is formed. We can see a systematic vortex dislocations are generated. In the dislocation, a vortex roll is divided into three parts from one side of the span. One of them connects with the upstream vortex, the second one joints to the adjacent downstream vortex, and the third part goes through the central and reaches the opposite side of the span. When the third one comes near the centre, it is joined by another pair of stream-wise vortex coming from downstream and upstream. Correspondingly, span similar process from the centre to the other side can occur in an inverted sequence.

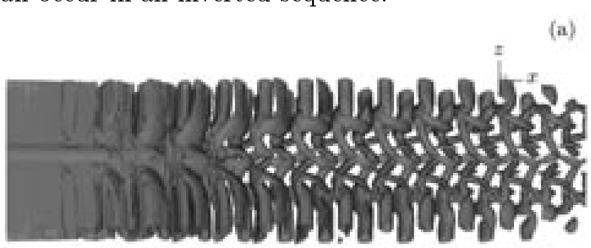

Fig. 1. Iso-surface of $|\omega|=0.12$ in the full calculation region at $t=300$.

The overview of vortex dislocations has been given by iso-vorticity contour, but the details of the vortex linkages in vortex dislocations, especially the connections near different sign regions, are still unclear. The linkages in vortex dislocations cannot be clearly identified from those contour surfaces. In fact, there have been few works studying such problems in the preview experiments or numerical analyses for observing and depicting such phenomena because they are very difficult. In order to study the dynamical process of vortex dislocation and to understand the real information on spatial linkages in vortex dislocations, the characteristics of vortex line are studied in the flow. The vortex lines will be drawn from different positions in a vortex and their tracks are traced. Because the tangent of vortex line at any point represents the vector direction of vorticity at the point, the modification of vortex lines shows the characteristics of vortex structure and its connections with adjacent vortices. Now we study the characteristics of vortex line starting or ending at some representative points in the concentrated region of vorticity at different downstream locations.

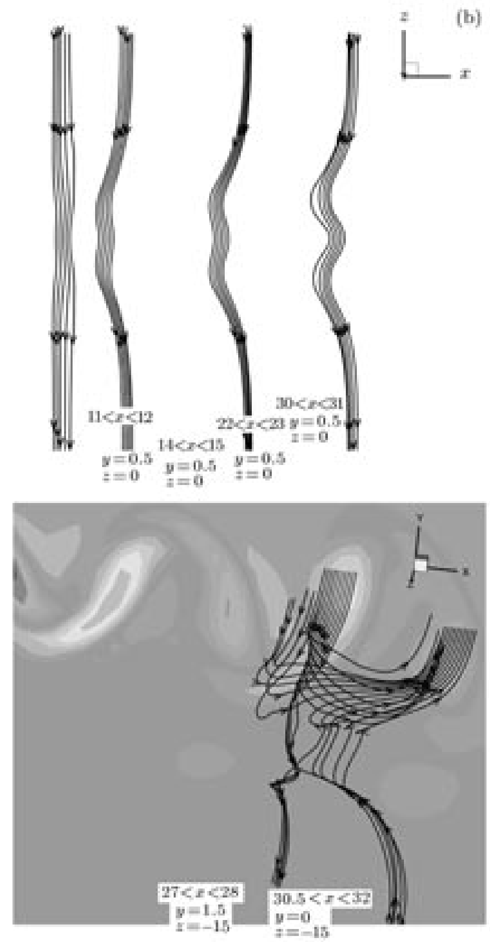

Fig. 2. Vortex lines with different beginning or ending positions. The background is the counter of ω_z on the $z=-15$ plane.

In Fig. 2(a) several clusters of vortex lines are displayed, which pass through several special places on the center plane at $z=0$ from upstream to downstream in the initial stage of flow. In the early initial stage, which is the flow in the region of $11<x<12$, the vortex lines are nearly parallel to the span direction; and a little downstream, i.e., $14<x<15$, although the large part of the vortex lines points to the z direction, the vortex lines undergo a large distortion near the central area, the vortex lines first bend to upstream, after passing the centre plane they bend to the downstream. Such a kind of the change become stronger at the downstream, such as in $22<x<23$ and $30<x<31$. Along the span-wise direction, there are two regions, which are the central region and near the central region, where the direction of vortex varies rapidly. This also indicates that the phase of vortex changes rapidly. Thus, the morph and tracing of vortex lines are in token of the actual connection and transition between the vortices with different phases. The stream-wise vorticity and vertical vorticity must occur in the region where the direction of vortex lines varies rapidly.

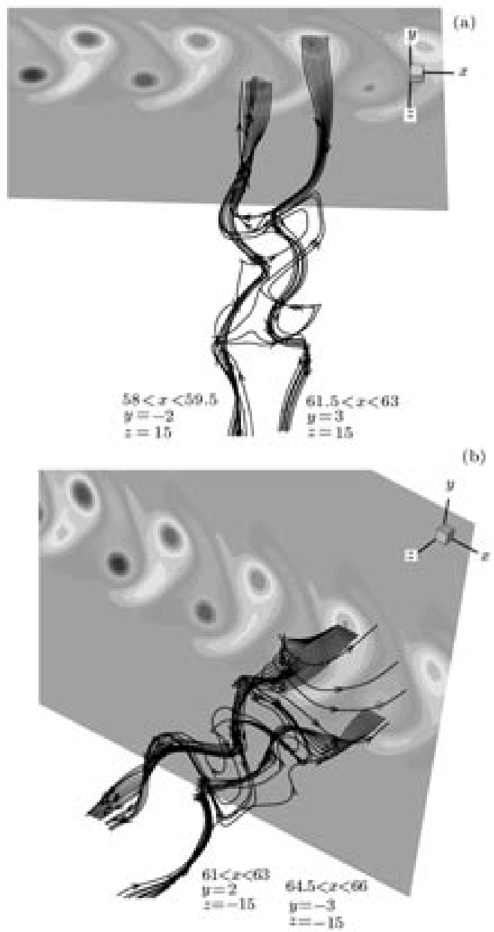

Fig. 3. Vortex lines with different beginning or ending positions. The background is the counter of ω_z on the $z = -15$ plane.

Another kind of the modification of vortex lines are found when the vortex lines emit from one span-wise end at $z = -15$. As Fig. 2(b) shown, two clusters of vortex lines from a positive vortex core are drawn in the region $23.5 < x < 24$. One of them passes across the span with a big distortion near the central area, the behaviour is the same as that in Fig. 2(a). The other cluster goes into the upstream adjacent vortex core, after turning 180° back. This means that some parts of the positive vortex are connected with adjacent negative vortex, as if they are "*split*" from that vortex. The connection between the positive vortex and its adjacent vortex with negative sign takes place by a cross-vortex street mode. The connection of cross-vortex street is three-dimensional, it does not occur on a plane, so there must have the stream-wise component ω_x and vertical component ω_y. This is the characteristic of vortex line started from one end at $z = -15$. If the vortex line is started from the other end at $z = +15$, the characteristic would be the same except that the direction is reverse.

Next to the vortex forming region is the mature region where the vortex dislocations have developed. The vortex lines drawing from a positive vortex core in the region $58 < x < 66$ and its adjacent vortex core is shown in Fig. 3. A part of them pass through the central region to the other-hand side of the span. This shows a span-wise vortex with great distortion, whose characteristics are the same as those in Fig. 2(a). The second part (Fig. 3(a)) join the upstream adjacent negative vortex core after bending to the upstream, turning 180° at a span location, which is as the same as that shown in Fig. 2(b). The third part (Fig. 3(b)) join the downstream negative vortex core after bending to the downstream, turning 180° at another span location. In addition, the distortion of some vortex lines is so large that they wind to the adjacent vortex core.

Our results have shown that the vortex linking in vortex dislocations described by vortex lines has two characteristics: the first one is that a part of the vortex lines traverse the whole span-wise but with large spatial distortion in the direction of stream-wise to form a "*wavy*" span-wise vortex. Along the span there are two regions in which the distortion is particularly large, one is the centre region and the other is slightly far from but near the centre. The second one shows that in those regions, a part of vortex lines after bending back become to be the opposite direction vortex lines and then join the adjacent with the opposite sign vortex by the cross-vortex street mode. This is the basic characteristic of the splitting process and reconnection in vortex dislocations. From these basic characteristics of vortex lines it is clear that there is a distorted span-wise vortex core, and at the largest distortion area the vortex can divide into the stream-wise and vertical branches connecting with adjacent vortex cores.

In summary, the present DNS results have shown systematic large-scale vortex dislocations generated in the non-uniform region of the wake-type flows. The structure and the physical dynamical process of the formation of the vortex dislocations are described by modifications of vortex line behaviour. The vortex splitting and vortex linkages in dislocations are described in details by tracing vortex line tracks. This shows that the vortex reconnection between vortex rolls with opposite signs is realized through vorticity diversion and transition, which is difficult to be provided by inviscid vortex dynamics.

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