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Plastic constitutive behavior of short-fiber/particle reinforced composites

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Abstract

A cylindrical cell model based on continuum theory for plastic constitutive behavior of short-fiber/particle reinforced composites is proposed. The composite is idealized as uniformly distributed periodic arrays of aligned cells, and each cell consists of a cylindrical inclusion surrounded by a plastically deforming matrix. In the analysis, the non-uniform deformation field of the cell is decomposed into the sum of the first order approximate field and the trial additional deformation field. The precise deformation field are determined based on the minimum strain energy principle. Systematic calculation results are presented for the influence of reinforcement volume fraction and shape on the overall mechanical behavior of the composites. The results are in good agreement with the existing finite element analyses and the experimental results. This paper attempts to stimulate the work to get the analytical constitutive relation of short-fiber/particle reinforced composites. © 2002 Elsevier Science Ltd. All rights reserved.

Keywords: B. Constitutive behavior; Particulate reinforced material

1. Introduction

Various methods for the production of reinforced metal alloys have been developed in the industries and spaceflight, in order to improve the mechanical properties of the materials and thus obtain a weight reduction in structural applications. Metals reinforced by short-fiber/particles, e.g., SiC particles, have the advantage of being machinable and workable using conventional processing techniques, especially short fiber reinforcements have more favorable influence on the stiffness and elastic–plastic tensile properties. If good fiber alignment is obtained the tensile properties

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are much improved (McDanels, 1985). Fiber alignment is often obtained during processing by using either contracting flow or expanding flow in the extrusion. The mechanism of strengthening and the mechanical properties of metal matrix composites have attracted a considerable number of investigations (see, for example, Li and Ponte Castañeda, 1993; Zhu and Zbib, 1995; Kailasam and Ponte Castañeda, 1998; Llorca and González, 1998; Shu and Barlow, 2000; Wickowski, 2000; Bruzzi et al., 2001; Chaboche et al., 2001; Carmai and Dunne, in press; Naboulsi and Palazotto, in press).

Theory for determining the overall elastic properties of two-phase composites is very well developed. Hashin and Shtrikman (1963) and Hashin (1965) have established important results for the bounds on elastic moduli of multiphase composites with an arbitrary phase geometry. And methods based on the Eshelby's (1957) solution of a single inclusion embedded in an infinite matrix have been founded, for example, the self-consistent method (Christensen and Lo, 1979), the Mori–Tanaka method (Mori and Tanaka, 1973), and the differential method (Boucher, 1976; Mclaughlin, 1977).

Nonlinear behaviors of metal matrix composites have been the subject of increased interest since 1970s (Pindera et al., 1991). Talbot and Willis (1987) developed bounds and self-consistent estimate for the effective behaviors of nonlinear composites. A similar method for evaluating the overall properties of elasto-plastic composites has been proposed by Teply and Dvorak (1988). Using minimum energy principles of plasticity they derived upper and lower bounds on instantaneous stiffness. Ponte Castañeda (1991) proposed a new variational procedure to obtain the upper and lower bounds and to estimate the properties of composites. Weng and his co-workers (Zhao and Weng, 1990; Qiu and Weng, 1991; Li and Weng, 1998) and Hu (1996) developed an average method to predict the nonlinear constitutive relation of composite material based on the modified Mori and Tanaka procedure.

A self-consistent method was proposed by Duva (1984) and Duva and Hutchinson (1984) based on the solution of a kernel problem, in which an isolated inclusion is embedded in an infinite matrix of nonlinear material. This method was developed by He (1990) and Lee and Mear (1992). An alternative self-consistent method was proposed by Stringfellow and Parks (1991). Zhu and Zbib (1995) developed a mathematical model capable of capturing the basic features of plastic properties of the particulate-reinforced metal matrix composites based on a finite axisymmetric unit cell. The predicted results of composites containing particles with sharp corners has a comparatively large difference with experimental results.

Due to the complexity of geometry of the reinforcements and the nonlinear mechanical behavior of the matrix, the theoretical modelling of the constitutive relation of composites is rather difficult, and many analyses are carried out based on the finite element method. The work of Bao et al. (1991) and Bao (1992) combined the finite element methods with theoretical analysis, and approximated the parameters of the models that can reflect the effect of volume fraction and the shape of particles. A micromechanical model was proposed by Llorca and González (1998), in which the perfectly bonded or damaged interface was introduced in the axisymmetric unit cell. Tvergaard (1990) analyzed the tensile properties of ductile metal reinforced by a periodic array of short fibers assuming that the whisker ends are not perfectly aligned but staggered (see also Levy and Papazian, 1990).

Experimental studies always give the enlightenment of the theoretical analysis and give the basic parameters of the mathematical model. The mechanical properties of various discontinuous reinforced aluminum composites were measured by Nieh and Chellman (1984) and McDanels (1985). An experimental study and a detailed finite element analyses of the tensile properties of the particle or whisker-reinforced metal–matrix composite was investigated by Christman et al. (1989a,b).

In the present paper a cylindrical unit cell is used to analyze the tensile properties of a metal–matrix composite reinforced by short-fibers/particles. Unlike the previous work, the cell model in this paper can simulate the composites reinforced with short-fibers or particles with sharp corners. The results of present work are semi-analytic and the calculating work load is much lighter compared with the FEM method which would consume very large computing time and would not get analytical results. This paper will stimulate the work, attempting to give an analytical constitutive relation of composites reinforced with short-fibers/particles.

2. Theoretical framework

Generally the reinforcements are randomly orientated and distributed in the composites, and the size and geometry of the reinforcements are inhomogeneous. It is impossible to consider all of the actual factors in the model as the problem would become rather complex and difficult. In this paper, low volume fraction of reinforcement is assumed ($f \leqslant 0.2$) and the composites were idealized as uniformly distributed periodic arrays of aligned hexagonal unit cells, and each unit cell consisted of an elastic cylindrical fiber surrounded by an elasto-plastically deforming matrix. Under the same loading condition, each hexagonal unit cell behaved identically. A cylindrical cell was introduced as an approximation to the hexagonal cell for computational reasons, the stress or strain distribution was axisymmetric if the reinforcement and the external loads were axisymmetric (see Fig. 1). The volume fraction

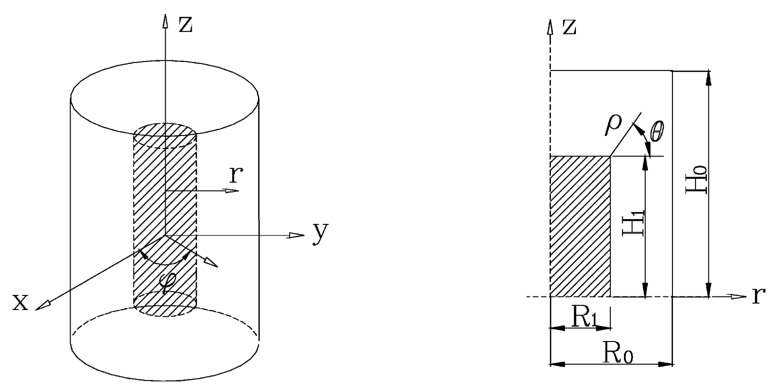

Fig. 1. The illuminating of the cell model.

of the reinforcement f was taken as the ratio of the reinforcement volume to the cell volume in the model.

Suppose that the macro elasto-plastic constitutive relation of composites can be written as following

$$E_{ij} = E_{ij}^{e} + E_{ij}^{p} = C_{ijkl}\Sigma_{kl} + \frac{3}{2}\frac{E_e^p}{(E_e^p)}S_{ij} \tag{1}$$

where C_{ijkl} is the macro elastic compliance tensor of composites, which can be easily obtained by solving a pure elasticity problem by having the unit cell as an elastic deformation. Σ_{kl} is the macro stress tensor of the unit cell, S_{ij} is the deviatoric part of the macro stress, E_{ij}, E_{ij}^{e}, E_{ij}^{p} are the macro strain, macro elastic strain and macro plastic strain, respectively, and $E_e^p = \left(\frac{2}{3}E_{ij}^{p}E_{ij}^{p}\right)^{1/2}$ is the macro equivalent plastic strain. In Eq. (1), function $A(E_e^p)$ describes the work hardening properties of the composites. The constitutive relation of the composites will be completely determined if function $A(E_e^p)$ is obtained. In this paper, our attention will focus on evaluating function $A(E_e^p)$. Eq. (1) will describe the constitutive behavior well and truly when composites undergoes elastic deformation or large plastic deformation. Since the composites reinforced with particles and short fibers have isotropic macroscopic mechanical characters (at low volume fraction of reinforcement), we assume that the composites also obey the J_2 flow theory and Mises yielding criterion in the paper.

Duva and Hutchinson (1984) have shown that the constitutive relation of composites could be expressed with the macro strain energy density. The macro energy density of composites is noted as

$$W = \frac{1}{V}\sum_{k=0,1}^{n}\int_{V_k}\omega_k \mathrm{d}V \tag{2}$$

where V_0 is the volume of the matrix in the composites, w_0 is the strain energy density of the matrix, and V_k, w_k ($k \geqslant 0$) are the volume and the strain energy density of the kth reinforcement component, respectively.

To simplify the problem of determination of function $A(E_e^p)$, the elastic strain in both the matrix and the fiber was neglected, i.e. the matrix is assumed as rigid plastic and the fiber is rigid. If the plastic deformation of composites is large enough, the simplicity has sufficient accuracy. Matrix material obeys the following power law equation,

$$\frac{\sigma}{\sigma_0} = \left(\frac{\varepsilon}{\alpha\varepsilon_0}\right)^{n} \tag{3}$$

where σ_0 is the tensile yield stress, $\varepsilon_0 = \sigma_0/E$, E is Young's modulus and n is the strain hardening exponent. Coefficient α is taken to be 3/7 by Ramberg and Osgood. For multiaxial stress states, the constitutive relation takes the form

$$s_{ij} = \frac{2}{3}\frac{\sigma_0}{\varepsilon_e}\left(\frac{\varepsilon_e}{\alpha\varepsilon_0}\right)^n \varepsilon_{ij} \tag{4}$$

where ε_e is the effective strain.

Considering inclusion is assumed as rigid, the macro strain energy density of composites is evaluated as

$$W = \frac{1}{V}\left(\frac{\sigma_0(\alpha\varepsilon_0)^{-n}}{n+1}\right)\int_{V_0} \varepsilon_e^{n+1}\mathrm{d}V \tag{5}$$

so the macro plastic constitutive relation for the composites material is expressed as

$$S_{ij} = \frac{\partial W}{\partial E_{ij}} \tag{6}$$

The boundary condition at the outer surface of the unit cell links the macroscopic value of the strain tensor to the microscopic displacement field through the compatibility requirement

$$u_i = E_{ij}x_j, \quad \text{at outer boundary} \tag{7}$$

where $E_{11} = E_{22} = -\frac{1}{2}E_{33}$, the macro strain of unit cell in the analysis satisfies the incompressible condition.

The inner boundary condition for the displacement field in the matrix, with respect to a reference frame fixed at a point on the reinforcement, is given by

$$u_i = 0, \quad \text{at inner boundary} \tag{8}$$

Let $(x^1, x^2, x^3) = (x, y, z)$ be the Cartesian coordinate system with the origin located at the cylinder center and the x^3-axis coinciding with the axis of revolution, and in the cylindrical coordinate $(x^1, x^2, x^3) = (r, \varphi, z)$, we define the aspect ratio of cell and reinforcement χ as

$$\chi = \frac{H_0}{R_0} = \frac{H_1}{R_1} \tag{9}$$

Since the reinforcement is short-fibers and particles, we assume $0.1 \leqslant \chi \leqslant 10$.

For plastically incompressible matrix material under axisymmetric loading conditions, a simple form of obtaining the physical components of the displacement vector **u** is to employ a displacement potential function ζ such that $u = \nabla \times (0, \zeta/\sqrt{g_{22}}, 0)$, (see, for example, Lee and Mear, 1992), yielding

$$u_r = \sqrt{\frac{g_{11}}{g}}\frac{\partial\zeta}{\partial z}, u_\varphi = 0, u_z = -\sqrt{\frac{g_{33}}{g}}\frac{\partial\zeta}{\partial r} \tag{10}$$

where g_{11}, g_{22}, and g_{33} are the covariant components and g is the determinant of the metric tensor in cylindrical coordinate system.

In this paper, we suppose that the displacement potential function is

$$\zeta = (\theta - \pi)^2(\theta + \pi/2)^2\left[\rho^2 r^2 z\bar{E}/\eta + \rho^2(r - R_0)^2(z - H_0)^2\sum_{k=1}^{\infty}\sum_{m=1}^{\infty}\beta_{km}z^k r^m\bar{E}\right] \quad (11)$$

where factor $(\theta - \pi)^2(\theta + \pi/2)^2$ was introduced to satisfy the inner boundary condition (8), and for eliminating the strain singularity at the corner $\rho=0$, the factor ρ^2 was introduced. The coefficients η and β_{km} in Eq. (11) will be fixed on later. ρ and θ were illustrated in Fig. 1, which can be expressed as

$$\rho = \sqrt{(r - R_1)^2 + (z - H_1)^2} \quad (12)$$

$$\theta = \arctan(r - R_1, z - H_1) \quad (13)$$

To obtain the solution of the strain field of the cell for a given aspect ratio, volume fraction of reinforcement and hardening exponent of matrix, the double series in Eq. (11) should be truncated, and the unknown coefficients β_{km} are determined by minimizing the plastic energy of the unit cell.

When the strain field of the unit cell is evaluated, the macro stress deviatoric tensor can be derived according to Eqs. (5) and (6), and then the macro effective stress and plastic strain relation of composites is obtained,

$$\frac{\Sigma_e}{\sigma_0} = \frac{\Sigma_n}{\sigma_0}\left(\frac{E_e}{\alpha\varepsilon_0}\right)^n \quad (14)$$

where

$$\frac{\Sigma_n}{\sigma_0} = \left(\frac{3}{2}\right)^{(n+1)}\frac{1}{V}\int_v \hat{\varepsilon}_e^{(n+1)}\mathrm{d}V \quad (15)$$

where $\varepsilon_e = \bar{E}\hat{\varepsilon}_e$, $\bar{E} = E_{33} - E_{11}$ and $E_e = \frac{2}{3}(E_{33} - E_{11})$. In this paper, $\frac{\Sigma_n}{\sigma_0}$ is called the strengthening factor of composites, which is an important reference coefficient to reflect the hardening effect of the reinforcement, and noted as the function of f, n, H_0/R_0,

$$F(f, n, H_0/R_0) = \frac{\Sigma_n}{\sigma_0} \quad (16)$$

At last, function $A(E_e^p)$ was obtained

$$A(E_e^p) = \sigma_0 F(f, n, H_0/R_0)\left(\frac{E_e}{\alpha\varepsilon_0}\right)^n \tag{17}$$

Substituting Eq. (17) into Eq. (1), the macro constitutive relation of composites is obtained. In addition, the macro elastic compliance C_{ijkl} in Eq. (1) could be calculated by solving a problem of elasticity.

3. Numerical analysis and results

In the general case of cylindrical reinforcement and power law matrix, the minimization of W and integration of $F(f, n, H_0/R_0)$ must be performed numerically. In both calculations, the central issue is to evaluate the integral

$$\hat{W} = \frac{1}{V}\int_V \hat{\varepsilon}_e^{(n+1)} \mathrm{d}V \tag{18}$$

Theoretically, one needs to consider a large number of terms in the series in Eq. (11) to ensure a good quantitative accuracy, which will bring us the nonlinear problem.

In this paper, the perturbation method is used to solve the nonlinear problem. First, the first order approximation of the displacement potential was employed, written as

$$\zeta_0 = \rho^2 r^2 z(\theta - \pi)^2(\theta + \pi/2)^2 \bar{E}/\eta \tag{19}$$

A primitive displacement field of the unit cell can be obtained by substituting Eq. (19) into Eq. (10), written as follows

$$\begin{aligned} u_{r_0} &= \frac{T}{\eta}[2rz(r - R_1)(2\theta - \frac{\pi}{2}) + r(2z(-H_1 + z) + \rho^2)T]\bar{E} \\ u_{z_0} &= -\frac{T}{\eta}[2rz(H_1 - z)(2\theta - \frac{\pi}{2}) + 2z(r(r - R_1) + \rho^2)T]\bar{E} \end{aligned} \tag{20}$$

where $T = (-\pi + \theta)(\frac{\pi}{2} + \theta)$, and η can be evaluated by

$$\eta = -\frac{1}{\pi R_0^2 H_0 E_{11}}\int_0^{H_0}\int_0^{2\pi} u_{r_0}(R_0, z)\eta\cos^2\varphi R_0 \mathrm{d}\varphi \mathrm{d}z \tag{21}$$

It can be proved that Eq. (20) satisfied both the external and inner boundary conditions. So the primitive strain field $\hat{\varepsilon}_{ij0}$and the primitive value of $\frac{\Sigma_n}{\sigma_0}$ can be easily calculated. An approximate analytical plastic constitutive relation of composites is obtained.

The macro stress–strain curve of the cell was predicted by using the approximate constitutive relation for a representative case of $f=0.1$, $n=0.2$ and $H_1/R_1=H_0/R_0=1$, i.e. the aspect ratio of the cell is taken to be equal to that of the reinforcement. Llorca and González (1998) had calculated this kind of ceramic inclusion using finite element code ABAQUS, its result was also depicted in Fig. 2. The behavior of the reinforcement was assumed to be linear elastic and isotropic. The matrix was modelled as an isotropically hardening elasto-plastic solid following J_2 deformation theory of plasticity, which was represented by the power law equation, i.e. α was selected as unity in Eq. (3). From Fig. 2, it can be seen that the primitive results have an approximate accuracy to the exact FEM results, which is within about 5%.

Since the primitive displacement can provide an approximate stress–strain relation of the unit cell, a more accurate description of the displacement field can be obtained by introducing the additional terms in displacement potential which is given as follows,

$$\tilde{\zeta} = \rho^2(\theta-\pi)^2(\theta+\pi/2)^2(r-R_0)^2(z-H_0)^2\sum_{k=1}^{\infty}\sum_{m=1}^{\infty}\beta_{km}z^k r^m \bar{E} \tag{22}$$

The perturbation method is used in determining the coefficients β_{km} in the trial displacement field and strain field of the unit cell under the minimum energy principle.

The normalized strain of the unit cell can be written as

$$\hat{\varepsilon}_{ij} = \hat{\varepsilon}_{ij0} + \Delta\hat{\varepsilon}_{ij} \tag{23}$$

then we can get

$$\hat{\varepsilon}_{ij}^2 = \hat{\varepsilon}_{ij0}^2 + 2\hat{\varepsilon}_{ij0}\Delta\hat{\varepsilon}_{ij} + (\Delta\hat{\varepsilon}_{ij})^2 \tag{24}$$

and

$$\begin{aligned}\hat{\varepsilon}_e^{n+1} &= \hat{\varepsilon}_{e0}^{n+1}\left(1+\frac{4}{3}\frac{\hat{\varepsilon}_{ij0}\Delta\hat{\varepsilon}_{ij}}{\hat{\varepsilon}_{e0}^2}+\frac{2}{3}\frac{\Delta\hat{\varepsilon}_{ij}\Delta\hat{\varepsilon}_{ij}}{\hat{\varepsilon}_{e0}^2}\right)^{\frac{n+1}{2}} \\ &\cong \hat{\varepsilon}_{e0}^{n+1}\left[1+\frac{2}{3}(n+1)\frac{\hat{\varepsilon}_{ij0}\Delta\hat{\varepsilon}_{ij}}{\hat{\varepsilon}_{e0}^2}+\frac{n+1}{3}\frac{\Delta\hat{\varepsilon}_{ij}\Delta\hat{\varepsilon}_{ij}}{\hat{\varepsilon}_{e0}^2}+\frac{n^2-1}{8}\left(\frac{4}{3}\frac{\hat{\varepsilon}_{ij0}\Delta\hat{\varepsilon}_{ij}}{\hat{\varepsilon}_{e0}^2}\right)^2\right]\end{aligned} \tag{25}$$

Substituting Eq. (25) into Eq. (18), and evaluating the partial derivative of $\hat{W}$ with respect to β_{km},

$$\frac{\partial\hat{W}}{\partial\beta_{km}} = \frac{1}{V}\int\hat{\varepsilon}_{e0}^{n+1}\left[\frac{2(n+1)}{3}\left(\frac{\hat{\varepsilon}_{ij0}}{\hat{\varepsilon}_{e0}^2}\frac{\partial\Delta\hat{\varepsilon}_{ij}}{\partial\beta_{km}}+\frac{\Delta\hat{\varepsilon}_{ij}}{\hat{\varepsilon}_{e0}^2}\frac{\partial\Delta\hat{\varepsilon}_{ij}}{\partial\beta_{km}}\right)+\frac{4(n^2-1)}{9}\frac{\hat{\varepsilon}_{ij0}\Delta\hat{\varepsilon}_{ij}}{\hat{\varepsilon}_{e0}^2}\frac{\hat{\varepsilon}_{ij0}}{\hat{\varepsilon}_{e0}^2}\frac{\partial\Delta\hat{\varepsilon}_{ij}}{\partial\beta_{km}}\right]\mathrm{d}\upsilon \tag{26}$$

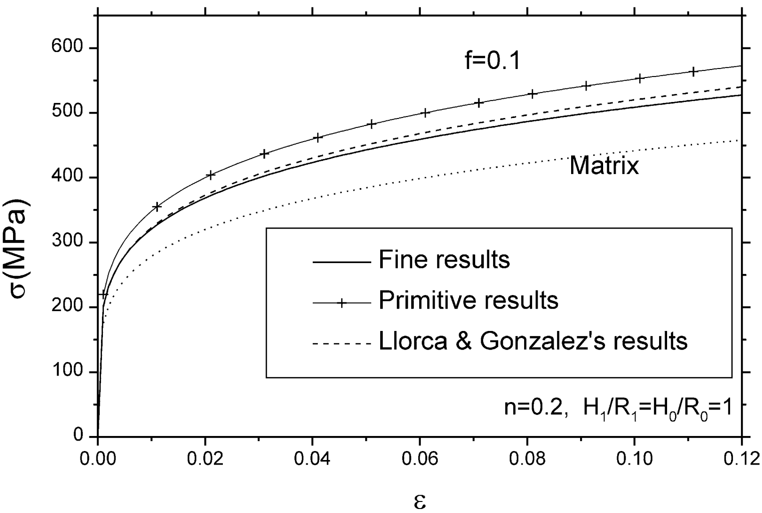

Fig. 2. Stress and strain curve of the unit cell, predicted fine results and primitive results compared with FEM results of Llorca and Gonzalez (1998), ($f = 0.1$).

Let the above equation be equal to zero, and we get the linear equations about β_{km},

$$\frac{\partial \hat{W}}{\partial \beta_{km}} = 0 \tag{27}$$

The displacement and strain field of the unit cell could be obtained by solving the linear equations, and the value of $\frac{\Sigma_n}{\sigma_0}$ is obtained according to Eq. (15). Since Eq. (27) is a linear equation, the computing work load is very light. In this paper, all the computing is done by engaging math software Mathematics 3.0.

The results presented below were obtained with $k = 1,...K$ and $m = 1,...M$, where $K = 8$ and $M = 8$. In the procedure of calculations, convergence studies were carried out and those studies indicated that the results to be presented are accurate to within approximately 1%. For the sake of the accuracy of the calculation, the Newton–Raphson method was employed to check the perturbation method also.

The stress–strain curve of the cell with multi-terms in the displacement field mode (which we call fine results) is compared with the primitive result and the FEM results of Llorca and González (1998), these curves are plotted in Fig. 2. From Fig. 2, it can be seen that the fine result is fairly close to the FEM results and the primitive results are indeed an approximation of the fine results which are within in about 5%, so the analytical constitutive relation could be founded based on the primitive results. It is noted that neither the present analyses nor the computations in Llorca and González (1998) have accounted for residual stress resulting from thermal contraction mismatch between fibers and the matrix. Although these residual stresses will result in early onset of plasticity locally, it is not expected that they will significantly change the shape of the tensile.

The role of inclusion shape on the macroscopic response of the two-phase composites is examined first for dilute concentrations of inclusions. The change in $\frac{\Sigma_n}{\sigma_0}$ due to the presence of the inclusions serves as a measure of the strength effect of the inclusions, the larger the magnitude of this quantity, the greater the strength provided by the inclusions. This quantity is plotted in Fig. 3 as a function of logarithm of aspect ratio H_0/R_0 of fiber for several hardening exponents. Fig. 3 shows the effect of the fiber reinforcement aspect ratio H_0/R_0 on the strengthening factor $\frac{\Sigma_n}{\sigma_0}$ for both prolate and oblate fibers, a large $\left|\log(H_0/R_0)\right|$ always indicates a shape far away from the unit cylinder. As can be seen from Fig. 3, fibers with large shape index $\left|\log(H_0/R_0)\right|$ (i.e. whisker or disks) exhibit much more reinforcement than that of unit cylinders. The figure also shows that prolate cylinders are more effective than oblate ones in strengthening the matrix, which is also observed by Lee and Mear (1992), Bao et al. (1991) and Yang et al. (1991). For example, when $f=0.2$ and $n=0.2$, the change in $\frac{\Sigma_n}{\sigma_0}$ due to the prolate cylinder fiber with an aspect ratio of 5, is 2.45 times larger than that provided by the unit cylindrical fiber. In addition to providing insight into the role of inclusion shape on the strength of two-phase composites, the tensile stress–strain relation curve was presented in Fig. 4(a)–(c). The stress and strain are normalized by σ_0 and ε_0, respectively, and the coefficient α in Eq. (3) was selected as 3/7. These curves described the role of matrix nonlinearity on the mechanical behaviors of the composites.

The predicted stress and strain curve and the experimental results of whisker reinforced composites for $H_0/R_0=H_1/R_1=5$ are plotted in Fig. 5. For this case, the composite parameters are given by $f=0.13$, and $n=0.1305$. It is observed that the present result is qualitatively in good agreement with the experimental results of Christman et al. (1989a), but quantitatively higher by about 10%, as shown in Fig. 5.

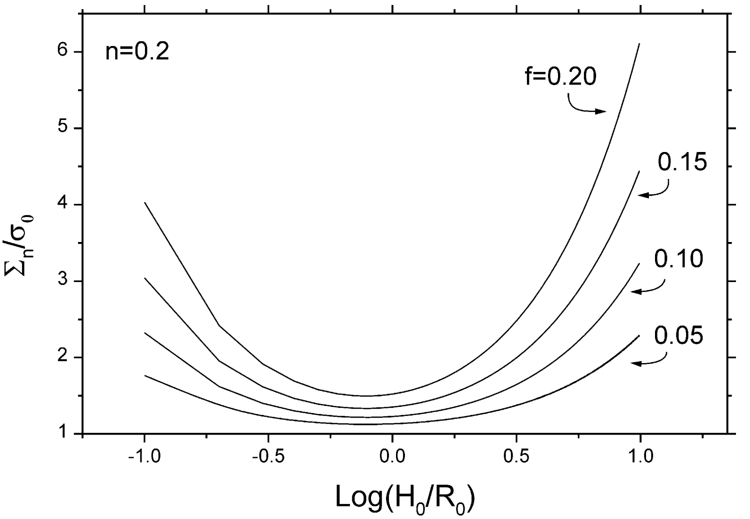

Fig. 3. Strengthening effect of the aspect ratio of fiber/particle reinforcement on the strength of composites.

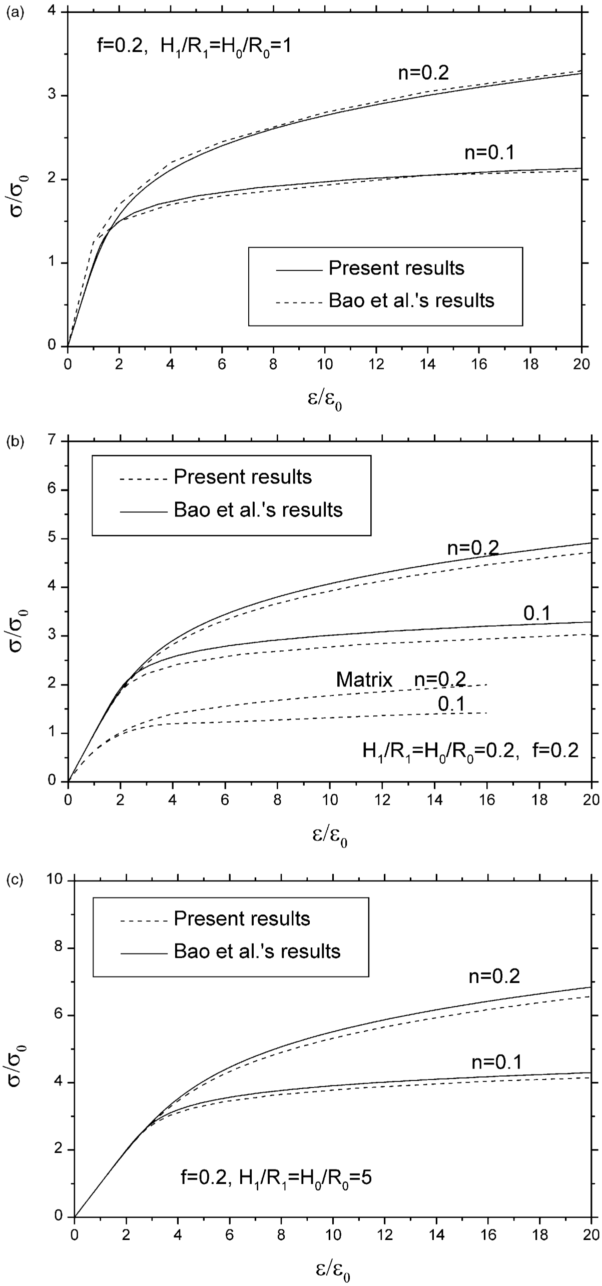

Fig. 4. (a) Tensile stress–strain curve for a matrix material reinforced by short fiber with unit aspect ratio and $f = 0.2$, present results compared with the results by Bao et al. (1991); (b) tensile stress–strain curve for a matrix material reinforced by aligned short fiber with aspect ratio $H_0/R_0 = 0.2$ and $f = 0.2$, present results compared with the results of Bao et al. (1991); (c) tensile stress–strain curve for a matrix material reinforced by aligned short fiber with aspect ratio $H_0/R_0 = 5$ and $f = 0.2$, present results compared with the results of Bao et al. (1991).

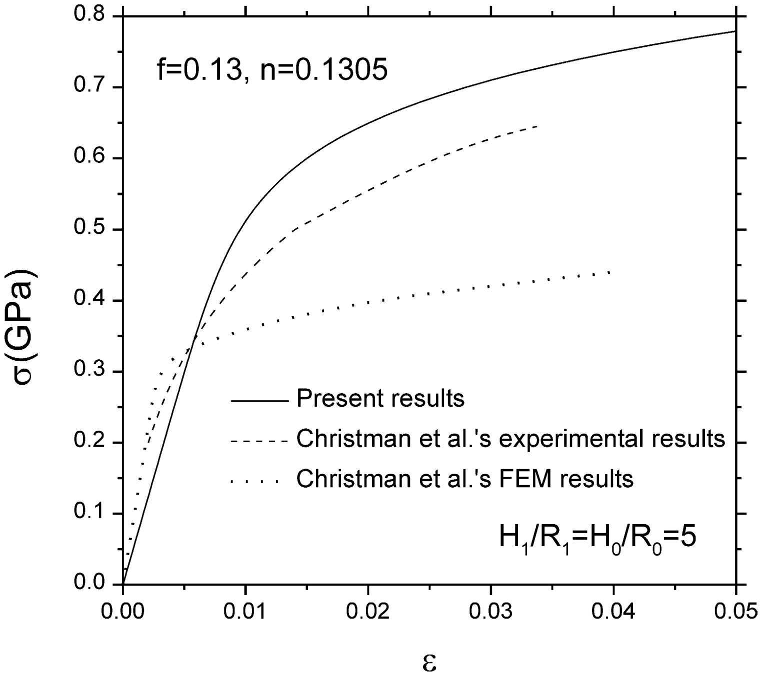

Fig. 5. Predicted uniaxial stress–strain curves for 2124 Al-SiC whisker reinforced composites compared with experimental and FEM results by Christman et al. (1989a,b).

The fiber reinforcements of material tested are randomly distributed and neighbour fibers are expected to be shifted relative to one another, both in the axial and transverse directions, maybe with a partial overlap between the fibers ends. It is the mechanism that causes the difference between the predicted results and experimental results.

A uniform distribution of aligned cylindrical particles, whose diameter equals their height, are far more effective reinforcing agents than spherical particles (Bao et al., 1991). At lower volume fraction the unit cylindrical particles ($H_1/R_1=1$) are approximately twice as effective as spherical particles at the same volume fraction, otherwise the volume of the smallest sphere which circumscribes the unit cylinder is 1.89 times that of the unit cylinder. In other words, the unit cylinder has almost the same effect as a spherical particle whose surface just circumscribes the cylinder. Fig. 6 shows the predicted results of this paper compared with the results of Zhu and Zbib (1995) and the experimental results of Nieh and Chellman (1984). From Fig. 6, it can be seen that whiskers can be modeled more accurately with the cylindrical cell model than the spherical cell model. This conclusion also agrees with results of Yang et al. (1991). The test results of Nieh and Chellman (1984) show that the hardening exponent of composites is remarkably higher than that of unreinforced matrix due to working process, so in the calculations, the hardening exponent of the matrix is selected as the same as that of composites, to consider the increase of the hardening exponent of the matrix during the process.

It is emphasized that the absolute strength of the composites due to the inclusions is strongly dependent upon the strain hardening exponent, and the calculation results were depicted in Figs. 7 and 8. From Fig. 7, it can be seen that strengthening factor $\frac{\Sigma_n}{\sigma_0}$ almost linear in n ($f < 0.2$), and the larger the volume fraction of inclusion, the steeper of the slope of the line. Fig. 8 shows that the degree to which the com-

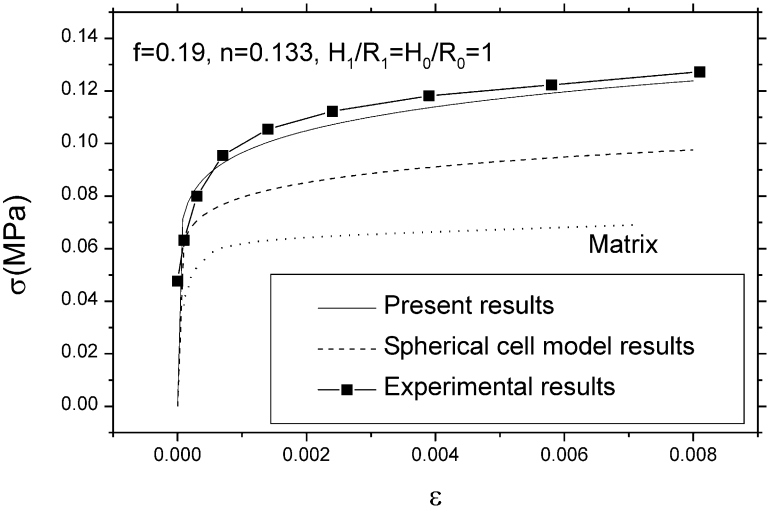

Fig. 6. Predicted uniaxial stress strain curves for particle reinforced composites compared with results of spherical cell model (Zhu and Zbib, 1995) and the experimental results of Nieh and Chellman (1984) on 1100 Al–SiC composites containing particles with sharp corner.

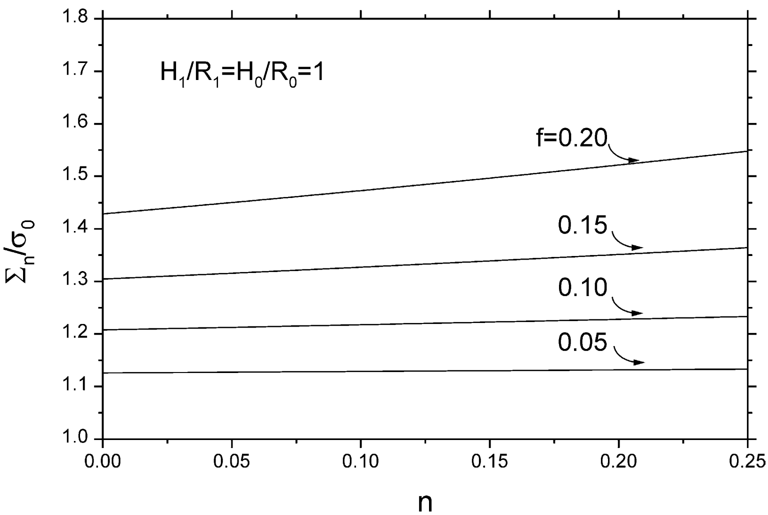

Fig. 7. Strengthening factor as a function of strain hardening exponent at a series of volume fraction of fiber reinforcement.

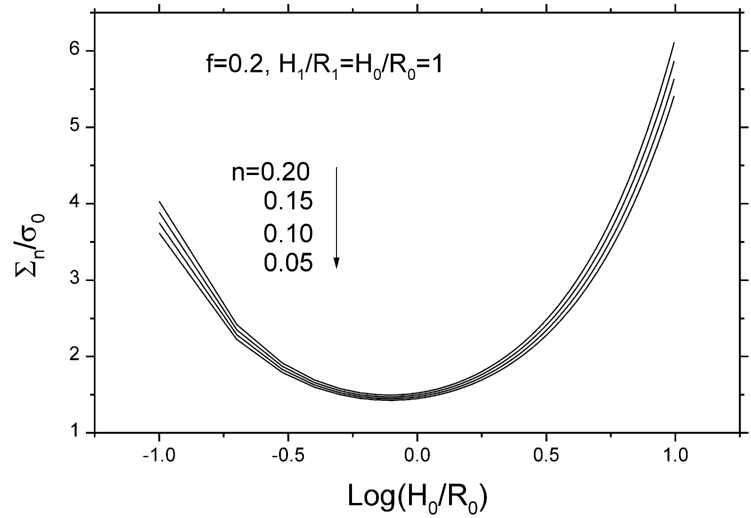

Fig. 8. Strengthening factor as a function of logarithm of aspect ratio (H_0/R_0) of fiber reinforcement at a series of strain hardening exponent of composites matrix.

posite strength depends upon inclusion shape is itself a function of matrix nonlinearity. And the effect of inclusion shape on the composites become more pronounced as the hardening exponent increases.

4. Discussion

A cell model to analyze the influence of microstructural factors on the mechanical response of short fibers/particles reinforced metal–matrix composites is developed in this paper. The work hardening function $A(E_e^p)$ was determined from the behavior of the unit cell using an isostrain approach in which the elastic strains were neglected. The plastic constitutive relation of fiber/particle reinforcement composites was established based on the cell model. The calculation results predicted by this model were proved by experimental results and finite element method results. It was concluded that the constitutive relation of fiber/particle reinforcement composites was dominated by the strain hardening exponent of matrix, the volume fraction and the aspect ratio of reinforcement. The cylindrical cell model can describe the mechanical behavior of short fiber or particle reinforced composites better than the spherical cell model.

We remark that since the elasticity of the matrix material and the inclusions have been neglected in determining the function $A(E_e^p)$, the constitutive relations which have been developed are expected to apply on the condition of the plastic strain within the matrix are sufficiently large, and the predicted results will be appreciated. One might argue that a lack of consideration of local effects due to size scale of the microstructure limits the usefulness of the model. Nevertheless, these generally good results in the paper suggest that the model can be effective in appropriate situations.

Finally, it should be indicated that the computing work load in this paper is much lighter than the FEM method although many terms of the displacement mode were selected in the calculations, as the perturbation method is employed in solving the nonlinear problem. The predicted constitutive relation of composites by primitive displacement field mode has been proved that it has approximate in 5% accuracy compared with the fine results with multi-terms displacement. Based on the primitive results of the constitutive relation of composites, an analytical constitutive relation of composites could be founded, which is very valuable to material scientists or engineers. However, further works should be done contributing to completed and perfect results.

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