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Asymptotic analysis for a crack on interface of damaged materials

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Abstract. An asymptotic analysis for a crack lying on the interface of a damaged plastic material and a linear elastic material is presented in this paper. The present results show that the stress distributions along the crack tip are quite similar to those with HRR singularity field and the crack faces open obviously. Material constants n, μ and m_0 are varied to examine their effects on the resulting stress distributions and displacement distributions in the damaged plastic region. It is found that the stress components σ_{rr}, $\sigma_{\theta\theta}$, $\sigma_{r\theta}$ and σ_e are slightly affected by the changes of material constants n, μ and m_0, but the damaged plastic region are greatly disturbed by these material parameters.

Key words: Damaged material, interface crack, asymptotic analysis.

1. Introduction

It is well known that interfaces play an important role in mechanical behavior of advanced materials such as structure ceramics, composites and multi-phase alloys. Due to the processing conditions and local stress or strain concentrations, interface regions are general sources of defects and a variety of voids or microcracks will initiate or grow on the interface. To evaluate the strength of such structure or materials, it is necessary to have good understanding of the mechanical behavior of a crack on a bimaterial interface.

Much work has been published on the crack tip fields at the interface of isotropic linear elastic media, such as the solutions by Williams (1959), Endogan (1965), Sih and Rice (1964), Rice and Sih (1965), England (1965). The comprehensive overviews on elastic fracture mechanics concepts and some recent developments for interfacial cracks can refer to the papers given by Rice (1988) and Hutchinson (1989).

The problem of interface crack in elastic-plastic materials has originally been investigated by Shih and Asaro (1988,1989). In their studies, a full numerical solution for a crack which lies along the interface of the elastic-plastic bimaterials has been presented. The exact asymptotic field analysis for a crack lying along the interface of dissimilar elastic plastic materials was first given by Wang (1990). A separable singular form of HRR-type field has been found in the plastic angular region in his solution. This problem has also been studied by Zwick and Park (1989), Gao and Lou (1990), Xia (1991), Sharma and Aravas (1991a,1991b), Aravas and Sharma (1991), Xia and Wang (1994).

Until now it was assumed that the materials both nearly and far away from the crack tip are nondamaged in elastic-plastic analysis of interface crack. In fact, strain concentration near the crack tip always leads to nucleation, growth and coalescence of microcracks and microvoids. Consequently, the constitutive behavior of the material near the crack tip becomes significantly different from that of the material far away from the crack tip. It is interest to introduce the damage mechanics approach into the study of interface crack in bimaterials. This approach

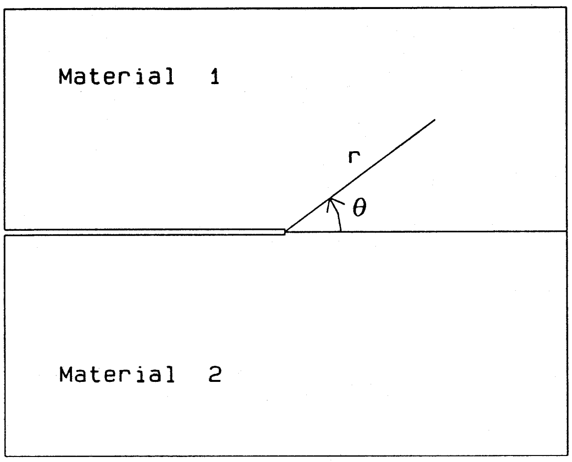

Figure 1. Geometry of a crack at bimaterial interfaces.

was first conceived by Kachanov (1958) in analyzing brittle creep rapture in which a continuous damage factor D is introduced to describe the degree of material degradation. Recently, a more realistic method which reflect the interaction between a macrocrack and distributed damages has been presented by Wang and Chow (1992).

Based on method of Wang and Chow (1992), this paper presents an exact asymptotic field analysis for a crack which lies along the interface of a damaged elastic-plastic medium and a linear elastic medium. Damage evolution equation and the constitute equation coupled with damage variable D are formulated within the framework of continuum mechanics. Both the continuity of displacements and tractions is satisfied across the interface. The asymptotic solutions of crack tip fields are solved numerically and the damage effects are examined.

2. Formulation of the problem

2.1. Constitutive relations and damage evolution equation

Consider a crack lying along the interface of medium 1 and medium 2 as shown in Figure 1. Medium 1 is a damaged elastic-plastic material with the region $0 \leqslant \theta \leqslant \pi$, and medium 2 is a linear material with the region $-\pi \leqslant \theta \leqslant 0$. Equilibrium conditions and geometric equations are taken to be linear in both mediums as,

$$\sigma_{ij,j} = 0, \qquad (i, j = r, \theta) \tag{1}$$

and

$$\varepsilon_{ij} = \tfrac{1}{2}(u_{i,j} + u_{j,i}), \qquad (i, j = r, \theta), \tag{2}$$

where the polar coordinates r and θ are centered at the right end of the crack as shown in Figure 1. σ_{ij}, ε_{ij} and u_i are respectively stress tensor, strain tensor and displacement components.

The constitutive relation for the material 2 can be expressed as

$$\varepsilon_{ij} = \frac{1+\nu}{E} S_{ij} + \frac{1-2\nu}{3E}\sigma_{kk}\delta_{ij}, \tag{3}$$

where E and ν are the Young's modulus and Poisson's ratio for the material 2.

The generalized Ramberg–Osgood stress-strain relation for virgin elastic-plastic materials is assumed as for the material 1,

$$\varepsilon_{ij} = \tfrac{3}{2}\alpha_0 \sigma_e^{n-1} S_{ij}, \tag{4}$$

where α_0 and n are material constants, S_{ij} is the deviatoric stress tensor and σ_e is the effective stress

$$S_{ij} = \sigma_{ij} - \tfrac{1}{3}\sigma_{kk}\delta_{ij}, \tag{5}$$

$$\sigma_e^2 = \tfrac{3}{2} S_{ij} S_{ij}. \tag{6}$$

Note that at the zone in the immediate neighborhood of the crack tip, the elastic strain can be negligible comparing to the plastic strains. By introducing Lemaitre's (1985) strain equivalence hypothesis, (4) becomes for the material 1,

$$\varepsilon_{ij} = \tfrac{3}{2}\alpha_0 \frac{\sigma_e^{n-1}}{(1-D)^n} S_{ij}, \tag{7}$$

where D is the damage parameter, $D < 1$.

For the sake of computational convenience, following Wang and Chow (1992), we introduce two independent material parameters β_1 and m_1 for the damaged material, and then reduce (7) to

$$\varepsilon_{ij} = \tfrac{3}{2}\alpha_0 \sigma_e^{n-1}(1 + \beta_1 D^{m_1}) S_{ij}, \qquad (i, j = r, \theta), \tag{8}$$

when D is not close to 1, (8) can yield a satisfactory approximation of (7). The damage equivalent stress σ_d also takes the form as

$$\sigma_d = \sigma_e \left[\tfrac{2}{3}(1-\mu) + 3(1+2\mu)\left(\frac{\sigma_m}{\sigma_e}\right)^2 \right]^{1/2}, \tag{9}$$

where σ_m is the hydrostatic stress and μ is a material constant reflecting the damage evolution mechanism. It is meant when $\mu = \frac{1}{2}$, the material damage is mainly caused by micro-cracks propagation under octahedral shear stress σ_e, while $\mu = 1.0$, the dominant mechanism of the damage is micro-voids dilatation under hydrostatic stress σ_m. So, $-\frac{1}{2} \ll \mu \ll 1.0$ is satisfied.

The damage evolution relation can be assumed as

$$D = \beta_0 \sigma_d^{m_0}, \tag{10}$$

where β_0 and m_0 are material constants.

2.2. Governing equations in plane strain problem

Introducing a stress function $\phi(r, \theta)$ and writing the stress components as following

$$\sigma_{rr} = \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2}, \tag{11a}$$

$$\sigma_{\theta\theta} = \frac{\partial^2\phi}{\partial r^2}, \tag{11b}$$

$$\sigma_{r\theta} = -\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\phi}{\partial\theta}\right). \tag{11c}$$

The constitute equations under plane strain condition in material 1 become

$$\begin{aligned}
\varepsilon_{rr} &= \tfrac{3}{4}\alpha_0\sigma_e^{n-1}(1+\beta_1 D^{m_1})\left(\frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2} - \frac{\partial^2\phi}{\partial r^2}\right)\\
\varepsilon_{\theta\theta} &= \tfrac{3}{4}\alpha_0\sigma_e^{n-1}(1+\beta_1 D^{m_1})\left(\frac{\partial^2\phi}{\partial r^2} - \frac{1}{r}\frac{\partial\phi}{\partial r} - \frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2}\right)\\
\varepsilon_{r\theta} &= -\tfrac{3}{2}\alpha_0\sigma_e^{n-1}(1+\beta_1 D^{m_1})\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\phi}{\partial\theta}\right).
\end{aligned} \tag{12}$$

The strain compatibility equation takes the form

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}(r\varepsilon_{\theta\theta}) + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}(\varepsilon_{rr}) - \frac{1}{r}\frac{\partial}{\partial r}(\varepsilon_{rr}) - \frac{2}{r^2}\frac{\partial}{\partial r}\left[[r\frac{\partial}{\partial\theta}(\varepsilon_{r\theta})\right] = 0. \tag{13}$$

Substituting (12) into (13), we obtain the stress function equation

$$L_0\phi + L_D\phi = 0, \tag{14}$$

where

$$\begin{aligned}
L_0\phi \;=\; \tfrac{3}{4}\alpha_0 \Bigg\{ & \frac{1}{r}\frac{\partial^2}{\partial r^2}\left[r\sigma_e^{n-1}\left(\frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial\phi}{\partial r} - \frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2}\right)\right]\\
&+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\left[\sigma_e^{n-1}\left(\frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2} - \frac{\partial^2\phi}{\partial r^2}\right)\right]\\
&-\frac{1}{r}\frac{\partial}{\partial r}\left[\sigma_e^{n-1}\left(\frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2} - \frac{\partial^2\phi}{\partial r^2}\right)\right]\\
&+\frac{4}{r^2}\frac{\partial}{\partial r}\left[r\frac{\partial}{\partial\theta}\left(\sigma_e^{n-1}\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\phi}{\partial\theta}\right)\right)\right]\Bigg\}
\end{aligned} \tag{15}$$

$$\begin{aligned}
L_D \;=\; \tfrac{3}{4}\alpha \Bigg\{ & \frac{1}{r}\frac{\partial^2}{\partial r^2}\left[r\sigma_e^{n-1}\sigma_d^m\left(\frac{\partial^2\phi}{\partial r^2} - \frac{1}{r}\frac{\partial\phi}{\partial r} - \frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2}\right)\right]\\
&+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\left[\sigma_e^{n-1}\sigma_d^m\left(\frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2} - \frac{\partial^2\phi}{\partial r^2}\right)\right]\\
&-\frac{1}{r}\frac{\partial}{\partial r}\left[\sigma_e^{n-1}\sigma_d^m\left(\frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2} - \frac{\partial^2\phi}{\partial r^2}\right)\right]\\
&+\frac{4}{r^2}\frac{\partial}{\partial r}\left[r\frac{\partial}{\partial\theta}\left(\sigma_e^{n-1}\sigma_d^m\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\phi}{\partial\theta}\right)\right)\right]\Bigg\}
\end{aligned} \tag{16}$$

where $\alpha = \alpha_0 \beta_1 \beta_0^{m_1}$, $m = m_0 m_1$.

Limit our consideration in the immediate vicinity of the crack tip. According to the computational results for a blunting interface crack given by Shigeru Aoki et al. (1992), there exists a damage dominant region. So it is reasonable to assume that the term $L_D\phi$ in (14) caused by damage is dominant at the immediate vicinity of the crack tip and the $L_0\phi$ can be neglected. Hence, the governing equation is reduced to

$$L_D\phi = 0. \tag{17}$$

Assume that the stress fields in the whole tip zone have the same singularity for material 1 and 2, so that the tractions can be continuous across the interface. According to the assumption of Hutchinson (1968) and Rice and Rosengren (1968), let

$$\phi = Kr^s F(\theta). \tag{18}$$

Substituting (18) into (11) yield

$$\begin{aligned} \sigma_{rr} &= Kr^{s-2}\tilde{\sigma}_{rr}(\theta) \\ \sigma_{\theta\theta} &= Kr^{s-2}\tilde{\sigma}_{\theta\theta}(\theta), \\ \sigma_{r\theta} &= Kr^{s-2}\tilde{\sigma}_{r\theta}(\theta) \end{aligned} \tag{19}$$

$$\begin{aligned} \sigma_m &= Kr^{s-2}\tilde{\sigma}_m(\theta) \\ \sigma_e &= Kr^{s-2}\tilde{\sigma}_e(\theta), \\ \sigma_d &= Kr^{s-2}\tilde{\sigma}_d(\theta) \end{aligned} \tag{20}$$

$$D = \beta_0 K^{m_0} r^{(s-2)m_0}\tilde{D}(\theta), \tag{21}$$

where

$$\begin{aligned} \tilde{\sigma}_{rr}(\theta) &= sF + F'' \\ \tilde{\sigma}_{\theta\theta} &= s(s-1)F \\ \tilde{\sigma}_{r\theta}(\theta) &= (1-s)F' \\ \tilde{\sigma}_m(\theta) &= (\tilde{\sigma}_{rr} + \tilde{\sigma}_{\theta\theta})/2 \\ \tilde{\sigma}_e(\theta) &= [\tfrac{3}{4}(\tilde{\sigma}_{rr} - \tilde{\sigma}_{\theta\theta})^2 + 3\tilde{\sigma}_{r\theta}^2]^{1/2}, \end{aligned} \tag{22}$$

$$\tilde{\sigma}_d(\theta) = \tilde{\sigma}_e(\theta)[\tfrac{2}{3}(1-\mu) + 3(1+2\mu)(\tilde{\sigma}_m/\tilde{\sigma}_e)^2]^{1/2} \tag{23a}$$

$$\tilde{D}(\theta) = \tilde{\sigma}_d^{m_0}, \tag{23b}$$

in which '$'$' $= \mathrm{d}/\mathrm{d}\theta$.

Substituting (18) into (17), we get

$$\left\{\frac{\partial^2}{\partial\theta^2} - G(s-2)[G(s-2)+2]\right\}\{\tilde{\sigma}_e^{n-1}\tilde{\sigma}_d^m[s(2-s)F + F'']\}$$

$$+4(s-1)[G[G(s-2)+1](\tilde{\sigma}_e^{n-1}\tilde{\sigma}_d^m F')' = 0. \qquad (24)$$

where $G = n + m$.

In material 2, we have the equation

$$F'''' + [(s-2)^2 + s^2]F'' + s^2(s-2)^2 F = 0. \qquad (25)$$

The traction free conditions on the crack faces require that

$$F(\pi) = F'(\pi) = 0. \qquad (26)$$

$$F(-\pi) = F'(-\pi) = 0. \qquad (27)$$

According to (12) and (18) and neglecting the effect of the term $L_0\phi$ in the immediate vicinity of the crack tip, the strain fields in the damage zone have the forms

$$\varepsilon_{rr} = \alpha K^G r^{(s-2)G}\tilde{\varepsilon}_{rr}(\theta) = \alpha K^G r^{(s-2)G}\{\tfrac{3}{4}\tilde{\sigma}_e^{n-1}\tilde{\sigma}_d^m[F'' - s(s-2)F]\}$$

$$\varepsilon_{\theta\theta} = \alpha K^G r^{(s-2)G}\tilde{\varepsilon}_{\theta\theta}(\theta) = \alpha K^G r^{(s-2)G}\{\tfrac{3}{4}\tilde{\sigma}_e^{n-1}\tilde{\sigma}_d^m[s(s-2)F - F'']\} \qquad (28)$$

$$\varepsilon_{r\theta} = \alpha K^G r^{(s-2)G}\tilde{\varepsilon}_{r\theta}(\theta) = \alpha K^G r^{(s-2)G}[\tfrac{3}{2}\tilde{\sigma}_e^{n-1}\tilde{\sigma}_d^m(1-s)F'].$$

The corresponding damage displacements in the damage region have the forms,

$$u_r = \alpha K^G r^{(s-2)G+1}\tilde{u}_r(\theta), \qquad (29a)$$

$$u_\theta = \alpha K^G r^{(s-2)G+1}\tilde{u}_\theta(\theta), \qquad (29b)$$

where

$$\tilde{u}_r(\theta) = \tilde{\varepsilon}_{rr}(\theta)/[(s-2)G+1], \qquad (30a)$$

$$\tilde{u}_\theta(\theta) = \left[2\tilde{\varepsilon}_{r\theta}(\theta) - \frac{\partial\tilde{u}_r}{\partial\theta}\right]\Big/(s-2)G\ . \qquad (30b)$$

At the interface in the plastic region side, the material system responds as that of a plastic material bonded to a perfect/elastic substrate as pointed out by Shih and Asaro (1988) and the results of Wang (1994). Hence at the interface we have

$$\tilde{u}_r(0^+) = \tilde{u}_\theta(0^+) = 0, \quad \text{at } \theta = 0^+. \qquad (31)$$

Using (28), (29) and the conditions of stress continuity, the interface stress and displacement boundary conditions can be expressed as following

$$F(0^+) = F(0^-), \qquad (32a)$$

$$F'(0^+) = F'(0^-), \tag{32b}$$

$$F''(0^+) - s(s-2)F(0^+) = 0, \tag{32c}$$

$$F'''(0^+) + \{4(s-1)[1 + G(s-2)] - s(s-2)\}F'(0^+) = 0. \tag{32d}$$

Equations (24), (26) and (32) are the governing equations for the asymptotic damage fields in the plastic region side of the crack tip.

3. Asymptotic solutions for the problem

The $\tilde{\sigma}_e$ and $\tilde{\sigma}_d$ in (24) can be expressed by $F(\theta)$ using (11, 18–23). Then the (24) is an nonlinear equation for the unknown function $F(\theta)$.

The nonlinear equations (24) for plane strain with boundary conditions (26) and (32) were numerically solved using the fourth order Runge–Kutta and Newton method (Fox, 1961). Note that the governing differential equation for $F(\theta)$ has an equi-dimensional character, in the sense that if $F(\theta)$ is a solution, then so is any constant times $F(\theta)$. Therefore, The initial value of $F(0^+)$ is taken as unity for the simpleness. By adjusting the initial value of $F'(0^+)$ and eigenvalue s so that the boundary condition (26) will be exactly met, the solution of (24) can be accurately obtained from $\theta = 0$ to $\theta = \pi$.

After the solution F in the damaged plastic region was obtained, the solution in the linear elastic region, $-\pi \leqslant \theta \leqslant 0$, can be easily derived. In material 2, the solution of (25) can be represented as

$$F = B_1 \cos(s-2)\theta + B_2 \sin(s-2)\theta + B_3 \cos(s\theta) + B_4 \sin(s\theta), \quad -\pi \leqslant \theta \leqslant 0. \tag{33}$$

The unknown coefficients $B_i (i = 1, 2, 3, 4)$ can be determined from the continuity of tractions on the interface and the boundary condition (27).

Like the mixity parameter M^p introduced by Shih (1974) which ranges from one for Mode I to zero for Mode II, we introduce a similar parameter M^d at the region of the crack tip, that is

$$M^d = \frac{2}{\pi} \tan^{-1}\left(\frac{\tilde{\sigma}_\theta(0)}{\tilde{\sigma}_{r\theta}(0)}\right). \tag{34}$$

It is worth noting that since the J-integral loses its path independent for the damaged materials as proved by Wang and Chow (1992) the parameter M^d is only calculated for reference.

Figure 2(a–d) show the angular variation of stresses $\tilde{\sigma}_{ij}(\theta)$ for $n = 6$ with different other material constants and comparisons of the present results to the solution for the bimaterial with dissimilar elastic plastic media. It can be seen that the present results are quite similar to those with the HRR singularity fields. The stress magnitude of present solutions $\tilde{\sigma}_{ij}(\theta)$ distributed along the crack tip both in the damaged plastic region and in the linear elastic region varies with the change of material parameters in the damaged plastic region. The parameter M^d is also relative to material constants used.

Figure 3(a–d) illustrate the comparisons of the angular variation of corresponding displacements $\tilde{u}_r(\theta)$ and $\tilde{u}_\theta(\theta)$ in the damaged plastic region in present results to solution for the bimaterial with dissimilar elastic-plastic madia in elastic plastic region. From these figures,

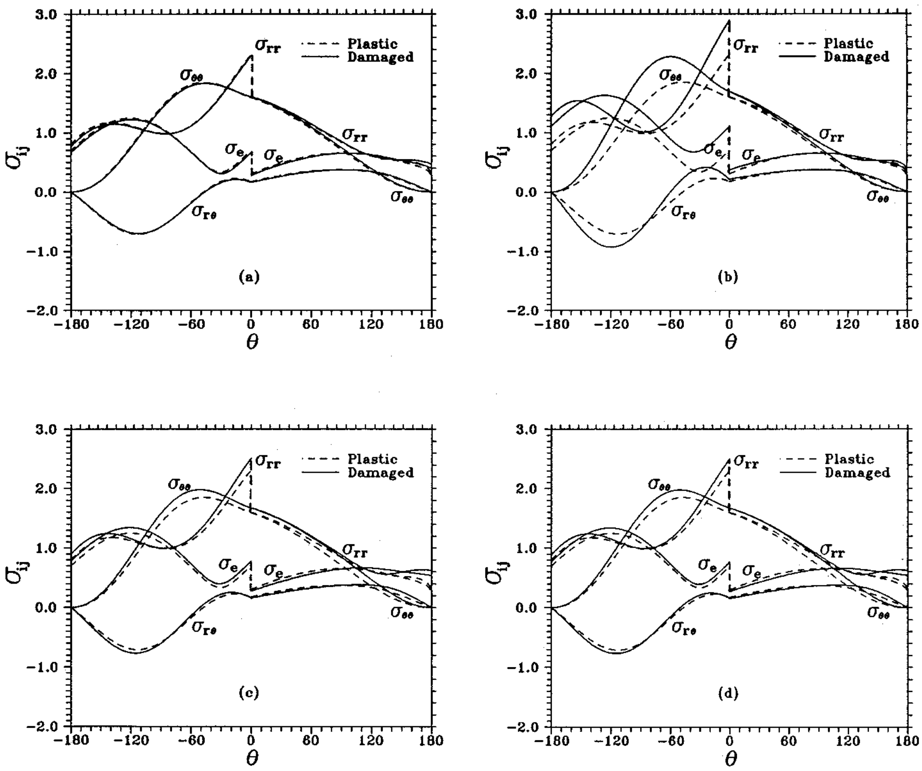

Figure 2. Distributions of stresses $\tilde{\sigma}_{ij}$ near the tip of the crack for $n = 6$; (a) $\mu = 0.5, m_0 = m_1 = 0.5(M^d = 0.936006)$; (b) $\mu = -0.5, m_0 = m_1 = 1.5(M^d = 0.920294)$; (c) $\mu = 0.5, m_0 = m_1 = 1.5(M^d = 0.940386)$; (d) $\mu = 1.0, m_0 = m_1 = 1.0(M^d = 0.940666)$.

it is clearly seen that the conditions (31) are exactly met and the crack face obviously open. Due to the effect of the damage equivalent stress $\tilde{\sigma}_d$ on the displacements, the magnitude of the displacements $\tilde{u}_r$ and $\tilde{u}_\theta$ is sensitive to the material parameters for the damaged material. Figure 4 shows the angular variation of the corresponding damage equivalent stress $\tilde{\sigma}_d$ in the damaged plastic region. From (28) and (30), the effect of the damage equivalent stress $\tilde{\sigma}_d$ on the displacements can also be easily derived.

4. Discussions

4.1. Effects of varying n

Figures 5 and 6 show angular variation of the stresses $\tilde{\sigma}_{ij}(\theta)$ for $n = 3$ and $n = 9$ respectively. Comining with Figure 2(a) (for $n = 6$), it is common that when the crack surface is approached ($\theta \to \pi$), all the three stress components $\tilde{\sigma}_{rr}$, $\tilde{\sigma}_{\theta\theta}$, $\tilde{\sigma}_{r\theta}$ assume positive values and the traction-free conditions on the crack faces are exactly met. The stress magnitude of the present results slightly increases as n increases, particularly in the linear elastic region. However, the parameter M^d decreases as n increases, from $M^d = 0.978574$ for $n = 3$, $M^d = 0.936006$ for $n = 6$ to $M^d = 0.920764$ for $n = 9$.

Figure 7 shows the distributions of displacements $\tilde{u}_r$ and $\tilde{u}_\theta$ along the crack tip in the damaged plastic region for $n = 3, 6, 9$ respectively. It can be observed that the displacement

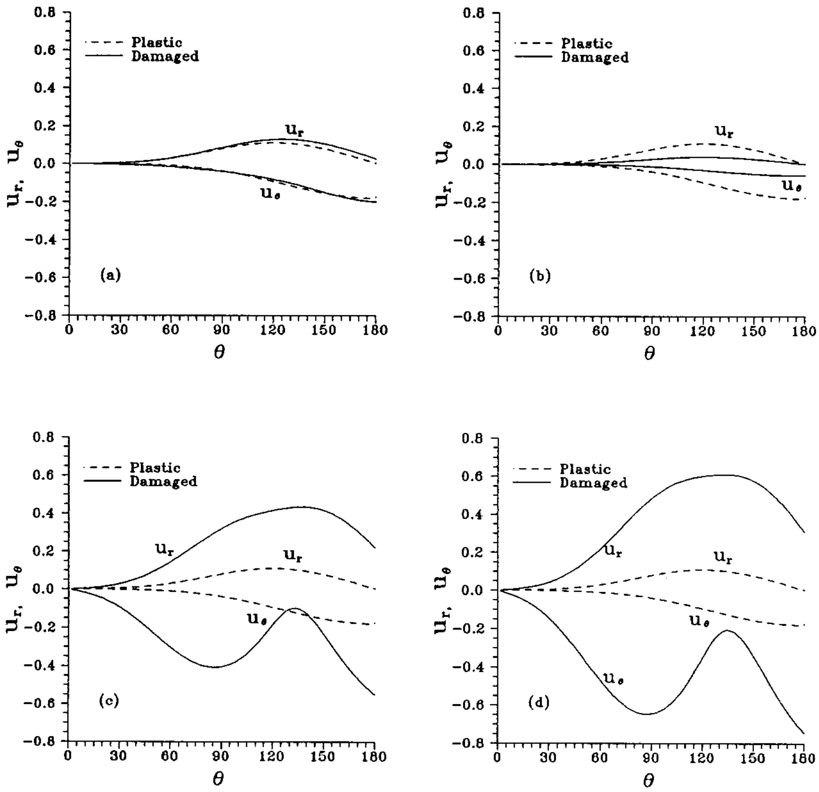

Figure 3. Displacement distributions in the damaged plastic region near the crack tip crack for the same material constants as Figure 2(a–d).

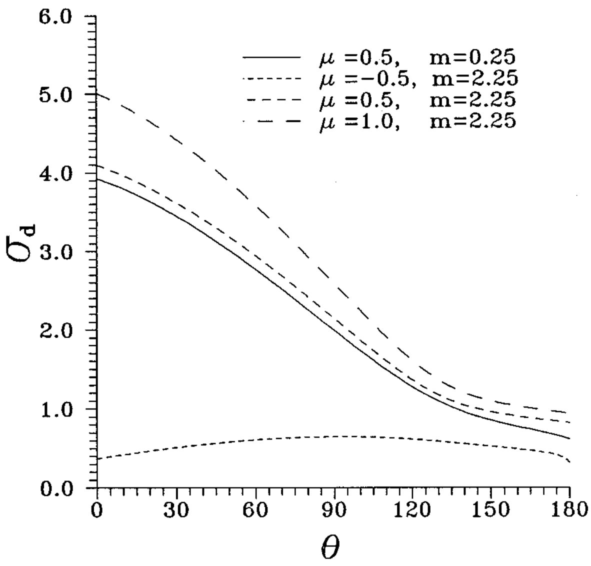

Figure 4. The corresponding damage equivalent stresses $\tilde{\sigma}_d$ to Figure 2 ($m = m_0 m_1$).

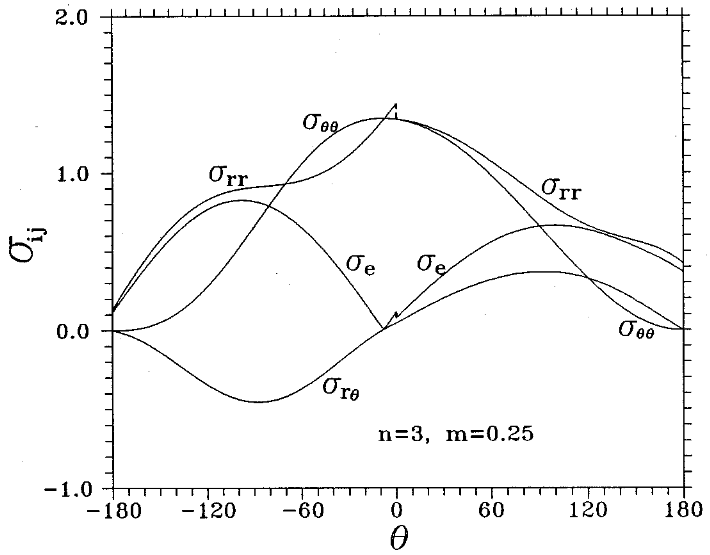

Figure 5. The angular variation of the stresses $\tilde{\sigma}_{ij}$ near the tip of the interface crack for $n = 3$, $\mu = 0.5$, $m_0 = m_1 = 0.5 (M^d = 0.978574)$.

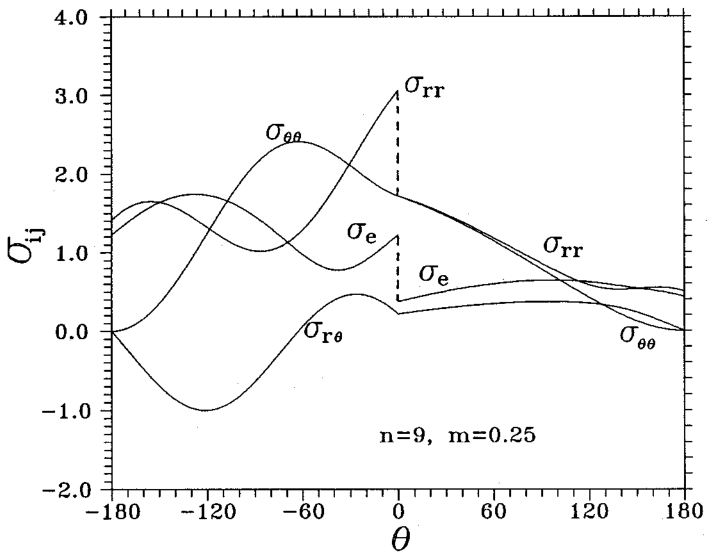

Figure 6. The angular variation of the stresses $\tilde{\sigma}_{ij}$ near the tip of the interface crack for $n = 9$, $\mu = 0.5$, $m_0 = m_1 = 0.5 (M^d = 0.920764)$.

magnitude is obviously decreases with the increase of n, and the radial displacement $\tilde{u}_r$ always is positive when the crack surface is approached, that is, the crack faces always open. In the damaged plastic region, the angular distributions of the damage equivalent stress $\tilde{\sigma}_d$ and damage variable $\tilde{D}$ are shown in Figure 8 (a,b). The magnitude of $\tilde{\sigma}_d$ and $\tilde{D}$ increases as n increases at the interface of the crack tip, and flattens out towards the crack surface for all n.

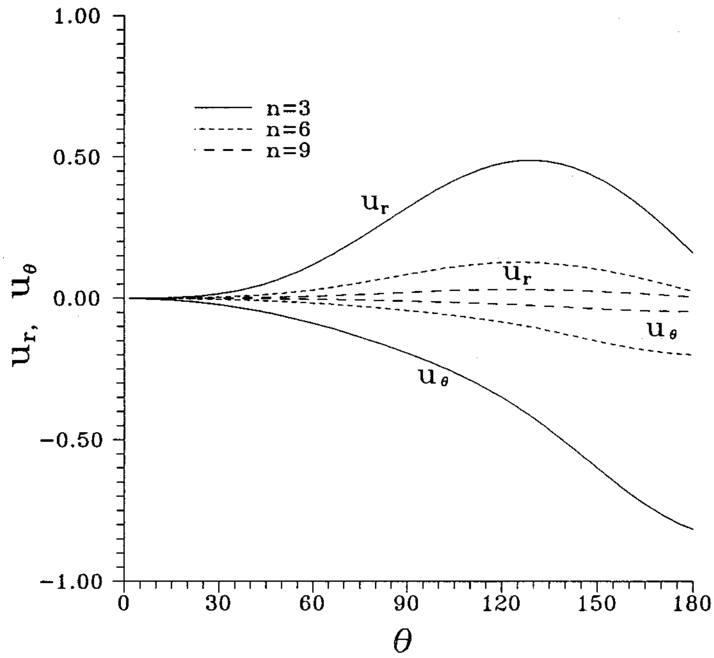

Figure 7. Displacement distributions around the crack tip in the damaged plastic region for $\mu = 0.5$, $m_0 = m_1 = 0.5$.

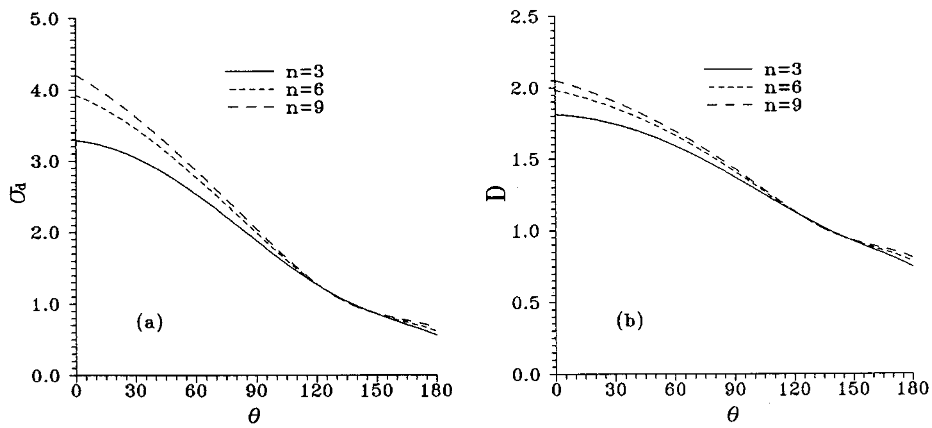

Figure 8. The damaged equivalent stress $\tilde{\sigma}_d$ and the damaged variable $\tilde{D}$ distribution around the crack tip in the damaged plastic region for $\mu = 0.5$, $m_0 = m_1 = 0.5$.

4.2. EFFECTS OF VARYING μ

Referring back to Figures 2 and 3(b,c,d) and Figure 4, the stress distributions along the crack tip are quite similar for different values of μ. However, the magnitude of the corresponding displacements $\tilde{u}_r$ and $\tilde{u}_\theta$ increases as μ increases. The effect of damage equivalent stress $\tilde{\sigma}_d$ for $\mu = 1.0$ is relatively much large than that for $\mu = 0.5$, and for $\mu = 0.5$ the angular variations of $\tilde{\sigma}_d$ is nearly a flat line and much lower than those for $\mu = 0.5$ or $\mu = 1.0$, as shown in Figure 4. This implies that the effect of micro-voids dilatation under hydrostatic

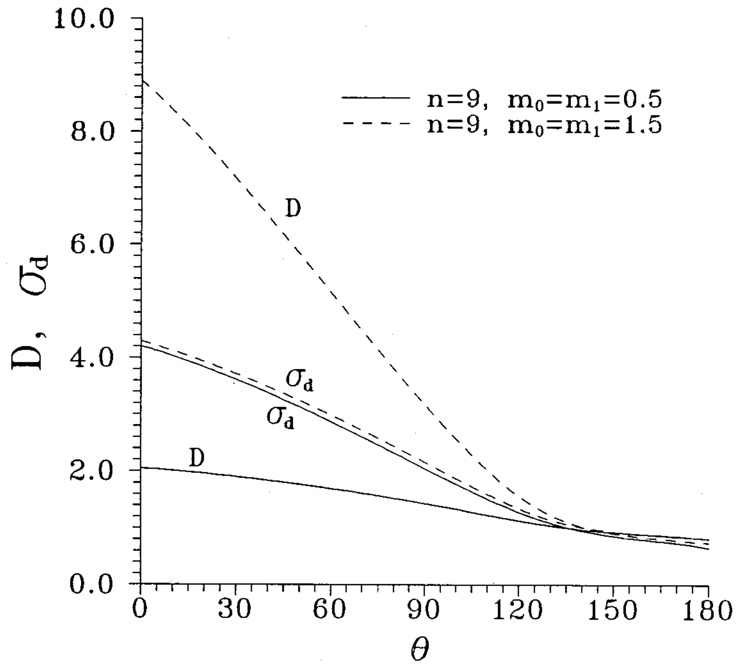

Figure 9. The angular distribution of $\tilde{\sigma}_d$ and $\tilde{D}$ in the damaged plastic region for $\mu = 0.5, m_0 = m_1 = 1.5$.

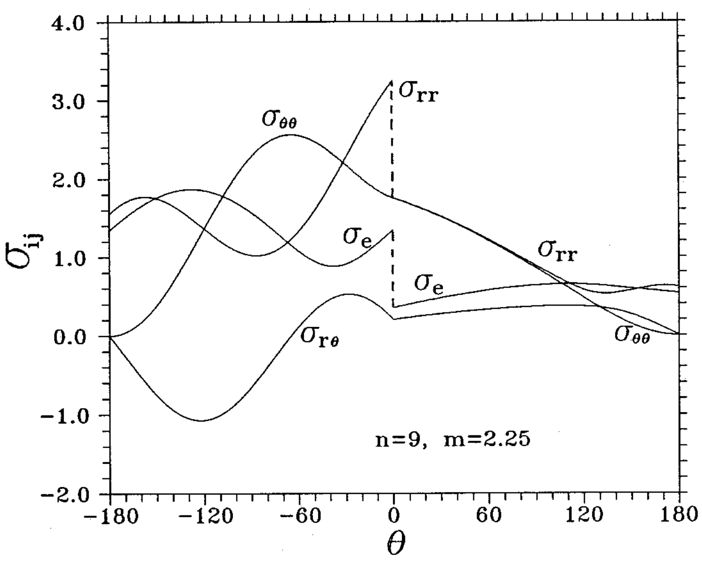

Figure 10. The angular variation of stresses $\tilde{\sigma}_{ij}$ near the crack tip for $\mu = 0.5, m_0 = m_1 = 1.5 (M^d = 0.925912)$.

stress σ_m is much than that of the micro-crack propagation under octahedral shear stress σ_e in the present damage evolution model.

It is interesting to note that for the bimaterial with dissimilar elastic-plastic media from Wang (1990), the mixity parameter $M^d = 0.930517$ for $n = 6$. When μ varies from -0.5 to 1.0, the present parameter M^d is equal to 0.920294 for $\mu = -0.5$, 0.940386 for $\mu = 0.5$ and

0.940666 for $\mu = 1.0$, respectively. M^p lies between the $M^d(\mu = -0.5)$ and $M^d(\mu = 0.5)$ for $m_0 = m_1 = 1.5, n = 6$.

4.3. Effects of varying m_0

From the damage evolution (10), it is clear to see that the larger the value of m_0, the greater the damage variable, and the greater the damage effect on the damaged plastic material, as shown in Figure 9. The Figure 9 shows the angular distributions of $\tilde{\sigma}_d$ and $\tilde{D}$ in the damaged plastic region for $m_0 = 0.5$ and $m_0 = 1.5$ with $n = 9$, $\mu = 0.5$ respectively. Figure 10 shows the corresponding stress angular distributions for $m_0 = 1.5$. Comparing Figure 10 with Figure 6, it can be observed that the distributions of the stress components $\tilde{\sigma}_{rr}$, $\tilde{\sigma}_{\theta\theta}$, $\tilde{\sigma}_{r\theta}$ and $\tilde{\sigma}_e$ are not greatly affected by the increase of m_0, but the parameter $M^d = 0.920764$ for $m_0 = 0.5$ and $M^d = 0.925912$ for $m_0 = 1.5$ slightly increase with the increase of m_0 and always are larger than that of the bimaterial with dissimilar elastic-plastic media, which $M^p = 0.91791$ for $n = 9$.

5. Conclusions

Based on the model introduced by Wang and Chow (1992) which reflects the interactions between a micro-crack and distributed damage in an elastic-plastic material, an asymptotic analysis for a crack lying on the interface of a damaged plastic material and a linear elastic material was presented. The present results show that the stress distributions along the crack tip are quite similar to those with HRR singularity fields and the crack faces open obviously.

Each of the values of material constants n, μ and m_0 was varied to examine their effects on the resulting stress distributions and displacement distributions in the damaged plastic region. It was found that the stress components $\tilde{\sigma}_{rr}$, $\tilde{\sigma}_{\theta\theta}$, $\tilde{\sigma}_{r\theta}$ and $\tilde{\sigma}_e$ are slightly affected by the changes of material constants n, μ and m_0, the damage equivalent stress $\tilde{\sigma}_d$ and damage variable $\tilde{D}$ increase as n and m_0 increase at the interfacial region of the crack tip, and increase significantly with the increase of μ. For $\mu = -0.5$, the angular variations of $\tilde{\sigma}_d$ and $\tilde{D}$ are nearly a flat line, which reveals that the effect of micro-voids dilatation on damage is much larger than that of micro-crack propagation on the damaged material. Due to the effect of damaged equivalent stress $\tilde{\sigma}_d$, the displacement magnitude in the damaged plastic region presents a decrease with the increase of n, and increases as μ and m_0 increase.

The mixity parameter M^d was also varied with the changes of material constants n, μ and m_0. For the same n, the parameter M^p from Wang (1990) lies between the parameter M^d (for $\mu = -0.5$) and M^d (for $\mu = 1.0$) for same small m_0 and m_1.

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