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EXTENDED SELF-SIMILAR SCALING LAW OF MULTI-SCALE EDDY STRUCTURE IN WALL TURBULENCE*

JIANG Nan (姜　楠)[1,2], WANG Zhen-dong (王振东)[1,2], SHU Wei (舒　玮)[1,2]

(1. Institute of Mechanics LNM, Chinese Academy of Science, Beijing 100080, P R China;
2. Department of Mechanics, Tianjin University, Tianjin 300072, P R China)

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Abstract: The longitudinal fluctuating velocity of a turbulent boundary layer was measured in a water channel at a moderate Reynolds number. The extended self-similar scaling law of structure function proposed by Benzi was verified. The longitudinal fluctuating velocity in the turbulent boundary layer was decomposed into many multi-scale eddy structures by wavelet transform. The extended self-similar scaling law of structure function for each scale eddy velocity was investigated. The conclusions are 1) The statistical properties of turbulence could be self-similar not only at high Reynolds number, but also at moderate and low Reynolds number, and they could be characterized by the same set of scaling exponents $\xi_1(n) = n/3$ and $\xi_2(n) = n/3$ of the fully developed regime. 2) The range of scales where the extended self-similarity valid is much larger than the inertial range and extends far deep into the dissipation range with the same set of scaling exponents. 3) The extended self-similarity is applicable not only for homogeneous turbulence, but also for shear turbulence such as turbulent boundary layers.

Key words: wavelet transform; eddy; scaling law
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Introduction

Much work has been devoted in the last few decades to the measurement and modeling of the scaling law of structure function of turbulent flows. The so-called "velocity structure function of order n" for turbulent flows is defined as $\langle \Delta V(r)^n \rangle$, where $\Delta V(r) = V(x+r) - V(x)$ is the velocity component increment parallel to the relative displacement r of two positions separated by a distance of r in the flow field.

Let us remember that the research of structure function scaling law is usually limited by the following assumptions: 1) in the full developed turbulent flow so that the Reynolds number is infinite; 2) local homogeneous and isotropic; 3) for r in the inertial range.

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Biography: JIANG Nan (1968～), Associate Professor, Doctor

The expectation of the scaling law is that

$$\langle \Delta V(r)^n \rangle \propto r^{\xi(n)} \qquad (\eta \ll r \ll L), \tag{1}$$

where η is the dissipate length, L is the integral scale and $\xi(n)$ is called scaling exponent. The scaling law is an indication of the existence of scale invariance in turbulence.

For the third-order structure function, one can deduce the following Kolmogorov relation within above assumptions from the Navier-Stokes equations:

$$\langle \Delta V(r)^3 \rangle = -\frac{4}{5}\varepsilon r + 6\nu \frac{\partial \langle \Delta V(r)^2 \rangle}{\partial r}, \tag{2}$$

where V is the kinematics viscosity, $\langle\rangle$ stands for ensemble averaging and ε is the average rate of energy dissipation per unit mass. Within the inertial range, the second term on the right-hand side in Eq. (2) can be neglected

$$\langle \Delta V(r)^3 \rangle = -\frac{4}{5}\varepsilon r. \tag{3}$$

This means that: $\xi(3) = 1$. (4)

The classical Kolmogorov theory predicts that: $\xi(n) = \frac{n}{3}$. (5)

Benzi et al[1] recently showed evidence for the so-called extended self-similarity in their measurements of turbulence generated either by a wake flow past a cylinder or by a jet at moderate Reynolds number:

$$\langle \Delta V(r)^n \rangle = A_n |\langle \Delta V(r)^3 \rangle|^{\xi_1(n)} = B_n \langle | \Delta V(r)^3 | \rangle^{\xi_2(n)}, \tag{6}$$

where A_n and B_n are two different sets of constant independent of r. Since the third-order structure function $\langle \Delta V(r)^3 \rangle$ is proportional to r, instead of plotting the nth-order structure function $\langle \Delta V(r)^n \rangle$ against r, they plotted the nth-order structure function $\langle \Delta V(r)^n \rangle$ against the third-order structure function $\langle \Delta V(r)^3 \rangle$. Eq. (6) is valid not only in the fully developed turbulence but also at moderate and low Reynolds number, even if no inertial range is established. Moreover, it has been shown that the range of scales where Eq. (6) valid is much larger than the inertial range and extends far deep into the dissipation range.

Stolovitzky (1993)[2] repeated the experiments of Benzi and presented their experimental results. They measured the time series of the fluctuating velocity at a moderate Reynolds number in a turbulent boundary layer over a flat plate and investigated the extended self-similarity of the structure function. They revealed that, for low-order moments, a single scaling law with the same scaling exponents not far from $\xi(n) = n/3$ for dissipate as well as inertial range. However, as the order of the moment increases, the scaling law in the dissipate region and in the inertial region separates out. Within the dissipate region, the scaling exponents are nearly given by $\xi(n) = n/3$ with $\xi_1(8) = 2.66$, $\xi_2(8) = 2.42$. For r in the inertial range, the plot of versus consists of another linear region of slope $\xi_1(8) = 2.05$, $\xi_2(8) = 2.12$ slightly less than $\xi(n) = n/3$, joined by a smooth transitional region. The difference between the two regimes becomes increasingly apparent for higher n.

1 Experimental Apparatus and Techniques

Experiments were conducted in a full-developed turbulent flow of a free-surface water channel. Velocity measurements in the water channel were taken by TSI anemometer system with a TSI model 1210-20W single-sensor hot-film probe and a TSI model 1218-20W single-sensor hot-film boundary probe. The coming stream velocity and intermediate Reynolds number were $U_\infty = 0.28$ m/s and $Re_\theta = 2570$ respectively. The hot-film probe was located at $y^+ = 16$ above the lower wall of the channel. Fig. 1 shows the longitudinal fluctuating velocity signal obtained from the hot-film probe located in the near wall region of a turbulent boundary layer.

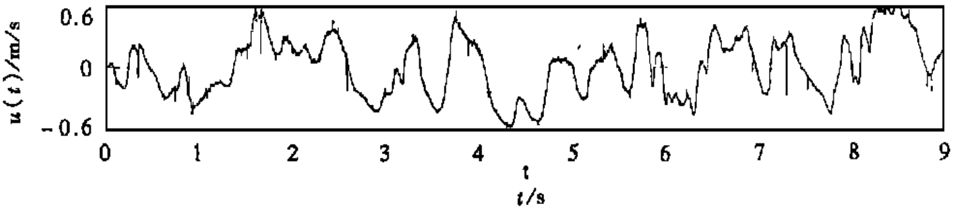


Fig. 1 Longitudinal fluctuating velocity in the near wall region of a turbulent boundary layer ($y^+ = 16$)

2 Extended Self-Similarity of Fluctuating Velocity Structure Function in Wall Turbulence

In Fig. 2, we show the structure function log| $\langle \Delta V(r)^n \rangle$ | against log| $\langle \Delta V(r)^3 \rangle$ | in inertial range and in dissipation range where it is shown that the nth-order structure function has apparently different scaling laws in dissipation range and in inertial range. The slopes in dissipation range are much smaller than those in inertial range. Fig. 3 is a plot of $\xi_1(n)$ in (6) against n in inertial range where the scaling exponents $\xi_1(n)$ are aligned on two curves of which one is larger than $\xi_1(n) = n/3$ for odd-order and the other is smaller than $\xi_1(n) = n/3$ for even-order.

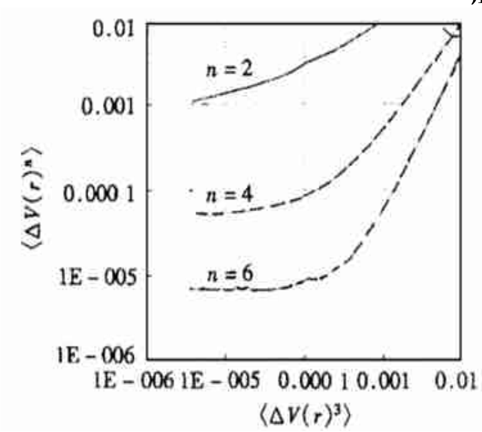


Fig. 2 Structure function log | $\langle \Delta V(r)^n \rangle$ | against log | $\langle \Delta V(r)^3 \rangle$ |

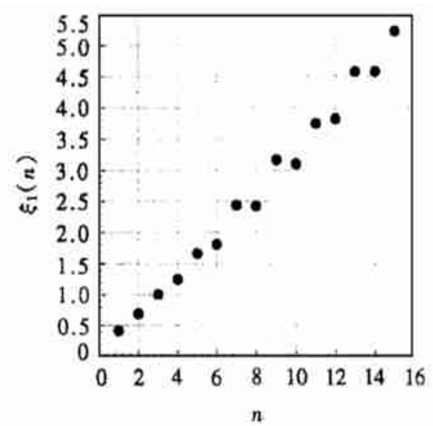


Fig. 3 Plot of the scaling exponents $\xi_1(n)$ against n

In Fig. 4, we show the structure function log$\langle | \Delta V(r)^n | \rangle$ against log$\langle | \Delta V(r)^3 | \rangle$. It is shown that the nth-order structure function has apparently different scaling laws in dissipation range and in inertial range. The slopes in dissipate range are much smaller than those in inertial range. Fig. 5 is a plot of $\xi_2(n)$ in (6) against n in inertial range where it is shown that $\xi_2(n) = n/3$.

3 Multi-scale Eddy Structure Extended Self-similarity Scaling Law

As far as turbulence is concerned, wavelet has special physical meaning. "Eddy" provides

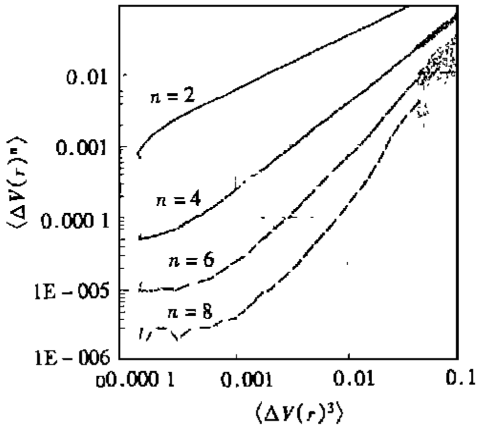


Fig.4 Structure function $\log\langle|\Delta V(r)^n|\rangle$ against $\log\langle|\Delta V(r)^3|\rangle$

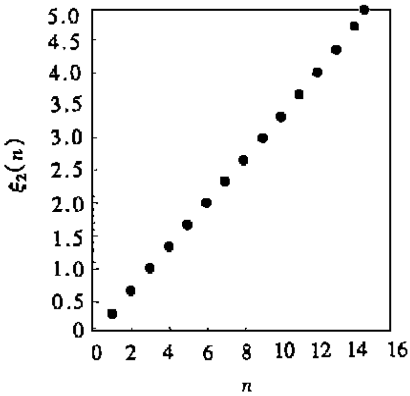


Fig.5 Plot of the scaling exponent $\xi_2(n)$ against n

the most suitable elementary decomposition of turbulence and is first introduced by Tennekes and Lumley[3]. According to them, eddies are fairly broad and self-similar contributions in the spectral domain that, unlike "waves", correspond to localized contributions in the physical space. They are localized both in spatial space and in time space. Fourier Transform does not take into account localized eddies in physics and therefore does not suit to decompose turbulence. This leads the physical meaning of Fourier Transform lost. Wavelet representation provides the decompositions of turbulence into eddy modes and the wavelet projection could be a very good alternative. "Eddies" are to turbulence study, what wavelets are. Fig.6 shows the typical shape of an "eddy" self-correlation function proposed by Tennekes and Lumley based on turbulence interpretation. Fig.7 shows the shape of an "eddy" self-correlation function of wall turbulence obtained by wavelet decomposition from experimental measured signals. As a new tool, wavelet transform can be devoted to the use for decomposing turbulence into eddies modes instead of Fourier Transform. Fig. 8 shows the reconstructed single scale eddy velocity by wavelet transformation of the sampled fluctuating velocity signal shown in Fig.1.

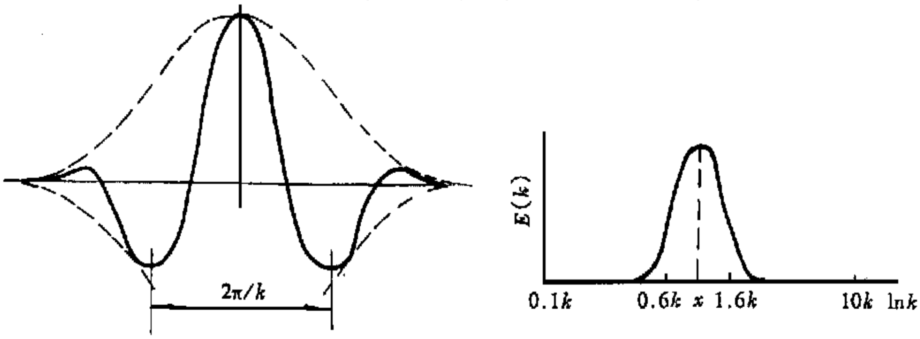


Fig.6 An eddy self-correlation function of wave number k and wavelength $2\pi/k$ [3]

In Fig.9, we show the structure function $\log|\langle\Delta V_l(r)^n\rangle|$ against $\log|\langle\Delta V_l(r)^3\rangle|$ where the nth-order structure function is aligned on a single line of slope $\xi_1(n) = n/3$. Fig. 10 is a plot of $\xi_1(n)$ in (6) against n where it is shown that $\xi_1(n) = n/3$.

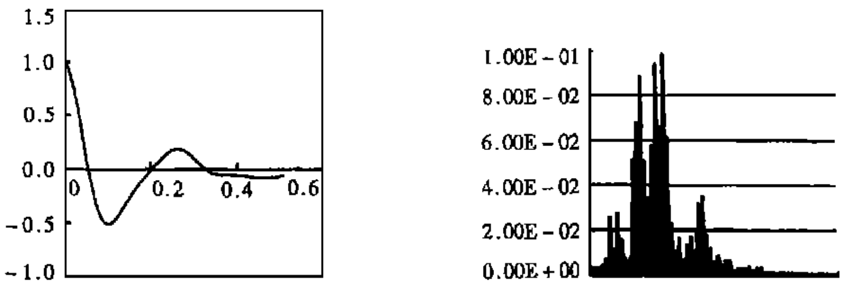


Fig. 7 An eddy self-correlation function obtained by wavelet transform

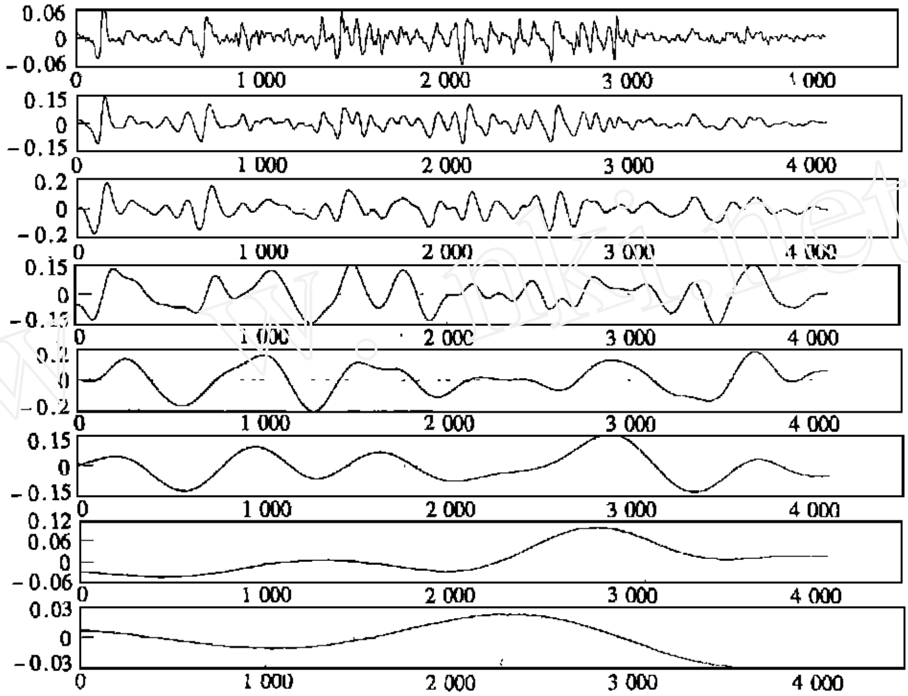


Fig. 8 Reconstructed velocity signal for each single scale eddy by wavelet decomposition

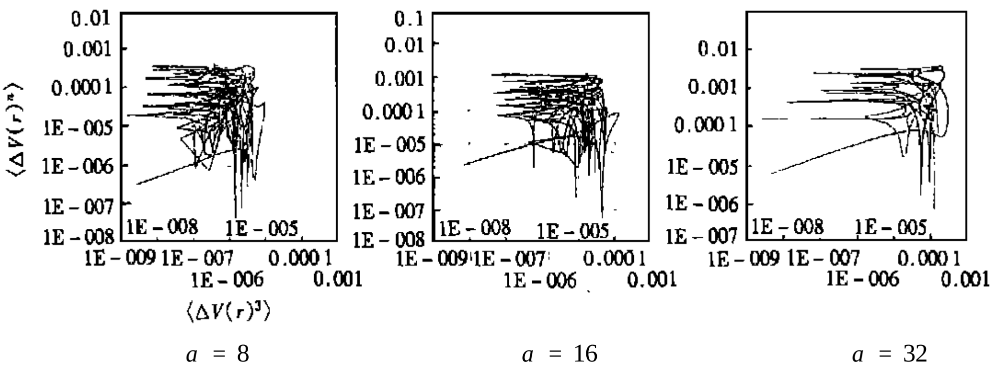
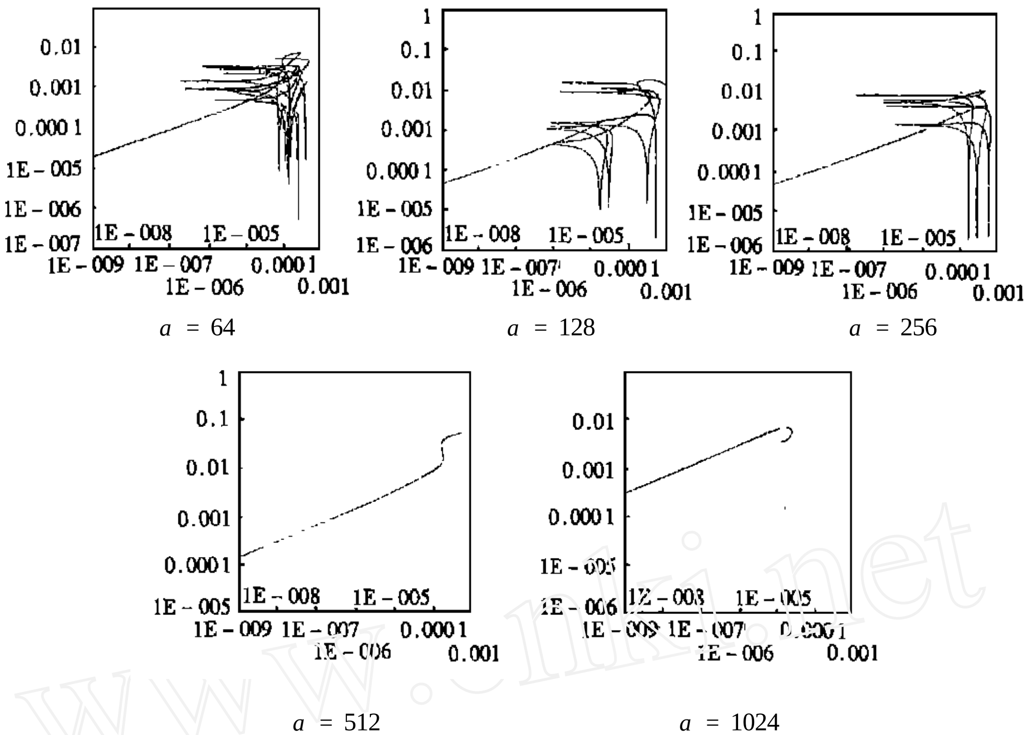


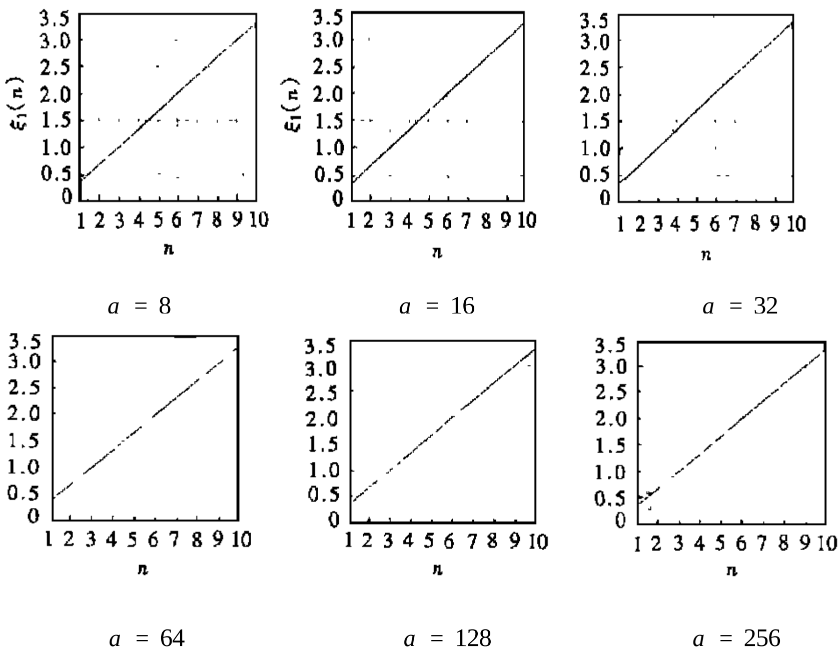
Fig. 9 The nth-order structure function $\log\langle|\Delta V_l(r)^n|\rangle$ against $\log\langle|\Delta V_l(r)^3|\rangle$ for each scale eddy



$a = 64$ $a = 128$ $a = 256$

$a = 512$ $a = 1024$

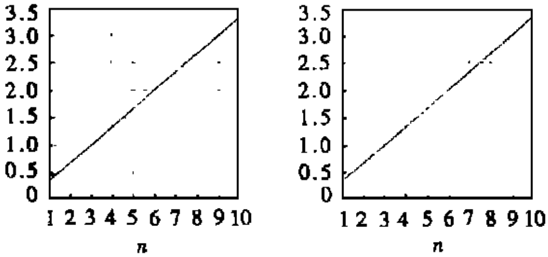
Fig. 9 The nth-order structure function $\log\langle|\Delta V_l(r)^n|\rangle$ against $\log\langle|\Delta V_l(r)^3|\rangle$ for each scale eddy



$a = 8$ $a = 16$ $a = 32$

$a = 64$ $a = 128$ $a = 256$

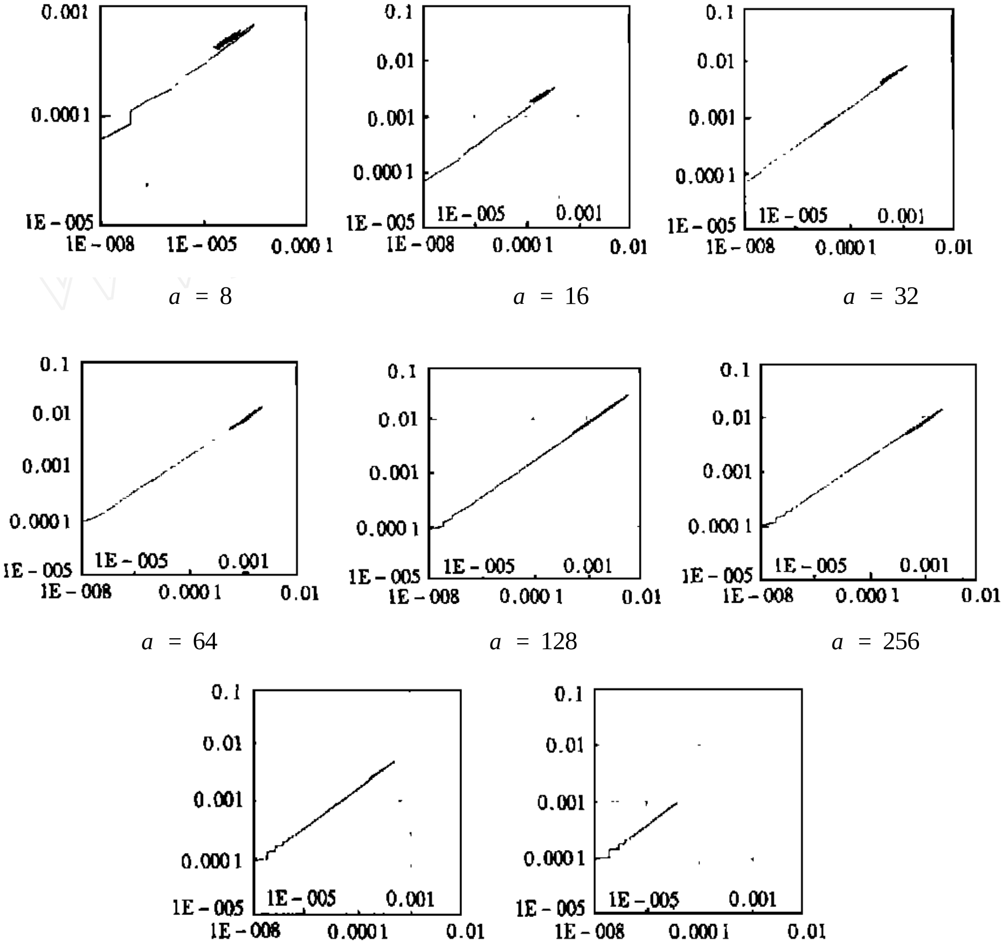
Fig. 10 Plot of $\xi_1(n)$ against n for each scale eddy



$a = 512$ $a = 1024$

Fig. 10 Plot of $\xi_1(n)$ against n for each scale eddy

Fig. 11 is a plot that $\log\langle | \Delta V_l(r)^n | \rangle$ versus $\log\langle | \Delta V_l(r)^3 | \rangle$ where the nth-order structure function is aligned on a single line of slope $\xi_2(n) = n/3$. Fig. 12 is a plot of $\xi_2(n)$ in (6) against n where it is also shown that $\xi_2(n) = n/3$.



$a = 8$ $a = 16$ $a = 32$

$a = 64$ $a = 128$ $a = 256$

$a = 512$ $a = 1024$

Fig. 11 The nth-order structure function $\log\langle | \Delta V_l(r)^n | \rangle$ versus $\log\langle | \Delta V_l(r)^3 | \rangle$ for each scale eddy

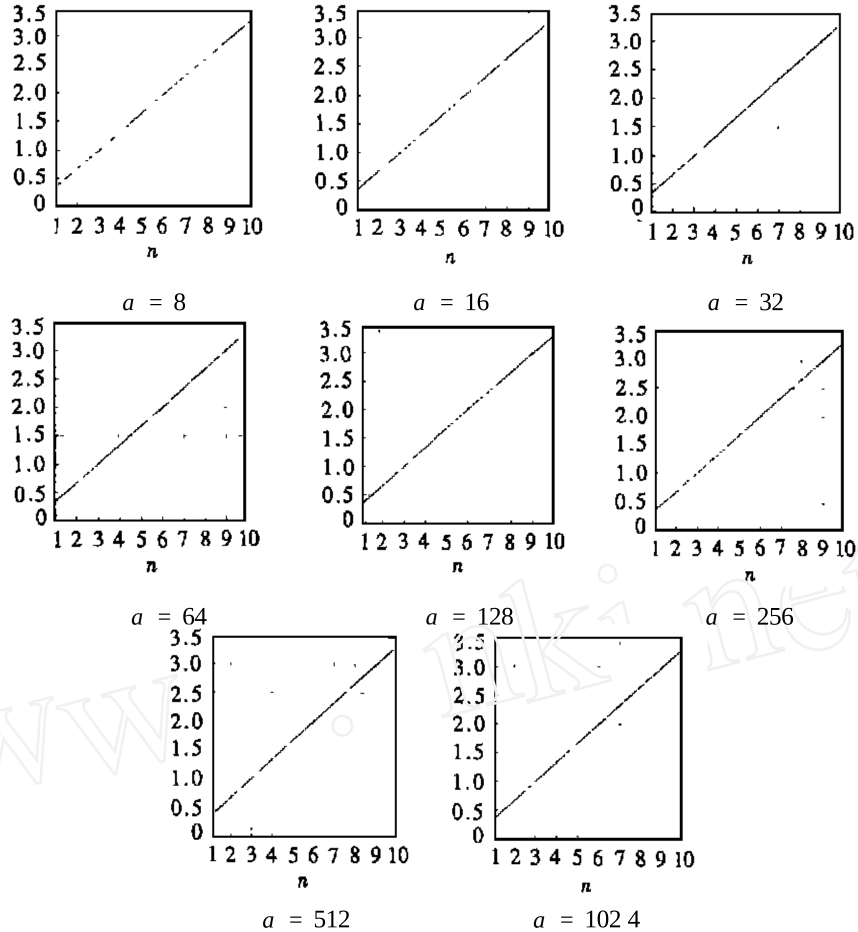


Fig. 12 Plot $\xi_2(n)$ against n for each scale eddy

4 Conclusions

1) The statistical properties of turbulence could be self-similar not only at high Reynolds number, but also at moderate and low Reynolds number, and they could be characterized by the same set of scaling exponents of the fully developed regime.

2) The range of scales where the extended self-similarity valid is much larger than the inertial range and extends far deep into the dissipation range.

3) The extended self-similarity is applicable not only for homogeneous turbulence, but also for shear turbulence such as turbulent boundary layers.

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