Simulations of oil spread on a water surface

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Abstract: Numerical simulation of an oil slick spreading on still and wavy surfaces is described in this paper. The so-called σ transformation is used to transform the time-varying physical domain into a fixed calculation domain for the water wave motions and, at the same time, the continuity equation is changed into an advection equation of wave elevation. This evolution equation is discretized by the forward time and central space scheme, and the momentum equations by the projection method. A damping zone is set up in front of the outlet boundary coupled with a Sommerfeld–Orlanski condition at that boundary to minimize the wave reflection. The equations for the oil slick are depth-averaged and coupled with the water motions when solving numerically. As examples, sinusoidal and solitary water waves, the oil spread on a smooth plane and on still and wavy water surfaces are calculated to examine the accuracy of simulating water waves by Navier–Stokes equations, the effect of damping zone on wave reflection and the precise structures of oil spread on waves.

Keywords: oil spread, free-surface waves, Navier–Stokes solver, wave simulation

NOTATION

D	discretized divergence operator
f	Coriolis parameter
f_i	ith component of body force, $f_1 = -fu_2, f_2 = fu_1$
g	acceleration due to gravity
G	discretized gradient operator
h	water depth
H	oil slick thickness
p	dynamic pressure of water
p_a	air pressure
p_o	pressure in oil slick
p_T	total pressure of water
r_s, r_e	horizontal coordinates of the start and end points respectively of the sponge layer
T_{oa}, T_{ow}, T_{wa}	values of the surface tension at the oil/air, oil/water and water/air interfaces respectively
u_i	ith velocity component
ζ	water surface elevation
μ_w, μ_o	viscosities of water and oil respectively
ν	kinematic viscosity of water $= \mu_w/\rho$
ρ, ρ_o	water and oil densities respectively
σ	transformed vertical coordinate
Σ	discretized integration operator in vertical direction
τ_i^f, τ_i^w	ith components of the wind and water stresses respectively acting on the oil slick
τ_{ij}	shear stress for water, and depth-averaged shear stress for the oil slick
ω	intermediate variable defined by equation (11)

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1 INTRODUCTION

The oil spread and drift on a water surface has attracted much attention since Fay's **[1]** pioneering work in 1971. In that work, Fay first divided the oil spread on a still water surface into three regimes: the gravity–inertia regime, the gravity–viscosity regime and the viscosity–surface tension regime. At first, many practical methods predicted the oil spread by the use of Fay's formulae, a slick drift following a current driven by the water and wind speed, and some physicochemical processes by experience. Two hydrodynamic models, namely the advection–diffusion (AD) model and the full dynamic (FD) model, have been developed since then. The AD model **[2, 3]** simulates the oil spread by the equation of AD of the oil thickness representing the current-driven (advection) and turbulent diffusion between the oil and the water. The FD model **[4]** solves the motion of an oil slick by depth-averaged Navier–Stokes (NS) equations in which the effect of the water current on the oil motion is taken into account through the friction expressed as the square of the velocity difference between the oil and the water at the interface. Both the AD and the FD models calculate the water current separately by depth-averaged or three-dimensional NS equations by assuming that the oil motion does not affect the water motion and that the wave effect is neglected because of its periodic nature; thus the interfacial conditions are greatly simplified.

In this paper, the numerical simulation of an oil slick spreading on still and wavy surfaces is described. For the water motions, the σ transformation is used to transform the time-varying physical domain into a fixed calculation domain and the continuity equation is changed into an advection equation of wave elevation. This evolution equation is discretized by the forward time and central space scheme **[5]**, and the momentum equations by the projection method **[6]**. A damping zone **[7, 8]** is set up in front of the outlet boundary coupled with a Sommerfeld–Orlanski condition **[9]** at that boundary to minimize wave reflection. The equations for the oil slick are depth averaged and solved simultaneously with the water motions. As examples, sinusoidal and solitary water waves are calculated to simulate water waves by NS equations and to assess the effect of the damping zone on wave reflection. The oil spread on a smooth plane and on still and wavy water surfaces is calculated to study the precise structures of oil spread on still water and waves.

2 GOVERNING EQUATIONS

In the simulation model, the following assumptions are used:

1. The oil slick is taken as a single-phase Newtonian fluid with constant properties.
2. The horizontal scale of the slick is much larger than its vertical, so that the equations of the oil slick's motion are depth averaged, the pressure distribution in the vertical direction is static, and the dispersion of non-uniformity of the velocity across the thickness of the slick is neglected.
3. As the deformation of the interfaces between air and oil and between oil and water is small, some of the expressions related to the surface deformation are more or less simplified.

The final expressions for the governing equations of oil slick are **[10, 11]**

$$\frac{\partial H}{\partial t}+\frac{\partial (Hu_j)}{\partial x_j}=0 \tag{1}$$

$$\frac{\partial u_i}{\partial t}+u_j\frac{\partial u_i}{\partial x_j}=f_i-\frac{1}{\rho_o}\frac{\partial}{\partial x_i}[p_a+(\rho_o g-T_{oa}\Delta)(H+\zeta)] + \frac{1}{\rho_o H}(\tau_i^f-\tau_i^w)+\frac{\partial}{\partial x_j}(H\tau_{ij}),\quad i,j=1,2 \tag{2}$$

where H is the thickness of the oil slick, u_j are the depth-averaged horizontal oil velocity components, $f_1=-fu_2$, $f_2=fu_1$, $f\approx 10^{-4}\ \text{s}^{-1}$ is the Coriolis parameter, p_a is the atmospheric pressure, ρ_o is the oil density, T_{oa} is the surface tension on the oil/air interface, ζ is the water surface elevation beneath the oil, $\Delta=\partial^2/\partial x^2+\partial^2/\partial y^2$, τ_i^w is the viscous stress of water approximated by $\tau_i^w\approx\mu_w\,\partial u_i^w/\partial z$, the subscript and superscript w refer to the variables for water, τ_i^f is the wind stress from the air above the oil slick and is expressed by $\tau_i^f=-\rho_a C_{Da}(u_i^w-u_i^a)|u_i^w-u_i^a|$, $C_{Da}=0.024$, $\tau_{ij}=\mu_o(\partial u_i/\partial x_j+\partial u_j/\partial x_i)$ are the depth-averaged shear stresses in oil slick and μ_o is the oil viscosity. In the discretization of the oil motion equations, a uniform rectangular mesh is used in the horizontal plane with staggered grids **[5]**. The time derivatives are approximated by a two-level forward scheme. It is explicit in general, however, for the viscous terms in the momentum equations the Douglas–Richford ADI scheme **[5, 12]** is used, in which each time step is further divided into two halves. In the first half step, the space derivative in the x direction is implicit, while that in the y direction is explicit. The reverse is true in the second half-step. For the spatial derivatives, the advection terms are approximated by the QUICK scheme **[5]**, which is of third-order accuracy and conservative, and the viscous terms are discretized by using central difference. The final difference form of the x-momentum equation is

$$A_W u_{I-1,j}^{n+1/2}+A_P u_{I,j}^{n+1/2}+A_E u_{I+1,j}^{n+1/2}=E_1 \tag{3}$$

$$A_P=1-A_W-A_E \tag{4}$$

$$A_B u_{I,j-1}^{n+1/2}+A_P u_{I,j}^{n+1/2}+A_T u_{I,j+1}^{n+1/2}=E_2 \tag{5}$$

$$A_P=1-A_B-A_T \tag{6}$$

The details of the expressions **[10]** are omitted here for simplicity. A similar scheme may be used for the y components of the velocities.

As for the water wave motions, the water surface is assumed to be single valued in the horizontal plane, and the so-called σ transformation

$$\sigma=\frac{x_3-\zeta(x_1,x_2,t)}{h+\zeta} \tag{7}$$

is used to replace the usual vertical coordinate z. This transformation is very popular in oceanography and meteorology for large scale fluid motion. Here it is used for general short wave motions, where $x_3=\zeta(x_1,x_2,t)$ is the water surface and h is the water depth. Under the σ transformation, (a) the time-varying physical domain becomes a time-independent fixed free surface boundary fitted calculation domain and (b) the equation of the surface elevation

$$\frac{\partial \zeta}{\partial t}+\frac{\partial}{\partial x_i}\int_{-1}^{0}(h+\zeta)u_i\,\mathrm{d}\sigma=0, \qquad i=1,2 \tag{8}$$

now is an evolution equation depending upon the spatial derivatives of the integration of the fluxes along the vertical direction. The continuity equation is replaced by

$$\frac{\partial}{\partial x_j}[(h+\zeta)u_j]+\frac{\partial}{\partial \sigma}\left[u_3-\sigma u_j\frac{\partial h}{\partial x_j}-(1+\sigma)u_j\frac{\partial \zeta}{\partial x_j}\right]=0, \qquad j=1,2 \tag{9}$$

The momentum equations for the three-dimensional water motions become

$$\begin{aligned}&\frac{\partial u_i}{\partial t}+u_j\frac{\partial u_i}{\partial x_j}+\omega\frac{\partial u_i}{\partial \sigma}+g\frac{\partial \zeta}{\partial x_i}+\frac{\partial}{\partial x_j}(p\delta_{ij}-\tau_{ij})\\&\quad-\frac{\sigma\,\partial h/\partial x_j+(1+\sigma)\,\partial\zeta/\partial x_j}{h+\zeta}\frac{\partial}{\partial\sigma}(p\delta_{ij}-\tau_{ij})\\&\quad+\frac{1}{h+\zeta}\frac{\partial \tau_{i3}}{\partial\sigma}-f_i=0, \qquad i,j=1,2\\&\frac{\partial u_3}{\partial t}+u_j\frac{\partial u_3}{\partial x_j}+\omega\frac{\partial u_3}{\partial\sigma}-\frac{\partial \tau_{j3}}{\partial x_j}+\frac{1}{h+\zeta}\frac{\partial}{\partial\sigma}(p-\tau_{33})\\&\quad+\frac{\sigma\,\partial h/\partial x_j+(1+\sigma)\,\partial\zeta/\partial x_j}{h+\zeta}\frac{\partial\tau_{j3}}{\partial\sigma}=0, \qquad j=1,2\end{aligned} \tag{10}$$

where

$$\omega=\frac{1}{h+\zeta}\left[u_3-\sigma u_j\frac{\partial h}{\partial x_j}-(1+\sigma)\left(\frac{\partial\zeta}{\partial t}+u_j\frac{\partial\zeta}{\partial x_j}\right)\right], \qquad j=1,2$$

$$\tau_{ij}=\nu\left[\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}-\frac{\sigma\,\partial h/\partial x_j+(1+\sigma)\,\partial\zeta/\partial x_j}{h+\zeta}\frac{\partial u_i}{\partial\sigma}\right], \qquad j=1,2$$

$$\tau_{j3}=\nu\left[\frac{\partial u_3}{\partial x_j}+\frac{\partial u_j}{(h+\zeta)\,\partial\sigma}-\frac{\sigma\,\partial h/\partial x_j+(1+\sigma)\,\partial\zeta/\partial x_j}{h+\zeta}\frac{\partial u_3}{\partial\sigma}\right], \qquad j=1,2$$

$$\tau_{33}=\frac{2\nu}{h+\zeta}\frac{\partial u_3}{\partial\sigma}$$

$$p=\frac{1}{\rho}[p_{\mathrm{T}}+\rho g(h+\zeta)\sigma] \tag{11}$$

p is the dynamic pressure and p_{T} is the total pressure, i.e. the sum of the static and the dynamic pressure.

The water/oil interface boundary conditions are, on $\sigma=0$,

$$u_3=\frac{\partial\zeta}{\partial t}+u_j\frac{\partial\zeta}{\partial x_j}, \qquad j=1,2$$

$$u_j=(u_j)_{\mathrm{oil}}$$

and

$$(p-p_{\mathrm{o}})+(\tau_{i,j}^{(\mathrm{o})}-\tau_{ij})n_i n_j=T_{\mathrm{ow}}\nabla\cdot\boldsymbol{n}$$

$$(\tau_{ij}^{(\mathrm{o})}-\tau_{ij})n_i t_j^k=0, \qquad i,j=1,2,3, \qquad k=1,2$$

where n_j and t_j^k are the unit normal and two perpendicular tangents respectively to the interface. It is assumed that the surface tension T_{ow} is constant in space and time. $\nabla\cdot\boldsymbol{n}$ denotes the curvature of the water/oil surface, p and τ_{ij} are pressure and shear stresses respectively for the water, while the subscripts and superscripts o refer to oil. The boundary condition on the water bottom is

$$u_3+u_j\frac{\partial h}{\partial x_j}=0 \qquad \text{on } \sigma=-1$$

The discretization of the equations is similar to that of the oil slick equations. The momentum equations (10) are numerically approximated by the projection method that can be described as

$$U^{n+1}-U^n+\Delta t\,G(p+g\zeta)^{n+1}=\Delta t\,F(U^n,U^*,\zeta^n) \tag{12}$$

where G is the discretized gradient operator, U^* is the intermediate velocity and F is a function of its augments. The above equation (12) can be split into an equation in which the intermediate velocity is partially implicit, given by

$$U^*-U^n=\Delta t\,F(U^n,U^*,\zeta^n) \tag{13}$$

and an equation for the velocity at the new time step, given by

$$U^{n+1}-U^*+\Delta t\,G(p^{n+1}+g\zeta^{n+1})=0 \tag{14}$$

The velocity at the new time step is substituted into the discretized equation of surface elevation [equation (8)], yielding

$$\zeta^{n+1} - \zeta^{n} + \Delta t\, D[\Sigma(h + \zeta^{n+1})U^{n+1}] = 0 \tag{15}$$

where D represents the discretized divergence operator. Σ denotes the discretized integration operator in the depth direction in equation (8), and the velocity at the new time step is substituted into the discretized continuity equation (9):

$$D[(h + \zeta^{n+1})U^{n+1}, U^{n+1}G(h, \zeta)] = 0 \tag{16}$$

giving the coupled equations for surface and pressure, which will be solved iteratively. Once the pressure and water elevation are determined, the velocities at the new time step can be calculated and the computation turns to the next time step.

In the calculation domain, the grids are non-uniform in the vertical direction to give higher resolution near the interface and the 'exact' interfacial conditions are carefully approximated. At the outlet boundary, a sponge layer [7, 8] is set up to add a damping term

$$a = -0.5\left[\frac{r(x, y, \sigma) - r_s}{r_e - r_s}\right]^2 (1 + \sigma)$$

in the vertical momentum equation, and the Sommerfeld–Orlanski condition [9] is still used at the boundary. r_s and r_e are the horizontal coordinates of the start and end points respectively of the sponge layer. Various boundary conditions on the interface are satisfied as exactly as possible in the pressure and momentum equations at each step in the calculation.

3 CALCULATION EXAMPLES

First, sinusoidal and solitary water waves are calculated to check the accuracy of the simulation of the water waves by numerical solution of the NS equations and the effect of the damping zone on wave reflection. It is found that for water wave simulation a small time step (one-thousandth of the wave period T) is necessary to reduce the numerical viscosity and that the sponge layer is effective in reducing the wave reflection from the outlet boundary. Figure 1 shows the time step influence and Fig. 2 shows the steady state of an incoming wave and the effect of the sponge layer. In the calculation, the length of the calculation domain is taken to be 900 m, in which the sponge layer occupies 300 m. The water depth is 30 m. The wavelength shown in Figs 1 and 2 is equal to 150 m, the wave height is 5 m and the wave period is 10.63 s. The kinematic viscosity of water is 1.0×10^{-6} m^2/s, the water density is taken to be 1000 kg/m^3 and the surface tension T_{wa} at the water/air interface is 0.0726 N/m. As for oil spreading, an initial circular oil slick with the thickness of the Gaussian distribution spreading under gravity on an infinite smooth plate is simulated. In order to avoid the complexity at the leading edge, a very thin (of the order of 1 mm) fictitious layer of oil is assumed on the plate such that the slip boundary condition is used on the plate and the minimum thickness of the spreading oil slick is taken to be the thickness of the fictitious layer. The time history of the spreading at several time steps is shown in Fig. 3 and is qualitatively in agreement with that found by Thomas *et al.* [13]. The figure shows the axial symmetry and the precise structure of the oil slick in the process of spreading. It is believed that the reliability of the code to simulate the motion of the oil slick has been verified.

Oil spread on still water is simulated as a second example, shown in Fig. 4. The initial thickness distribution is also of a Gaussian form with a thickness of 0.7979 m at the central point. The calculation domain for water is 900 m × 600 m and the initial depth is 30 m. At each side of the domain, there are sponge layers of width 225 m in the x direction and 150 m in the y direction. The calculation domain of oil is 450 m × 450 m lying in the central area of the water domain. A uniform square cell is used in the horizontal plane for both the oil and the water. The cell size of the water is 6.25 m by 6.25 m, while that of the oil is 1.25 m by 1.25 m. The time step for water motion calculations is 0531 s and that for oil is 0.005 31 s. The density and kinematic viscosity of the oil are 900 kg/m^3 and 1.5×10^{-3} m^2/s respectively. The surface tensions used are $T_{ow} = 0.028$ N/m, $T_{oa} = 0.030$ N/m and $T_{wa} = 0.0726$ N/m. From Fig. 4, the wavy configuration underneath the front edge of the oil slick is revealed to be similar to observations [14], and the reappearance of the wavy structure may be mainly due to the simultaneous solution of the oil and water equations. The spreading speed compared with Fay's [1] results in Fig. 5 shows that the calculation is in good agreement with his equation for the inertia range except during the initial stages. In the initial stage, the shape of the oil slick and its initial condition may have an important influence on its spreading and this can be used to explain the discrepancy during the initial stage. Figure 6 shows the velocity field and thickness contour at time $t = 95.67$ s. The outer edge of the oil slick and the velocity distribution are axisymmetric as expected while the thickness contour shows some symmetry with respect to the x and y directions. In fact, when the time t is small, all the velocity and thickness contours appear to be perfectly axisymmetric. However, at about $t = 50$ s, they start to be x and y symmetric. This may be due to the grid and boundary effect because the spreading is axisymmetric while the grids are rectangular in the x and y directions and, as the time progresses, the oil slick becomes larger

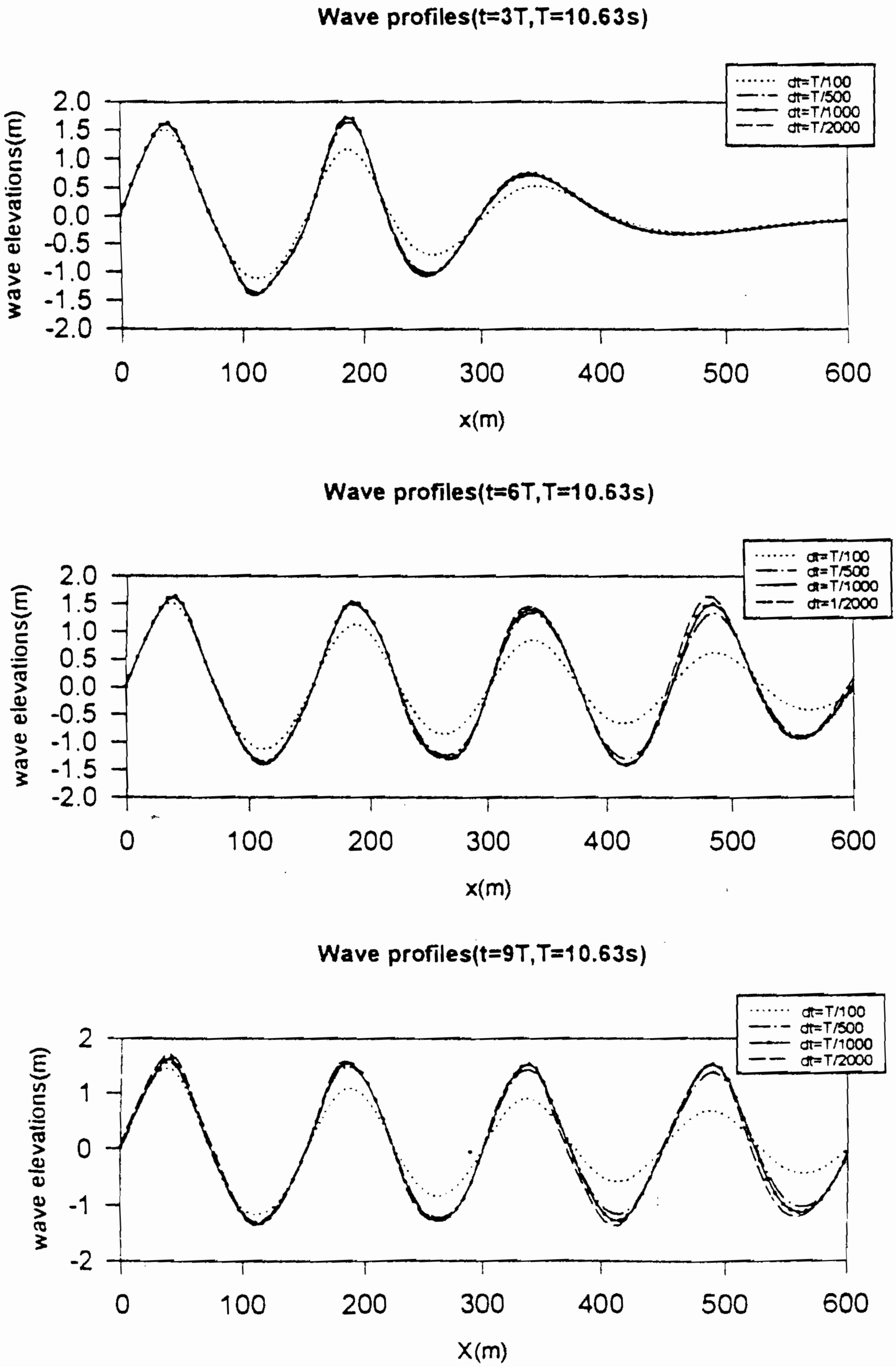

Fig. 1 Time step influence

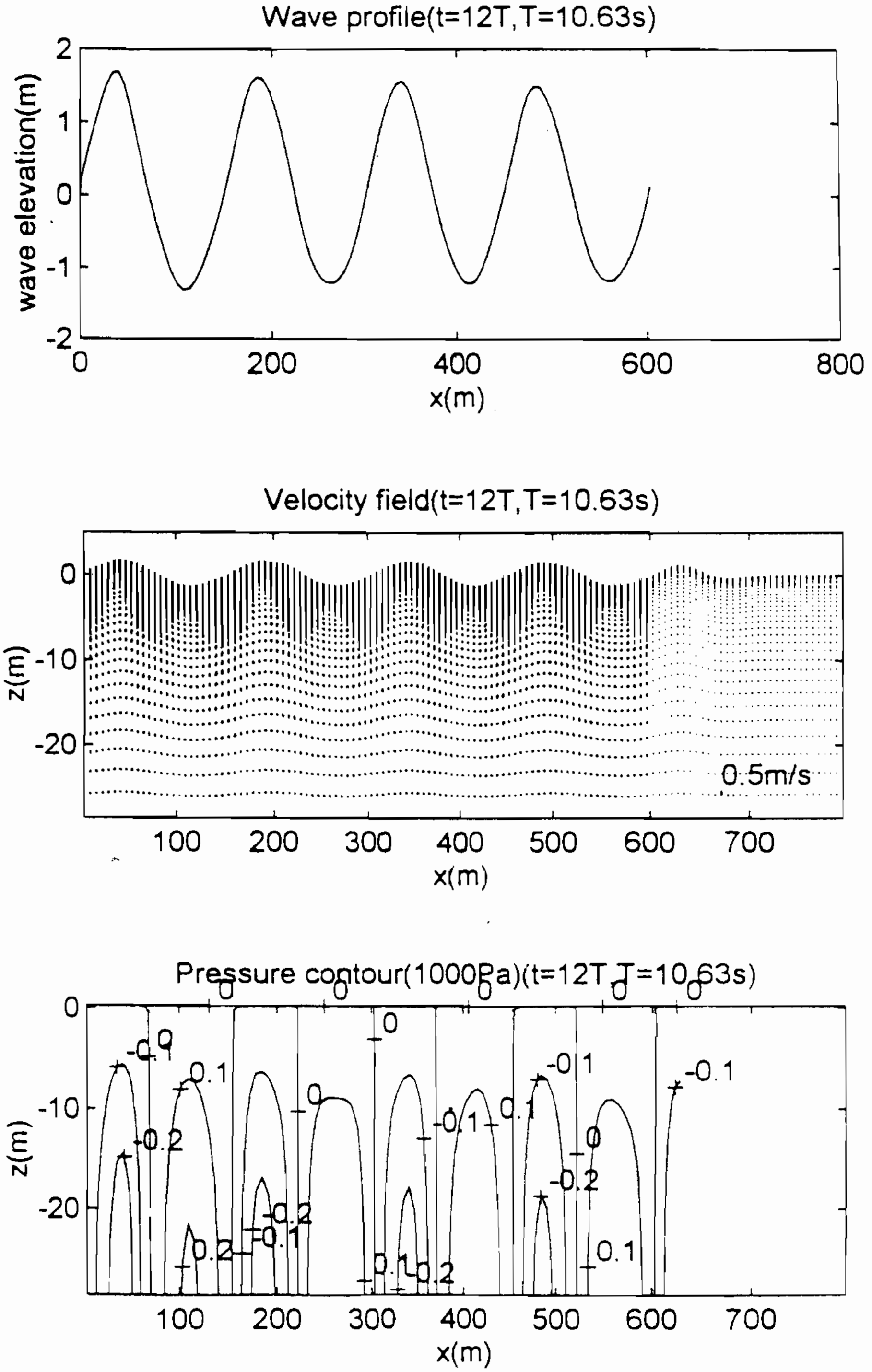

Fig. 2 Profile, together with the velocity field and pressure contours of an incoming wave train

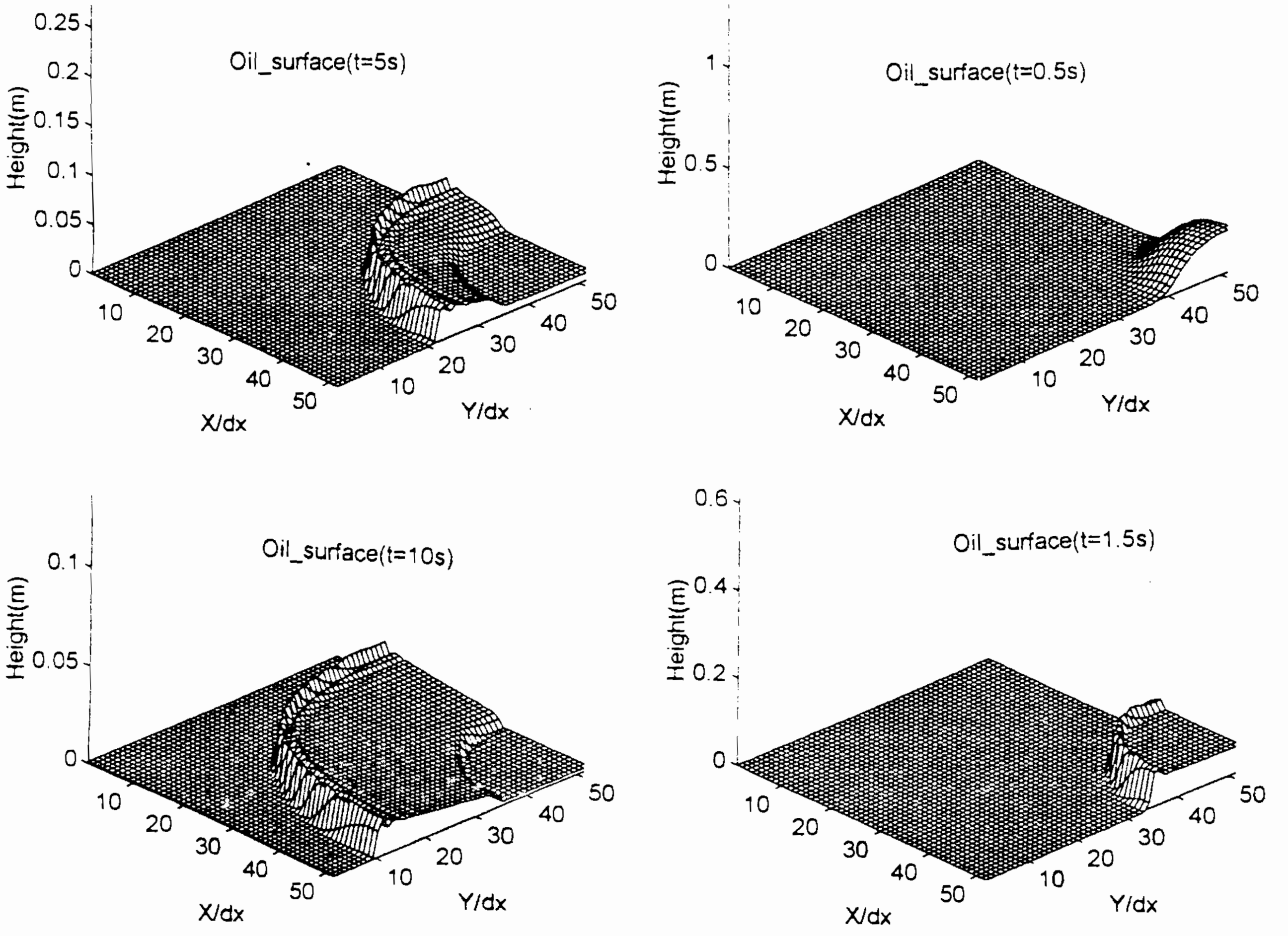

Fig. 3 Oil spread on a smooth plate

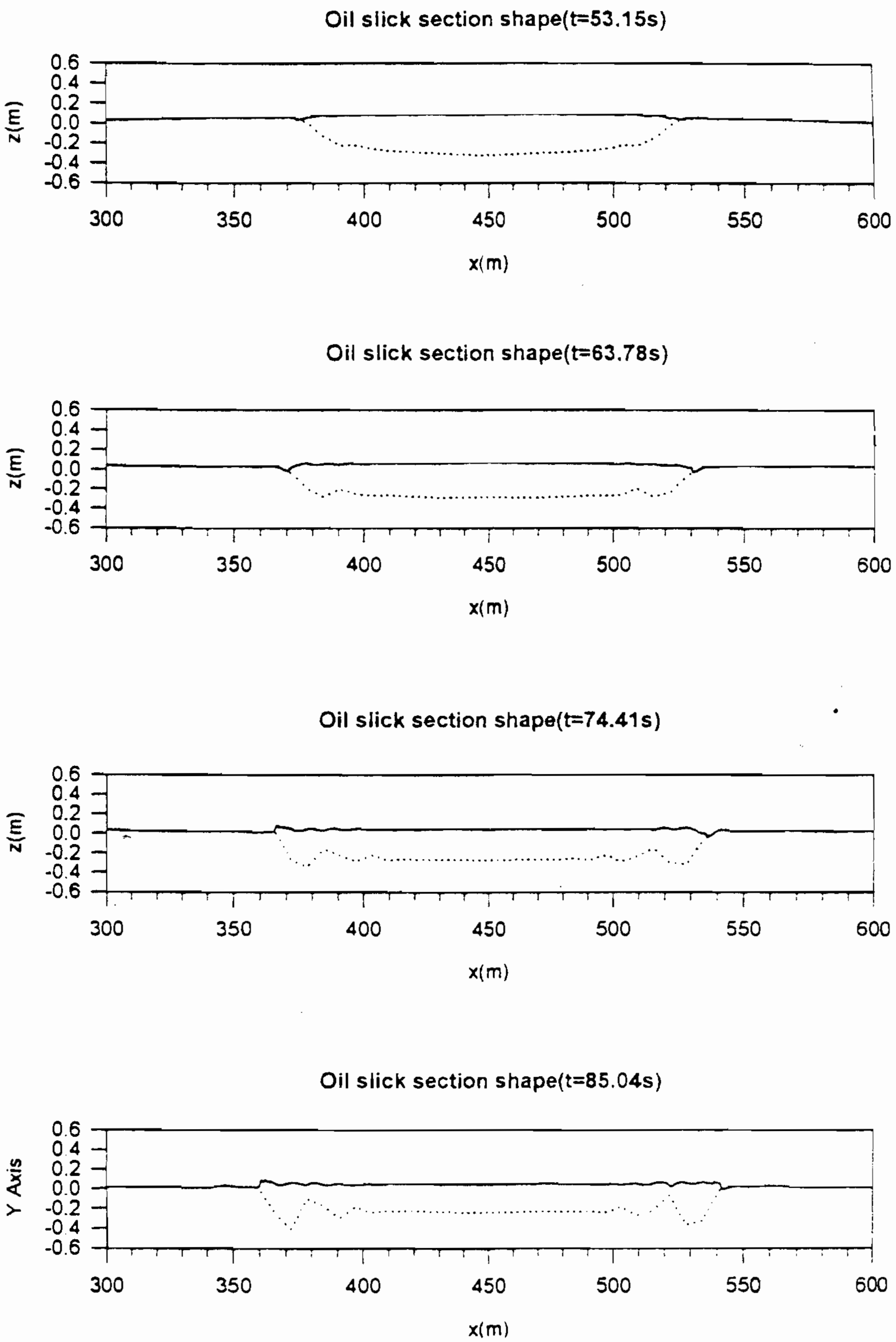

Fig. 4 Oil spread on still water

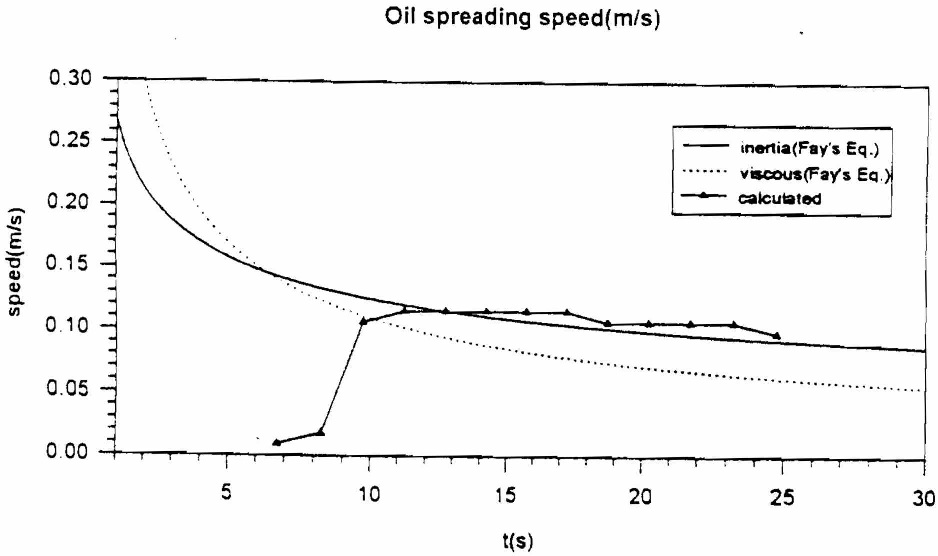

Fig. 5 Spreading speed of an oil slick on still water

and larger so that the outer boundaries will affect its spreading.

After having checked carefully the code for water motions, the spread of the oil slick on waves is calculated. Initially, the thickness of oil slick distributed in a Gaussian form rested on the still water surface and then a wave starts to propagate into the domain. The evolution of the slick profile is shown in Fig. 7 and the velocity field and thickness contours are shown in Fig. 8.

4 CONCLUSIONS

1. In order to reveal the precise structure of the oil spreading on still and wavy water surfaces, the simultaneous solution of the oil and water motion equations is necessary; in particular the interfacial conditions should be carefully approximated to simulate the interaction between oil and water. The oil slick equations may be depth averaged while the water motion equations should be three dimensional.
2. Water wave motions are basically dominated by inertia and gravity. When simulated by the use of NS equations, the reduction in the numerical viscosity is very important and the use of a small time step is an effective way to do this. For wave motion simulations, wave reflection from the outlet boundary usually prevents a further reduction in the time steps. A sponge layer coupled with the Sommerfeld–Orlanski condition is shown to be successful in eliminating wave reflection. The model and the code developed in this paper have given a quantitatively correct picture of the oil spreading on still and wavy water surfaces. However, many important factors have not been considered, even in the hydrodynamic sense, such as turbulent diffusion. Also, some detailed treatment of the methods may need improvement.

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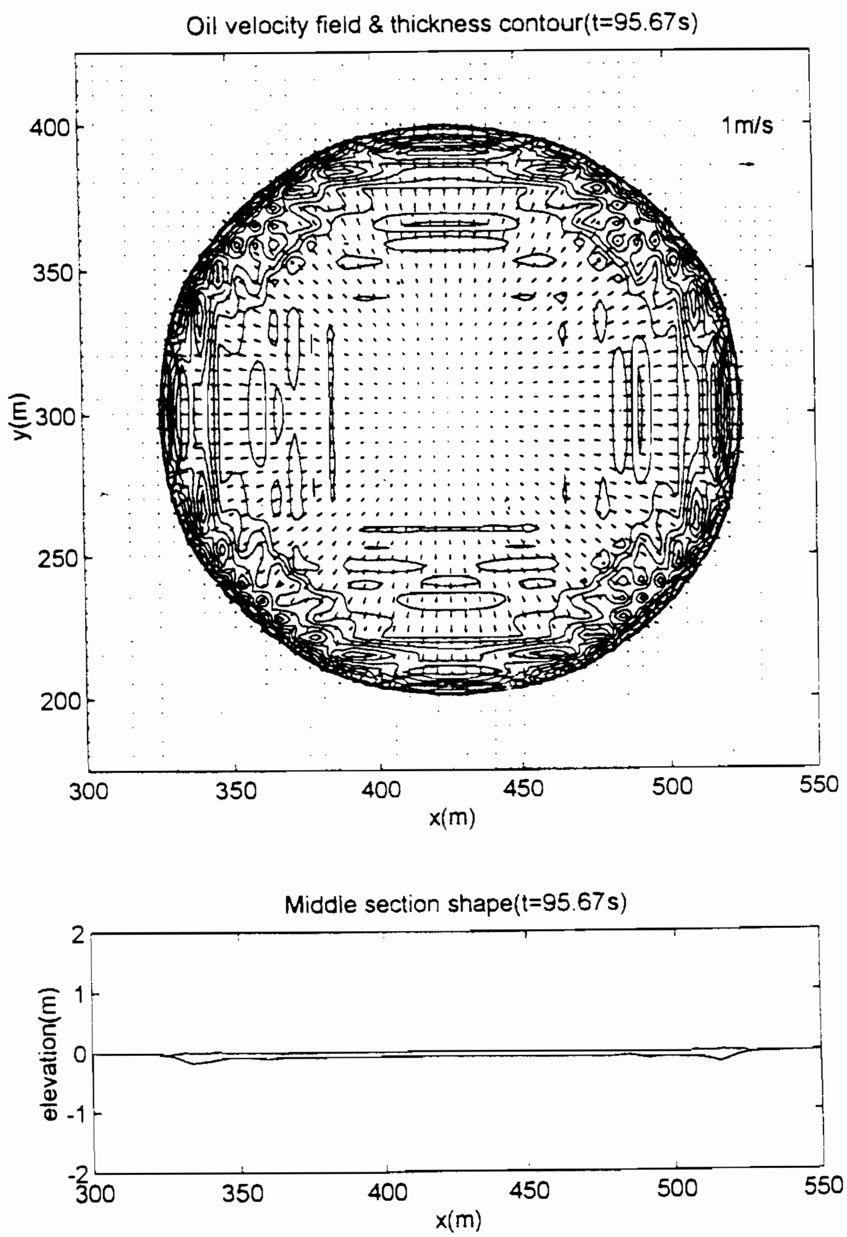

Fig. 6 Oil velocity and thickness contours on still water

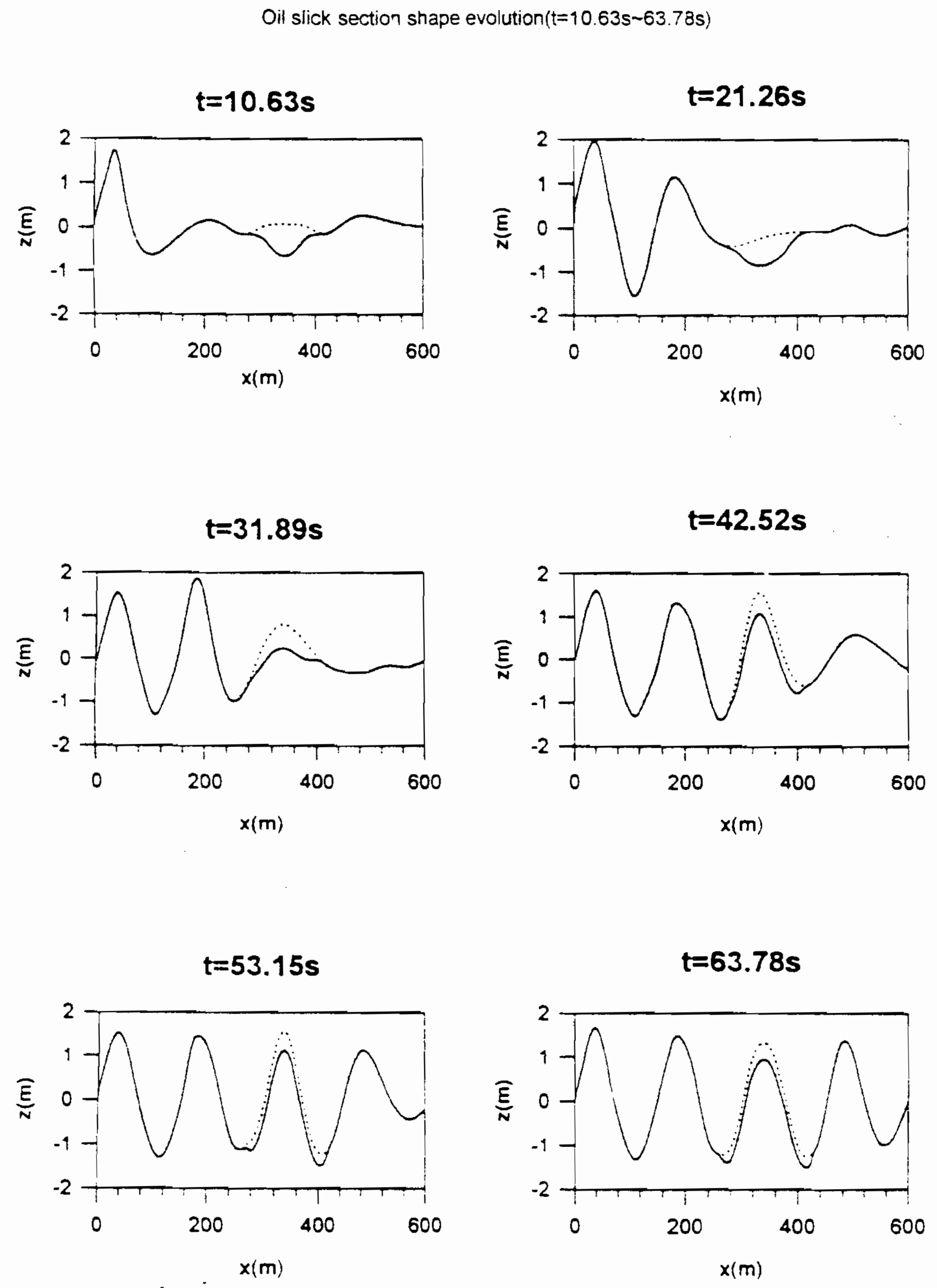

Fig. 7 Evolution of the slick's sectional shape on a wave

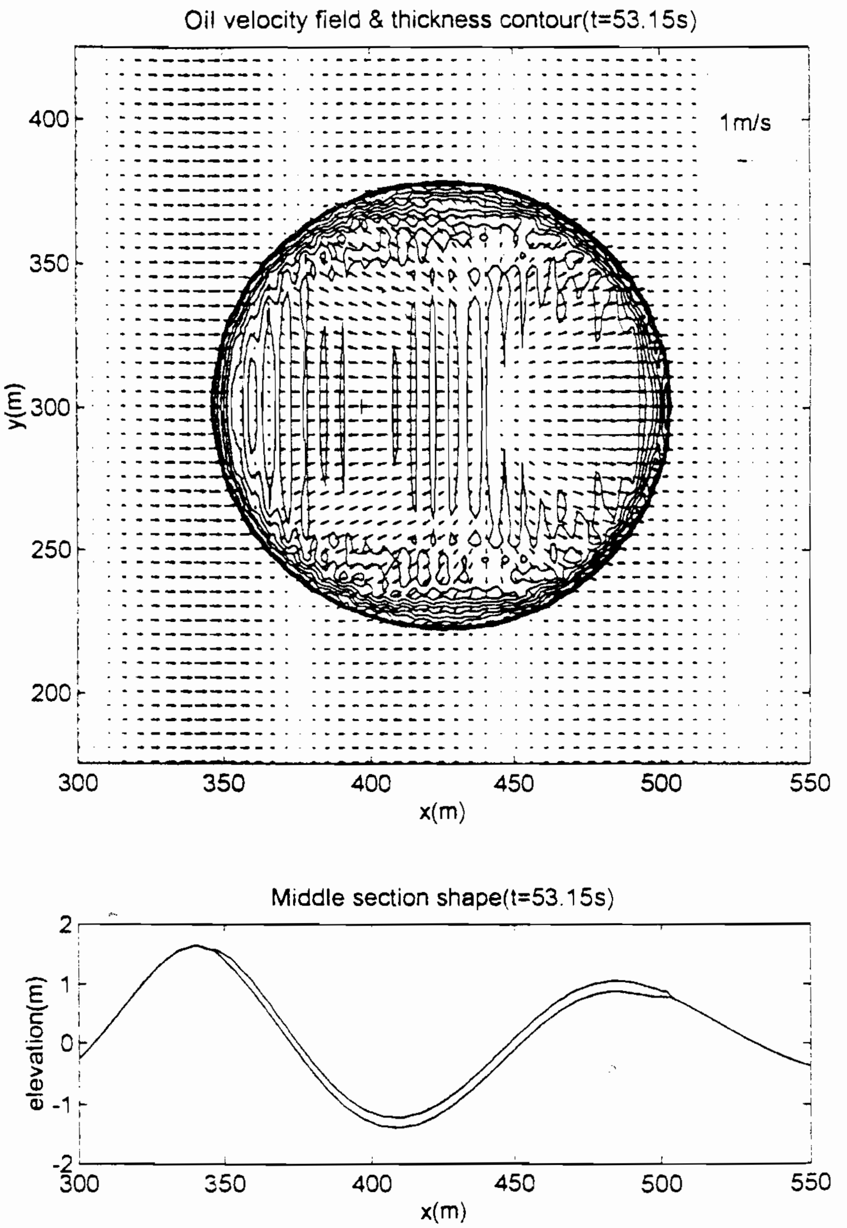

Fig. 8 Velocity field and thickness contours of an oil slick on a wave

