



Interface crack problems with strain gradient effects

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Abstract. In this paper, the strain gradient theory proposed by Chen and Wang (2001a, 2002b) is used to analyze an interface crack tip field at micron scales. Numerical results show that at a distance much larger than the dislocation spacing the classical continuum plasticity is applicable; but the stress level with the strain gradient effect is significantly higher than that in classical plasticity immediately ahead of the crack tip. The singularity of stresses in the strain gradient theory is higher than that in HRR field and it slightly exceeds or equals to the square root singularity and has no relation with the material hardening exponents. Several kinds of interface crack fields are calculated and compared. The interface crack tip field between an elastic-plastic material and a rigid substrate is different from that between two elastic-plastic solids. This study provides explanations for the crack growth in materials by decohesion at the atomic scale.

Key words: Finite element method, interface crack, strain gradient effects.

1. Introduction

Recent experiments have shown that materials display strong size effects when the characteristic length scale is on the order of microns (Fleck et al., 1994; Stolken and Evans, 1998; Ma and Clarke, 1996; McElhaney et al., 1998; Nix, 1989; Poole et al., 1996; Lloyd, 1994). The conventional plasticity theory, however, can not predict this size dependence because its constitutive model possesses no internal length scale.

Elssner et al. (1994) measured both the macroscopic fracture toughness and atomic work of separation of an interface between a single crystal of niobium and a sapphire single crystal. The macroscopic work of fracture was found to be two to three orders of magnitude higher than the atomic work of separation. This large difference between the macroscopic work of fracture and its counterpart at the atomic level was attributed to plastic dissipation in niobium, i.e., there must be significant plastic deformation associated with dislocation activities in niobium. However, Elssner et al. (1994) observed that the interface between two materials remained atomistically sharp. Meanwhile the stress level needed to produce atomic decohesion of a lattice or a strong interface is typically on the order of 0.03 times the Young's modulus, or 10 times the tensile yield stress. But the maximum stress level that can be achieved near a crack tip is not larger than 4 or 5 times the tensile yield stress of metals, according to models based on conventional plasticity theories (Hutchinson, 1997). This clearly falls short of triggering the atomic decohesion observed in Elssner et al.'s (1994) experiments. Attempts to link macroscopic cracking to atomistic fracture are frustrated by the inability of conventional plasticity theories to model stress-strain behavior adequately at the small scales involved in crack tip deformation.

In order to explain the size effect and the stress field at atomistically sharp crack tip, it is necessary to develop a continuum theory for the micron level application.

There are significant efforts to develop continuum plasticity models in order to explain the stress field at the atomistically sharp crack tip, such as SSV model (Suo et al., 1993) and EPZ model (Needleman, 1987; Tvergaard and Hutchinson, 1992, 1993) and a combined model of SSV and EPZ models (Wei and Hutchinson, 1999).

An alternative approach that has the potential to bridge the gap between the atomistic fracture and macroscopic cracking comes from strain gradient plasticity theories. Based on the notion of geometrically necessary dislocations in dislocation mechanics, several kinds of strain gradient theories have been developed (Fleck and Hutchinson, 1993, 1997; Fleck et al., 1994; Nix and Gao, 1998; Gao et al., 1999, Huang et al., 2000a, b). All these strain gradient plasticity theories introduce the higher order stress which is required for this class of strain gradient theories to satisfy the Clausius-Duhem thermodynamic restrictions on the constitutive model for second deformation gradients (Acharya and Shawki, 1995). In comparison, no higher-order stresses have been defined in the alternative gradient theories (Aifantis, 1984; Zbib and Aifantis, 1989; Muhlhaus and Aifantis, 1991; Acharya and Bassani, 1995; Chen and Wang, 2000a, 2001a, 2002a, b; Gao and Huang, 2001; Guo et al., 2001).

As direct application, strain gradient plasticity theory has been used to investigate fracture of materials. Huang et al. (1995, 1997, 1999), Xia and Hutchinson (1996), Wei and Hutchinson (1997), Chen et al. (1998, 1999), Chen and Wang (2000b, 2001b) have investigated the asymptotic field near a crack tip as well as the full-field solution. It is established that, for the couple stress theory of strain gradient plasticity proposed by Fleck and Hutchinson (1993) and Fleck et al. (1994), the stress level near a crack tip is not significantly increased as compared to that in classical plasticity. For the strain gradient theory proposed by Chen and Wang (2002a), the stress level near the crack tip is the same as that in HRR field. This is because the effect of stretch gradients, which is important near a crack tip, has not been accounted for. In order to incorporate this effect, Chen et al. (1999) have used Fleck and Hutchinson's (1997) theory to analyze the crack tip field. Indeed, stretch gradients can elevate the stress level near a crack tip, as also observed in steady-state crack propagation (Wei and Hutchinson, 1997). However, Chen et al. (1999) have clearly shown that the asymptotic crack tip field based on Fleck and Hutchinson (1997) theory gives a surprising compressive stress traction ahead of a mode I crack tip. This is physically unacceptable since a tensile crack tip should not have a compressive stress traction ahead of the crack tip. Shi et al. (2000a) also obtained the analytical full field solution that clearly showed the transition from a remote tensile stress field to a compressive crack tip in Fleck and Hutchinson (1997) strain gradient theory. Shi et al. (2000b) investigated the structure of asymptotic crack tip fields associated with the theory of mechanism-based strain gradient (MSG) plasticity (Gao et al., 1999; Huang et al., 2000a, b) and concluded the crack tip field in MSG plasticity does not have a separable form of solution. Using finite element method, Jiang et al. (2001) have successfully analyzed the crack tip field with MSG theory and have established that the stress level in MSG plasticity is significantly higher than its counterpart in classical plasticity. Using the new strain gradient theory (Chen and Wang, 2001a, 2002b), Chen and Wang (2001a, c, d) have successfully investigated the thin-wire torsion, ultra-thin beam bending, micro-indentation problems and size effects in particle reinforced metal-matrix composites, the theoretical predictions agree well with the experimental results. The crack tip field has also been successfully investigated by Chen and Wang (2002b) and the results are similar to that in Jiang et al. (2001).

The purpose of this paper is to study interface crack problem using the strain gradient theory proposed by Chen and Wang (2001a, 2002b) via the finite element method. We will focus on the increase of stress level around the interface crack tip due to strain gradient effects in order to explain the observed cleavage fracture in ductile materials (Elssner et al., 1994). In Section 2, the strain gradient theory used in the present paper is briefly summarized. Without accounting for the strain gradient effects, numerical results for the crack tip field around an interface crack between an elastic-plastic solid and a rigid substrate agree well with the results in Shih and Asaro (1988) in classical plasticity as shown in Section 3.1. The special limit of an interface crack in the bimaterial between an elastic-plastic solid and a rigid substrate are shown in Section 3.2. In Section 3.3, numerical results accounting for the strain gradient effects for an interface crack in a bimaterial of two general elastic-plastic solids are shown.

2. Summary of the strain gradient theory

The strain gradient theory proposed by Chen and Wang (2001a, 2002b) is briefly reviewed here. It preserves the basic structure of the general couple stress theory and involves no higher-order stress or higher-order strain rates. Its key features are that the rotation gradient influences the material behavior through the interaction between Cauchy stresses and couple stresses, while the stretch gradient explicitly enters the constitutive relations through the instantaneous tangent modulus. The tangent hardening modulus is influenced by not only the generalized effective strain but also the effective stretch gradient.

In a Cartesian reference frame x_i , the strain tensor ε_{ij} and the stretch gradient tensor η_{ijk} (Smyshlyaev and Fleck, 1996) are related to the displacement u_i by

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \quad \eta_{ijk} = u_{k,ij}. \quad (1)$$

The rotation gradient is related with the independent micro-rotation vectors ω_i

$$\chi_{ij} = \omega_{i,j}. \quad (2)$$

The effective strain, effective rotation gradient and effective stretch gradient are defined as

$$\varepsilon_e = \sqrt{\frac{2}{3}\varepsilon'_{ij}\varepsilon'_{ij}}, \quad \chi_e = \sqrt{\frac{2}{3}\chi'_{ij}\chi'_{ij}}, \quad \eta_1 = \sqrt{\eta_{ijk}^{(1)}\eta_{ijk}^{(1)}}, \quad (3)$$

where ε'_{ij} , χ'_{ij} are the deviatoric part of the counterparts and the definition of $\eta_{ijk}^{(1)}$ can be found in Smyshlyaev and Fleck, (1996).

The constitutive relation are as follows

$$\sigma_{ij} = \frac{2\Sigma_e}{3E_e}\varepsilon'_{ij} + K\varepsilon_m\delta_{ij}, \quad m_{ij} = \frac{2\Sigma_e}{3E_e}l_{cs}^2\chi'_{ij} + K_1l_{cs}^2\chi_m\delta_{ij} \quad (4)$$

$$\begin{cases} E_e^2 = \varepsilon_e^2 + l_{cs}^2\chi_e^2, & \Sigma_e = (\sigma_e^2 + l_{cs}^{-2}m_e^2)^{1/2}, \\ \sigma_e^2 = \frac{3}{2}S_{ij}S_{ij}, & m_e^2 = \frac{3}{2}m'_{ij}m'_{ij}, \end{cases} \quad (5)$$

E_e is called the generalized effective strain and Σ_e is the work conjugate of E_e ; l_{cs} is an intrinsic material length, which reflects the effects of rotation gradient on the material behaviors; K is the volumetric modulus and K_1 is the bend-torsion volumetric modulus.

In order to consider the influence of stretch gradient, the new hardening law (Chen and Wang, 2000a) is introduced

$$\begin{cases} \dot{\Sigma}_e = A'(E_e) \left(1 + \frac{l_1 \eta_1}{E_e}\right)^{1/2} \dot{E}_e = B(E_e, l_1 \eta_1) \dot{E}_e, & \Sigma_e \geq \sigma_Y, \\ \dot{\Sigma}_e = 3\mu \dot{E}_e, & \Sigma_e < \sigma_Y, \end{cases} \quad (6)$$

where $B(E_e, l_1 \eta_1)$ is the hardening function; l_1 is the second intrinsic material length associated with the stretch gradient, σ_Y is the yield stress and μ the shear modulus.

The equilibrium relations in V are

$$\sigma_{ij,j} = 0, \quad m_{ij,j} = 0 \quad (7)$$

The traction boundary conditions for force and moment are

$$\sigma_{ij} n_j = T_i^0 \quad \text{on } S_T, \quad m_{ij} n_j = q_i^0 \quad \text{on } S_q. \quad (8)$$

The additional boundary conditions are

$$u_i = u_i^0 \quad \text{on } S_u, \quad \omega_i = \omega_i^0 \quad \text{on } S_\omega. \quad (9)$$

3. Numerical analysis of interface crack tip field

In this section, the finite element method for the strain gradient theory (Chen and Wang, 2001a, 2002b) is used to analyze the interface crack tip field. The finite element formula for this kind of theory can be found in Chen and Wang (2002b) and here is omitted. Several kinds of interface cracks are considered such as interface crack in the bimaterial between an elastic-plastic solid and a rigid substrate, interface crack in the bimaterial of two different elastic-plastic solids.

3.1. COMPARISON WITH SHIH AND ASARO (1988)

In order to verify the finite element program, in this section the strain gradient is not considered, i.e., $l_1 = l_{cs} = 0$ and the results will be compared with those in Shih and Asaro (1988). The calculation model is similar to that used in Shih and Asaro (1988). A large plate is loaded by remote uniform stresses, in which the upper material is elastic-plastic solid and the lower material is rigid substrate and a center crack exists on the interface. Only the right half of the deformable medium need to be considered in the finite element analysis since the problem possesses reflective symmetry with respect to the vertical plane bisecting the crack. The half-crack length is a , and the half-width and height of the deformable slab is $100a$. The finite-element model is constructed using 9-node quadrilateral Lagrangian elements and 3×3 Gauss points are used. The remote loading is expressed by σ_{22}^∞ and σ_{11}^∞ . We take the following loading as that used in Shih and Asaro (1988),

$$\sigma_{11}^\infty = \frac{\nu_1}{1 - \nu_1} \sigma_{22}^\infty, \quad (10)$$

where ν_1 is the Poisson ratio and we take $\nu_1 = 0.3$ in the calculation.

The calculation results agree very well with those in Shih and Asaro (1988). When the external loading is very small, $\sigma_{22}^\infty / \sigma_Y = 2.0 \times 10^{-5}$, the plastic zone is confined to a distance of about $10^{-11}a$, the small strain asymptotic field for an interface crack is characterized by oscillatory stresses and the oscillate can be significant fractions of the crack length, which

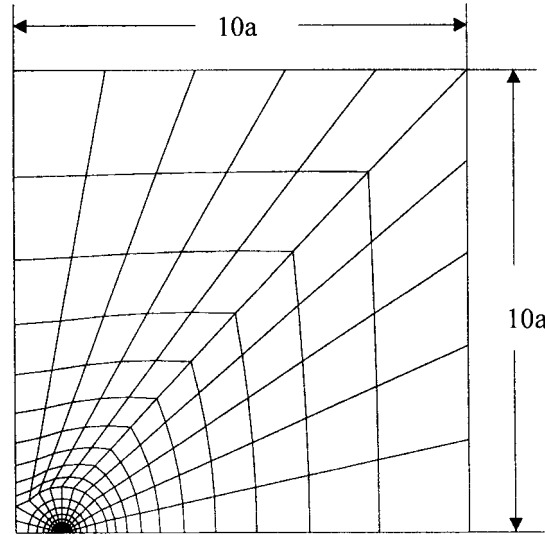


Figure 1. Finite element mesh for an interface crack between a deformable material and a rigid substrate and only half of the deformable material is needed.

has been obtained by Shih and Asaro (1988). The stress is negative within the plastic zone, the slope of the curve becomes positive for $r/a < 10^{-14}$. At a higher load level, $\sigma_{22}^{\infty}/\sigma_Y = 6.0 \times 10^{-3}$, the hoop stress increases monotonically over the entire distance under discussion, i.e., there is no trace of an oscillatory field.

3.2. INTERFACE CRACK IN A BIMATERIAL BETWEEN AN ELASTIC-PLASTIC SOLID AND A RIGID SUBSTRATE

The interface crack with a rigid substrate is often found in engineering problems and here this special kind of case is calculated and the strain gradient is considered. A finite square plate subject to uniform tensile is considered and the calculation model is shown in Figure 1, in which only the right half of the deformable medium is considered. The half of the crack length is a and the width of the calculated model is $10a$. On the boundary of $y = 0$, all the nodes are fixed in x and y direction. On the boundary of $y = 10a$, only normal stress σ_{22}^{∞} is imposed. On the left boundary, all the nodes are fixed in x direction for symmetry. Traction for both the force and moment is free on the right boundary. During the calculation the parameters of the upper material are $\sigma_Y/E = 0.2\%$, $\nu = 0.3$, $n = 0.2$.

In all the calculation, we take $l_{cs} = 1 \mu\text{m}$ and stresses σ_{ij} are normalized by the yield stress σ_Y in uniaxial tension, while the distance r to the crack tip is normalized by the internal material length l_{cs} in the strain gradient theory proposed by Chen and Wang (2001a, 2002b). The normalized remotely applied stress is $\sigma_{22}^{\infty}/\sigma_Y$.

Figure 2 shows the normalized effective stresses, σ_e/σ_Y , versus the non-dimensional distance to the crack tip, r/l_{cs} , ahead of the crack tip (at polar angle $\theta = 3.15^\circ$) predicted by the Chen and Wang's strain gradient theory. The remotely applied stress is $\sigma_{22}^{\infty}/\sigma_Y = \frac{1}{6}$. There are four kinds of cases in Figure 2, in which three results are corresponding to different relations of l_{cs} and l_1 , the other one is for the classical theory. Since the material length l_{cs} is taken prior, the value of l_1 for each case is known also. The horizontal line $\sigma_e/\sigma_Y = 1$ separates the elastic and plastic zones for each curve. Outside the plastic zone and $r/l_{cs} < 5$, both the present

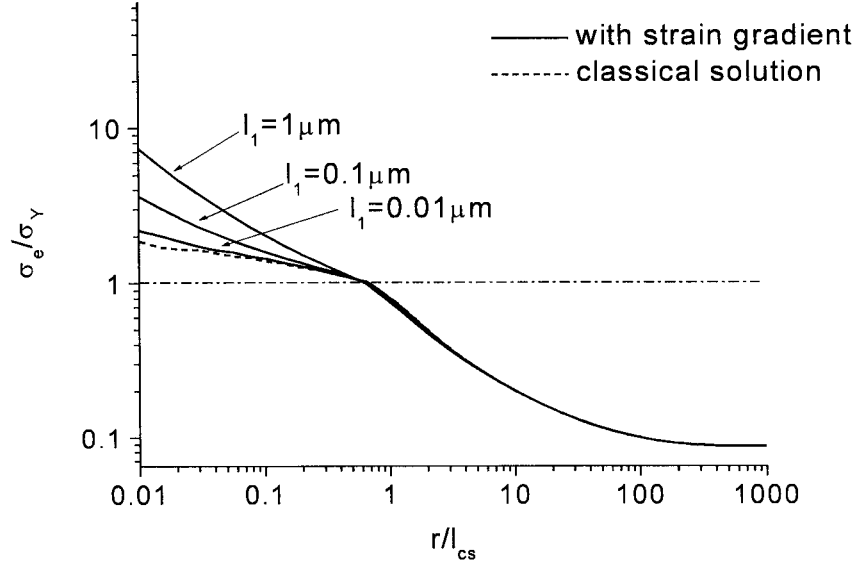


Figure 2. Plots of the normalized effective stresses, σ_e/σ_Y , near the polar angle of $\theta = 3.15^\circ$ versus the normalized distance r/l_{cs} with different material length l_1 .

strain gradient theory and the classical plasticity theory give the same straight line with slope $-\frac{1}{2}$, which corresponds to the elastic K field. As $r/l_{cs} > 5$, there is an elastic field, which is influenced by the outer boundary. From Figure 2, one can find the interface crack tip field is significantly influenced by the material length l_1 while $l_1 \geq 0.01 \mu m$. When $l_1 > 0.1 \mu m$, the HRR type field seems to vanish, there is no HRR type field, which is different from the results for crack tip field in homogeneous material (Jiang et al., 2001; Chen and Wang, 2002b). We call the plasticity zone in this kind of interface crack tip as the generalized plasticity zone. This interesting phenomena may be due to the intrinsic properties of the interface crack between an elastic-plastic solid and a rigid substrate. Since the lower material is rigid substrate, the material points along interface in both directions are fixed, which results in the greater strain gradient along y direction near the interface. Hence the strain gradient could be larger than that in the homogeneous material. From Figure 2, one can see that the length scale l_1 has no effect on the size of the plastic zone size while the applied stress is the same. Since there is hardly classical plasticity zone existing for the strain gradient theory, the generalized plastic domain like the classical plasticity zone will increase accompany with the increasing external load. While the value of material length l_1 decrease, the result tends to the classical solution, which reflects that the length scale l_1 plays an important role in crack tip field.

Figure 3 shows the normalized effective stress σ_e/σ_Y at polar angle $\theta = 3.15^\circ$, versus the non-dimensional distance to the crack tip r/l_{cs} for three levels of remotely applied stress, $\sigma_{22}/\sigma_Y = \frac{1}{12}$, $\frac{1}{6}$ and $\frac{1}{3}$. The relation $l_1 = l_{cs}$ is taken and the other parameters are the same as that in Figure 2. The deformation outside the plastic zone is essentially the elastic K field, as evidenced by the straight line with the slope of $-\frac{1}{2}$ for large r , then it tends to be the zone influenced by the out limit boundary. At a small distance r to the crack tip, all curves approach to another set of straight lines, with the absolute value of the slope larger or equal to $\frac{1}{2}$. This confirms that the singularity of the crack tip stress field in Chen and Wang's strain gradient theory is stronger than the classical field. It is also observed from Figure 3 that the plastic zone size increases rapidly with the applied loading and there is no classical HRR type field, it is

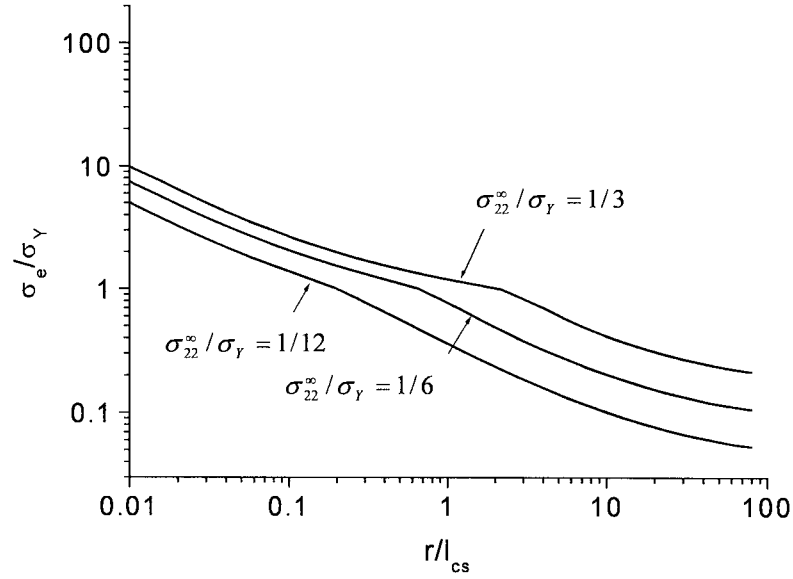


Figure 3. Plots of the normalized effective stress σ_e/σ_Y at $\theta = 3.15^\circ$ versus the distance to the crack tip r/l_{cs} for three levels of applied stress, $\sigma_{22}^\infty/\sigma_Y = \frac{1}{12}$, $\frac{1}{6}$ and $\frac{1}{3}$.

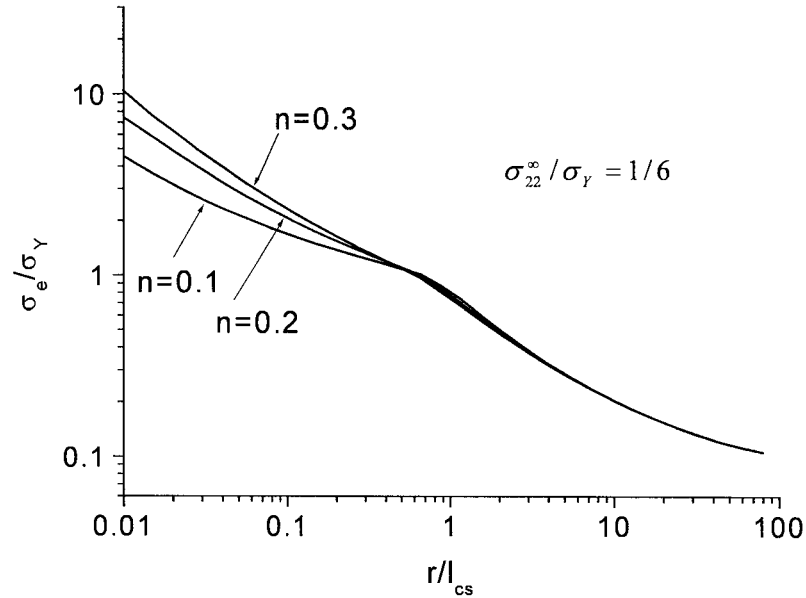


Figure 4. Plots of the effective stress distribution ahead of the crack tip with various hardening exponents $n = 0.1, 0.2, 0.333$.

different from that in Jiang et al. (2001) and Chen and Wang (2002b) for the crack tip field in homogeneous material and in which the strain gradient dominated zone increases relatively slow with the applied loading, while the whole plastic zone size increases rapidly with the applied loading.

Figure 4 shows the normalized effective stresses versus the non-dimensional distance to the crack tip r/l_{cs} with different hardening exponents $n = 0.1, 0.2$ and 0.333 , respectively. The

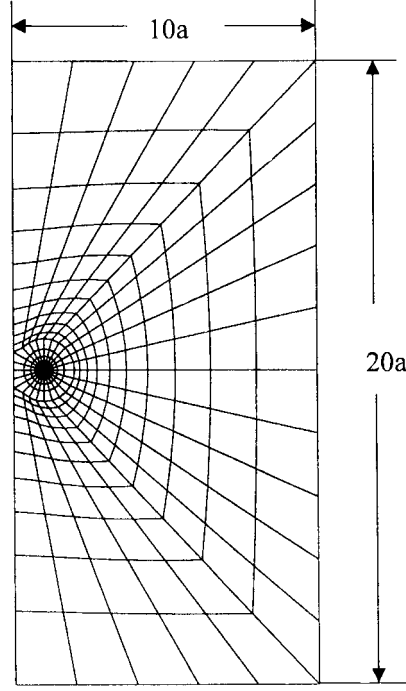


Figure 5. Finite element mesh of an interface crack in finite width plate and only half of the deformable medium is needed.

remotely applied stress is the same as that in Figure 2 and $l_1 = l_{cs}$. One can find the effect of plastic work hardening on the effective stress distribution ahead of the crack tip (polar angle $\theta = 3.15^\circ$). One interesting observation is that, at small distance r to the crack tip, all curves approach straight lines that have the same slope. This means the interface crack tip singularity in Chen and Wang's strain gradient theory is essentially independent of the plastic work hardening exponent, which is the same as that in Jiang et al. (2001) and Chen and Wang (2002b). Furthermore, the larger the hardening exponents, the higher the effective stress near the crack tip in the strain gradient dominated domain with the same external loading.

3.3. INTERFACE CRACK IN A BIMATERIAL OF TWO GENERAL ELASTIC-PLASTIC SOLIDS

In this section, the interface crack tip field in a bimaterial of two elastic-plastic solids is investigated. Also, we take a finite width plate as the calculation model, which is shown in Figure 5. Only half of the deformable medium needs to be considered in the finite element analysis due to the possesses reflective symmetry. The lower material is designated as material 1 and the upper material is designated as material 2. The mesh division is the same as that in Figure 1 except the lower deformable part is meshed.

Here, we assume that both materials have the same elastic properties but different plastic properties. The parameters are taken as follows: $\sigma_y/E = 0.2\%$, $\nu = 0.3$, $n_1 = 0.3$, $n_2 = 0.1$, $l_1 = l_{cs}$. The applied load is $\sigma_{22}^\infty = \sigma_Y/6$, the traction for moment vanishes along the whole outside boundary.

Figures 6 and 7 show the normalized effective stresses, σ_e/σ_Y , along the polar angle of $\theta = -3.15^\circ$ and $\theta = 3.15^\circ$ versus the normalized distance r/l_{cs} for the lower material 1 and upper material 2, respectively. The corresponding stress distribution in classical plasticity

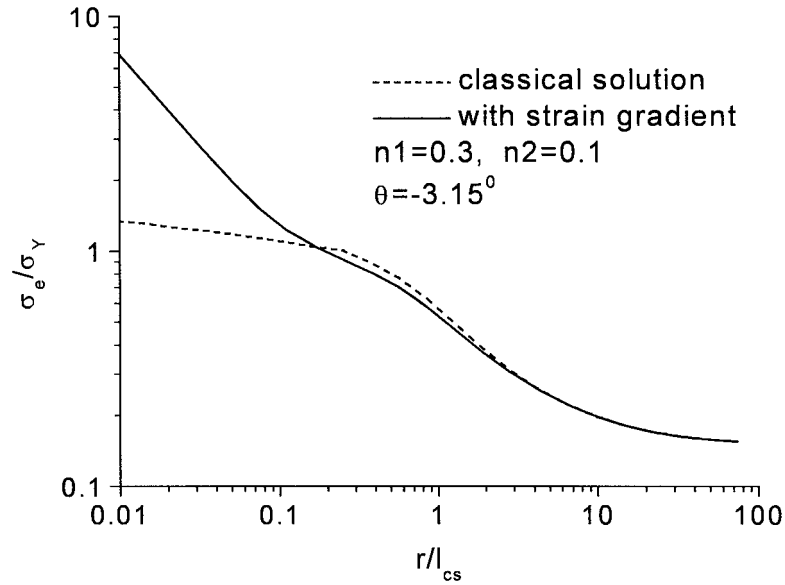


Figure 6. Plots of the normalized effective stress σ_e/σ_Y at $\theta = -3.15^\circ$ versus the distance to the crack tip r/l_{cs} in the lower deformable material.

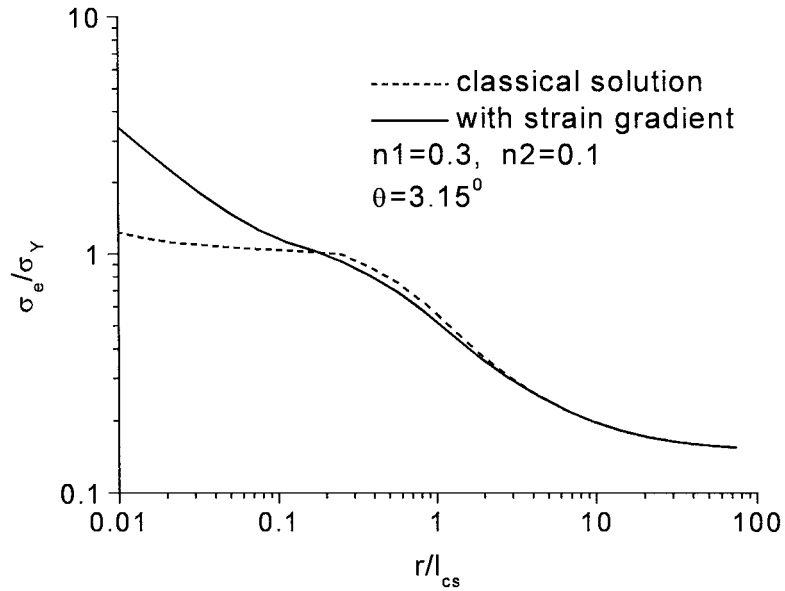


Figure 7. Plots of the normalized effective stress σ_e/σ_Y at $\theta = 3.15^\circ$ versus the distance to the crack tip r/l_{cs} in the upper deformable material.

(without strain gradient effects) is also shown in Figures 6 and 7. It is observed that, outside the plastic zone (as determined by $\sigma_e/\sigma_Y \leq 1$), both the gradient theory and classical plasticity theory give the same straight line slope $-\frac{1}{2}$, corresponding to elastic K field. But it is also different from the solution to crack tip field in homogeneous materials that there is no HRR type field within the plastic zone. Once the plasticity is produced the effects of strain gradient are very large, which is due to the stiffness difference between the upper and lower material. The generalized plastic zone size can not be predicted because it changes as

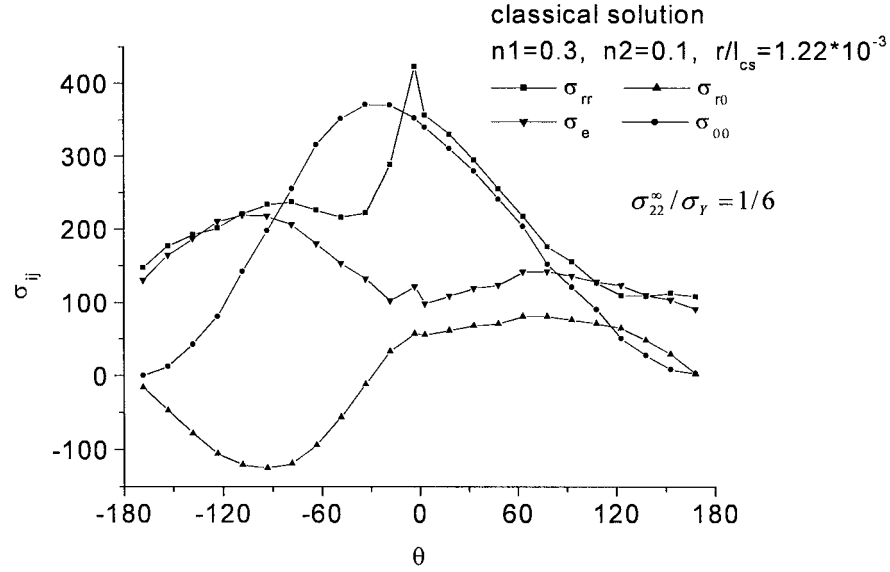


Figure 8. Plots of the stress components of classical plasticity solution at some distance from the crack tip versus the angle.

the external loading changes. Comparing Figure 6 with Figure 7, we can find that though the singularity in the strain gradient dominated domain is hardly related with the hardening exponents, the value of the effective stress is influenced by the hardening exponents and the larger the hardening exponents, the larger the effective stress in the strain gradient dominated domain for the interface crack tip field.

To develop a better understanding of the interface crack tip field with strain gradient, we give the angular distribution at some distance from the crack tip and within the generalized plastic zone. Figures 8 and 9 show the stress component distributions versus the polar angle without and with strain gradient effects, respectively, and the stress distributions in these two figures lie on the same radius. One can find that the absolute maximum value of each stress components increase significantly while the effects of strain gradient are considered. It should be pointed out that due to no higher order stresses included in the strain gradient theory (Chen and Wang, 2001a, 2002b), the stress components $\sigma_{\theta\theta}$ and $\sigma_{r\theta}$ ahead of the crack tip and on the crack free faces is the same as the corresponding stress traction. From Figures 8 and 9, we can find that both the stress components $\sigma_{\theta\theta}$ and $\sigma_{r\theta}$ on the crack free faces are zero strictly and meet the boundary conditions. Furthermore the stress components σ_{rr} and σ_e at the interface are discontinuous but the other stress components are continuous. Comparing these two figures, one can also find that the discontinuity of the stress components for the strain gradient theory are larger than that for classical plasticity theory, which denotes that the influences of the strain gradient effects on the stresses for materials with different hardening exponents are stronger than the influences of different hardening exponents on stresses.

4. Conclusions

This paper mainly presents a study of plane strain interface crack tip field at microscale based on the strain gradient theory proposed by Chen and Wang (2001a, 2002b). For remotely imposed tension loading, several kinds of full field solutions are obtained numerically for

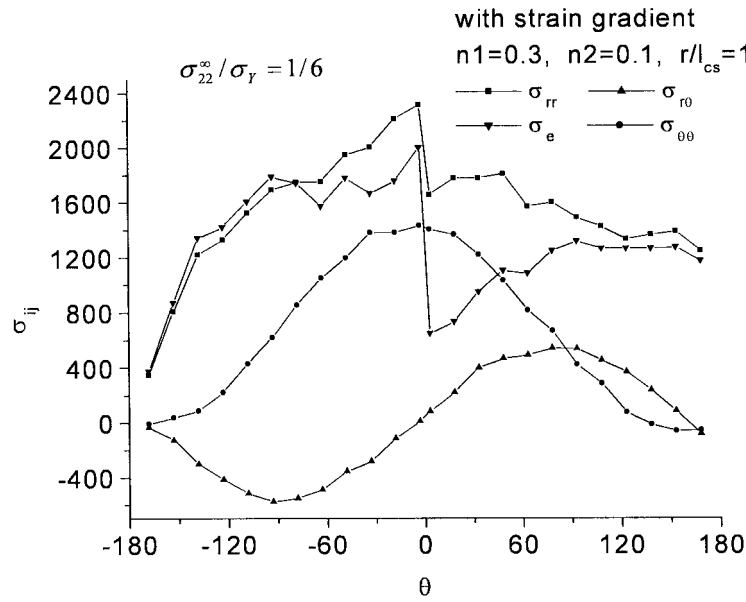


Figure 9. Plots of the stress components with strain gradient effects at some distance from the crack tip versus the angle.

different interface cracks, such as interface crack between two different deformable materials and interface crack between a deformable material and a rigid material.

It is found that the stresses near an interface crack tip are significant influenced by strain gradient effects. Under small scale yielding condition and $l_1 > 0.1 \mu\text{m}$, the remote classical K field goes directly to the near tip strain gradient dominated zone without a classical plasticity field. The material length scale l_1 has an important influence on the interface crack tip field and once the plasticity is produced, the effects of strain gradient dominate the field, which is different from that in homogeneous material. The singularity of the stress field in the strain gradient dominated domain is independent of the material hardening exponents and is equal to or larger than $r^{-1/2}$.

At a distance that is much larger than the dislocation spacing such that continuum plasticity is expected to be applicable. The near tip stresses predicted by the strain gradient theory are significantly higher than that in HRR field. The increase in the near tip stress level provides an explanation to the experimental observation of cleavage fracture in ductile materials (Elsner et al., 1994). The classical plasticity theories fail to predict the stresses needed for cleavage fracture, while the significant stress increase in Chen and Wang's strain gradient theory seems to be capable of bridging the gap between the macroscopic cracking and atomistic fracture.

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