

Experiments on Interaction Between Current-Induced Vibration and Scour of Submarine Pipelines on Sandy Bottom *

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Abstract — In order to understand the dynamic behavior of submarine pipelines exposed to current and the mechanism of the interaction between current-induced vibration and scour of pipelines on a sandy bottom, an experimental investigation is conducted with a small scale model. A test model which can be tested in the flume is set up by taking into account the typical working conditions of the pipelines and by applying the similarity theory. The interactions between the shape of the scour hole and the behavior of the pipeline as well as the flow patterns of the current are detailed, and the interaction mechanism outlined. The effect of vibration of the pipeline on the development of dynamic scour at different stages is found out. The proposed experimental method and test results provide an effective means for design of marine pipelines against scouring.

Key words: *dynamic scour; current-induced vibration; marine pipeline; shedding of vortex; lock-in vibration*

1. Introduction

Under the working condition of submarine pipelines for oil transportation, the development of scour-induced free span is a very serious problem. In a free span, vortex-induced vibration of pipelines frequently takes place, resulting in fatigue and damage of the structures and jeopardizing the safety of the pipelines. At present both the prediction of scour depth and the mechanism of hydro-elastic vibration in a free span are not fully understood. This paper deals with these problems.

To meet the requirement of engineering design of marine pipelines, two methods are used to study the hydro-elastic vibration in a free span. One is to perform field measurements (Bruschi *et al.*, 1989) and to conduct full scale tests for direct application purpose (Raven *et al.*, 1985), in which the length of the pipe span ranges from 40 to 50 meters and the diameter of the pipeline is around 0.5 m. The other is to perform a model test in the laboratory (Bryndum and Bonde, 1989), where flow patterns of the current acting on the model can be either stable current, or regular, irregular waves, or wave-current. The rigid pipe used in the test is about 3 m long, and the flexible one is over 10 m. It is evident that the above mentioned two methods need comparatively large equipment and are costly. Therefore, small scale model tests are usually used (Torum *et al.*, 1989; Jensen *et al.*, 1993; Anund and Torum, 1986), and corresponding analyses conducted (Kaye, 1993; Wang *et al.*, 1988).

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Scour below a fixed pipeline on a sandy bottom was studied early in 1970's, (Chao and Hennessy, 1977) and from then on the study has been improved continuously. However, current induced vibration of free spanning pipelines has significant effect on the scour depth in field conditions. For this purpose, some researchers are devoted to exploring the interaction mechanism between pipeline vibration and scour on a sandy bottom (Sumer *et al.*, 1980; Sumer, 1989), and some theoretical models were developed (Hansen, 1986). Through investigation of the intrinsic relationship between the horizontal and vertical vibration of the pipeline caused by the current, the causality between the pipeline vibration and scouring of the hole on a sandy bottom is found out based on the experiments performed in a small scale model. The present study provides a simple and valid means for selecting the required parameters for design of pipelines against scouring.

2. Determination of Similitude Ratio

It is very important to determine the similarity criterion in model tests. The related factors include the flow pattern, the viscous force of the fluid, the gravity, the inertial force, etc. The corresponding parameters are listed below:

$$\text{Reynolds Number: } Re = \frac{VD}{\nu} \quad (\text{inertial force} / \text{viscous force})$$

$$\text{Froude Number: } F = \frac{V}{\sqrt{gD}} \quad (\text{inertial force} / \text{gravity})$$

$$\text{Reduced Velocity: } V_r = \frac{V}{f_n D}$$

where, V is the velocity of fluid flowing in a single direction, m/s; D is the pipe diameter, m; ν stands for the dynamic viscosity of the fluid, m^2/s ; g is the acceleration of gravity, m/s^2 ; and f_n is the natural frequency of the pipeline system in still water, Hz.

To ensure that the test model can simulate the actual vortex around the pipeline prototype, the parameters of the model listed above should be kept in consistence with those of the prototype. However, that is hard to be fully satisfied due to the restriction of the actual test conditions. This difficulty can be solved on the basis of the similitude ratio detailed below, where the subscripts p and m stand for the cases of the prototype and the test model respectively.

2.1 Range of Reynolds Number

In the actual condition, scour happens when the sand and the soil are raised by the turbulence that is induced by the circling flow of the current. In the model test, it must be ensured that the vortex path is completely in turbulent condition. That is to say, the Reynolds number should be larger than or equal to the Critical Reynolds number Re_{cr} , i. e. $Re_m \geq 4000$, where 4000 is the smallest value to ensure turbulent condition for the vortex path.

2.2 Vortex-Induced Vibration

To simulate the vortex-induced vibration of the pipeline in the actual hydro-elastic system, it should also be ensured that the vortex-induced vibration happens in the model test. In general, the condition for vortex-induced vibration is to keep $V_r = 4 \sim 7$, and the best condition for the vi-

bration is $V_r = 5$. That is to say, $V_{rp} = V_{rm}$ should be preserved to make V_{rm} in the model test approximate to V_{rp} in the actual hydro-elastic system so that V_{rm} could vary in the range of the 4~7.

2.3 Froude Number

The Froude number in the model test must be in consistence with the prototype when the gravity has an effect on the flow of the current, i. e. $F_p = F_m$. Then the velocity ratio can be defined as:

$$\lambda_v = \frac{V_p}{V_m} = \left(\frac{D_p}{D_m} \right)^{1/2} = (\lambda_l)^{1/2}$$

where λ_v and λ_l represent the velocity ratio of the fluid and the model length ratio respectively.

3. The Testing System and Its Physical Model

For simulation of the actual dynamic behavior of the submarine pipeline, it is necessary to describe the state of the pipeline once current-induced scour takes place. Fig. 1 schematically shows the situation of the pipeline system, where z stands for the depth of the scour hole. The sand below the pipeline is swashed away, and the free span is gradually increased with the development of the induced scour. The pipe is supported on the span shoulder. As far as the scour problem is concerned, most attention is paid to the vibration of the pipeline. Such a vibration system is simplified to a test model, which is composed of a rigid pipe supported by a horizontal spring k_h (in-line vibration) and a vertical spring k_v (transverse vibration). Here, k_h and k_v are used to simulate the stiffness of elastic pipeline plus the stiffness of soil in the span shoulder. The support points are at both ends of the rigid pipe with the diameter shown by D . The model test system can be such designed to simulate the pipeline vibration in a free span, and the parameters of the model will be determined by the similarity criterion as mentioned above. The main parameters of the test equipment are shown below:

The width and the length of the flume are 0.5 m and 17 m respectively. The pipe is located in the middle of the flume and the sands cover the bottom of the flume acting as sandy bed. The mean diameter of sand grain is 0.4 mm, the thickness of the sandy bed is 16 cm, and the highest flow rate designed for the flume is $32000 \text{ cm}^3 / \text{s}$. The related parameters can be changed in accordance with the requirement of the experiment.

3.1 Basic Parameters for the Prototype Pipeline System

— The submarine pipelines used in engineering have different diameters: 15 cm, 20 cm, 30 cm, etc. Taking into account the thickness of the covering layer, the authors select the pipelines with larger outer diameters ranging from 36 cm to 64 cm.

— The current velocity near the seabed is 1~3 m/s.

3.2 Parameters for the Testing System

First, the turbulent state of the current in the flume should be preserved according to the principle of keeping the test condition at $Re \geq 4000$, in which the geometric ratio of prototype to

model satisfies the basic condition of $\lambda_l \leq (V_p D_p / 4000\nu)^{2/3}$. Assuming the prototype parameters $V_p = 1 \text{ m/s}$ and $D_p = 0.36 \text{ m}$, $\nu = 0.0000010 \text{ m}^2/\text{s}$, then $\lambda_l \leq 20$. By setting $\lambda_l = 9$, the outer diameter of the model can be determined as $D_m = 4 \text{ cm}$ with the condition of $D_p = 36 \text{ cm}$. And according to the principle that the ratio of length to diameter should be larger than or equal to 10, the pipe length is calculated to be $l = 42 \text{ cm}$.

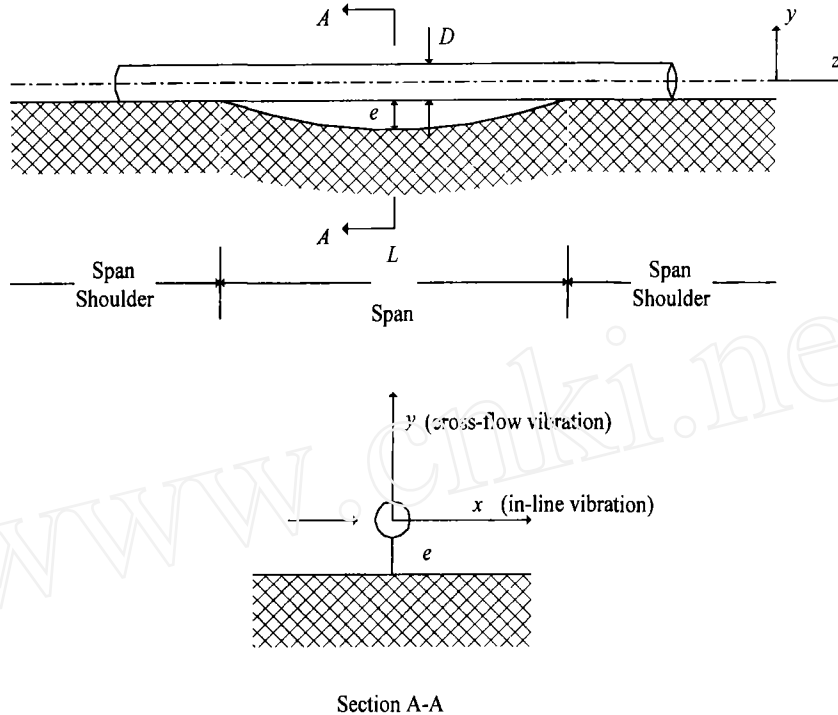


Fig. 1. Scouring state of the pipeline.

As is known, in deep waters, waves have little effect on the flow of the current. This paper will mainly deal with the hydro-elastic vortex motion in the stable flow in single direction. According to the similarity principle, two important parameters should be considered. One is the Reynolds number of the model, which must satisfy $Re_m \geq 4000$, as emphasized above. The other is the reduced velocity of the prototype, which should close to that of the model, i. e. $V_{rm} = V_{rp}$. This is the basic requirement of keeping the system in the state of vortex-induced vibration. Here $V_{rm} = 3 \sim 8$ is taken and the following test parameters are determined accordingly:

Current velocity: $V = 12 \sim 36 \text{ cm/s}$

Water depth: $h = 16 \text{ cm}$

Reynolds number of the test model: $Re_m = (0.48 \sim 1.4) \times 10^4 > 4000$

Natural frequency of the model in still water, vertically: $f_{nv} = 1 \text{ Hz}$

Natural frequency of the model in still water, horizontally: $f_{nh} = 2 \text{ Hz}$

The mass of the test model in horizontal direction: 1 kg

The mass of the test model in vertical direction: 3.4 kg

The mass of the water added to pipeline is calculated by the following formula:

$$m_{ad} = \frac{\pi}{4} \rho C_m D^2 l \sin(i, l)$$

where, l is the pipe length; ρ is the water density; i indicates the vibration direction, the symbol (i, l) shows the angle between l and i directions, and C_m is a coefficient for the added mass. By setting $C_m = 1$, the added mass is calculated to be 0.53 kg.

The natural frequency of the test system defined above gives $k_h = 1.536$ N/mm and $k_v = 0.155$ N/mm, which represent the stiffness in the horizontal and vertical directions respectively. The testing system so established is shown in Fig. 2.

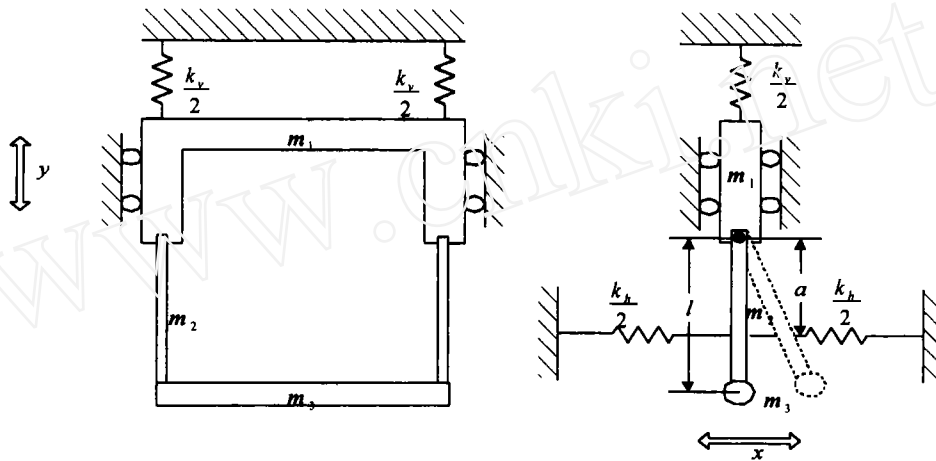


Fig. 2. Vibrating system of the pipeline.

4. Lock-In Vibration of the Pipeline in the Free Span

Field observation shows that the free span is initiated when scour is induced, and the excitation is in two directions due to the alternative vortex shedding in the span. The pipeline vibrates in both the vertical (transverse) and the horizontal (in-line) directions. The vertical vibration is caused mainly by the fluctuant lift force acting on the pipeline, and is synchronous with the alternate vortex shedding. The frequency in this direction is denoted by f_s . The in-line (horizontal) vibration is caused mainly by the fluctuation of the compressive force synchronous with the single vortex shedding of the pipeline. This frequency is two times of that of vertical vibration, i. e., $2f_s$. When the natural frequency of the system in the horizontal direction is two times of that in the vertical direction, the resonant vibration induced by vortex shedding will take place in these two directions at the same time, and the pipeline vibrates seriously.

The phenomena observed in the test support the above conclusions. The damped free vibration curves and the auto-power spectrum of the vibration system are shown in Fig. 3. It can be seen that the horizontal natural frequency is 2 Hz and the damping ratio is 0.038, while the verti-

cal natural frequency is 1 Hz and the damping ratio 0.042.

The lock-in vibration of the hydro-elastic system has also been investigated through experiment. The axial line of the pipe is 6 cm to the sandy bottom surface and 10 cm to the surface of the water. The results are listed in Table 1.

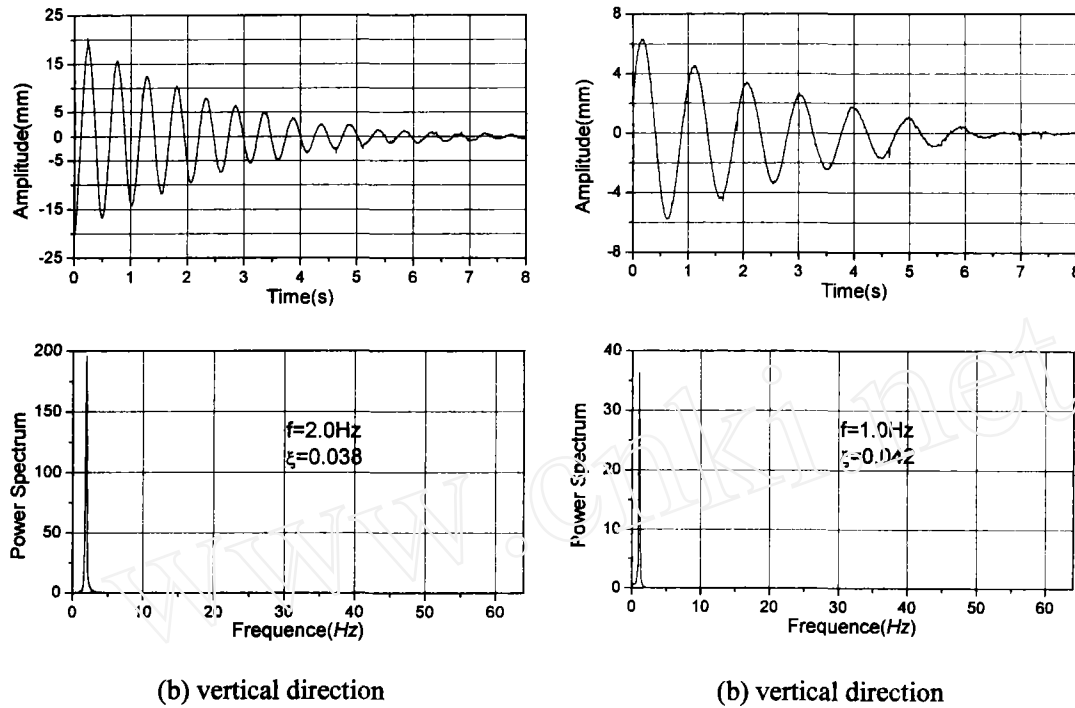


Fig. 3. The damped free vibration time histories and the auto-power spectrum of the system.

A conclusion can be drawn from Table 1 that if the flow velocity is lower than 16 cm/s, unstable small vibration occurs; if the flow velocity is higher than 16 cm/s, the pipe vibrates with small amplitudes. The amplitude increases slowly until vibration reaches a stable state. Especially when the flow velocity is in the range of 29~34 cm/s, the pipe vibrates drastically with a vertical amplitude of 50~56 mm and a horizontal amplitude of 25~30 mm. Although the flow velocity increases from 16 cm/s to 34 cm/s, the vibration frequency of the system changes only a little, about 15%, showing that the hydro-elastic system vibrates in the vicinity of its natural frequency.

As analyzed above, the lock-in vibration takes place once the vortex shedding frequency coincides with the natural frequency of the spring-supported pipe system. During the lock-in period, the amplitude of vibration increases gradually with the increase of flow velocity, and energy dissipation also increases. More energy is needed to compensate for the energy loss. But the lift force can not be increased infinitely, and the vibration stays finally at a limited state. Here the maximum peak-peak value of the vertical amplitude is 55 mm. In addition, with the increase of flow velocity, the exciting frequency increases slightly. And when the velocity reaches 35 cm/s, the vortex shedding frequency becomes 1.2 Hz, evidently different from the natural frequency of

1 Hz of the system. Therefore resonance can not be kept and the lock-in vibration ends. The reduced velocity V_r , corresponding to the lock-in state ranges from 4 to 8.

Table 1 Lock-in vibration of the pipeline system

Flow velocity (cm/s)	Ratio of reduced velocity ($V_r = V / f_n D$)	Vibration states of pipeline			
		Amplitude of displacement (mm)		Frequency at peak (Hz)	
		Vertical	Horizontal	Vertical	Horizontal
< 16	< 4	1~2	< 1		
16	4.0	10	4	1	2
20	5.0	20	6	1	2
25	6.3	35	15	1.05	2.10
27	6.8	40	18	1.05	2.10
29	7.3	50	25	1.10	2.15
31	7.8	50	25	1.10	2.15
32.5	8.1	55	30	1.15	2.25
34	8.5	55	30	1.15	2.25
35	8.5	2	1	1.20	

5. Test of Scour on A Sandy Bottom Below Pipeline

5.1 Static Scour

The concept of static scour is proposed in comparison with dynamic scour that will induce pipeline vibration during scour development. The characteristic of static scour is that the pipeline is fixed and keeps still during the scouring period. Generally, static scour reflects the scour on a sandy bottom before a free span begins to form in the actual pipeline. Although the problems related to static scour have been studied extensively, an experiment is still carried out on static scour for comparison with dynamic scour. The experimental results are shown in Table 2.

5.2 Dynamic Scour

Dynamic scour is a phenomenon of scour on a sandy bottom that happens at the lock-in vibration stage in the pipe-fluid system. The pipeline vibration interacts with scour on a sandy bottom, enhancing the scouring intensity. And in turn, the scour intensifies the vibration of the pipeline. Therefore, the condition for dynamic scour is a certain flow velocity and the formation of a free span of the pipeline. The typical process of dynamic scouring can be detailed as follows.

a) At the beginning of the test, lay the pipe horizontally on the sandy bottom in the flume, and keep the sandy bottom smooth. The pipe is perpendicular to the flow direction. At this moment most of the pipe weight is supported by springs and the sandy bottom takes only 3% of the whole weight so that a close contact can be ensured between the pipe and the sandy bottom

during the whole experiment.

b) Switch on the pump and adjust the flow velocity to the predetermined value (25 cm/s here). With the water brushing, sands rise from the surface of the bed at the beginning and the pipe vibrates horizontally at a very small amplitude. With the development of scour, the pipe sags down. When the sagging height reaches about 7 mm, the pipe hangs freely, and stays at this balance position.

Table 2 Scour on a sandy bottom below pipeline

Flow velocity (cm/s)	Vibration state of pipe during scouring			Final scour depth (mm)	
	Amplitude of displacement (mm)		Description	Static scour	Dynamic scour
	Vertical	Horizontal			
14.9			No evident vibration	8	10
19.0			Small vibration	9	12
21.5	5	2	Critical vibration	18	26
23.5	50	19	Lock-in vibration starts, x-y curve exhibits 8-shaped pattern	21	31
25.6	50	25	x-y curve exhibits 8-shaped pattern	22	34
27.1	56	30	x-y curve exhibits 8-shaped pattern	26	40
35.7	2	1	Small vibration	32	33

c) If the scour develops further, the pipe starts to vibrate vertically at small amplitude. At the same time the horizontal amplitude increases continuously, and the scour depth is about 10 mm at this moment.

d) After insignificant vibration in the vertical direction lasts a period of time, the scour depth begins to increase, and quickly, vertical vibration at a large amplitude takes place with the horizontal amplitude increasing to the maximum.

e) The pipe starts to vibrate with an approximately periodic motion in both horizontal and vertical directions, showing an 8-shaped pattern. From then on the amplitudes in both directions keep unchanged.

The development of scour exhibits the following characteristics. The hole depth increases at the beginning. When it reaches about 20 mm, the hole wall turns steep and the downstream slope turns to a hump-shaped pattern. This results from the alluviation of the sand grains raised by scouring. With the increase of the vibration amplitude, the rate of scouring is increased rapidly and sand grains are completely suspended in the flow. A cloud of sand grains appears whenever the vibrating pipe moves downwards. This kind of movement keeps about 20 minutes. Then the sharp edge of the downstream slope is quickly flattened into a small hump. With time passing the hump moves forward gradually until it disappears. When the hole depth reaches 35~40 mm, a dynamic balance is reached between sand rising and settling. The scour test lasts for about 2 hours at this flow velocity. In the last hour, the hole depth nearly keeps unchanged.

The changing of the maximum amplitude of the pipe vibration during scouring is shown in Fig. 11. It can be seen that in the first 40 minutes, the typical free span is not formed, and the

pipe vibrates with a small amplitude in both horizontal and vertical directions. At the 45th minute, vibration with a large amplitude suddenly takes place and it keeps stable. Fig. 4~10 show the vibration curves, auto-power spectrum and x - y curves. The data are collected at seven stages during the development of scour. These figures show that at the 5thth, 24th and 40th minute from the start, the amplitude is small and chaotic. The main frequencies of vibration are 1.25 Hz in horizontal direction and 2.25 Hz in vertical direction respectively, both of which are slightly larger than the natural frequencies of the system. After the 46th minute, the motion of the pipe becomes regular gradually. Especially at the 51st minute, 70th minute and 106th minute, the x - y curves becomes an 8-shaped pattern, and the vertical amplitude is about two

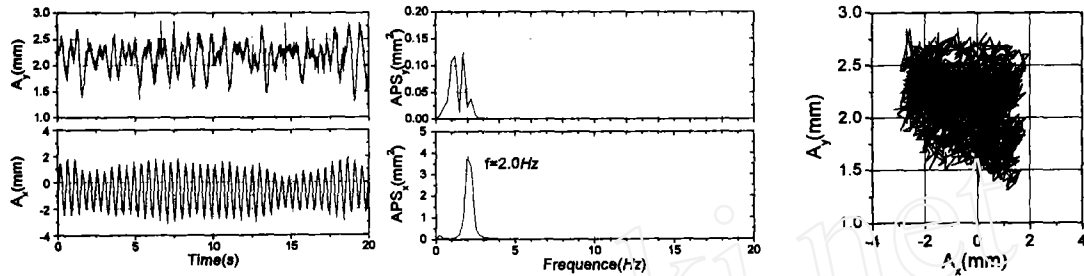


Fig. 4. V235 vibration curves, auto-power spectrum and x - y curves ($T = 5$ min).

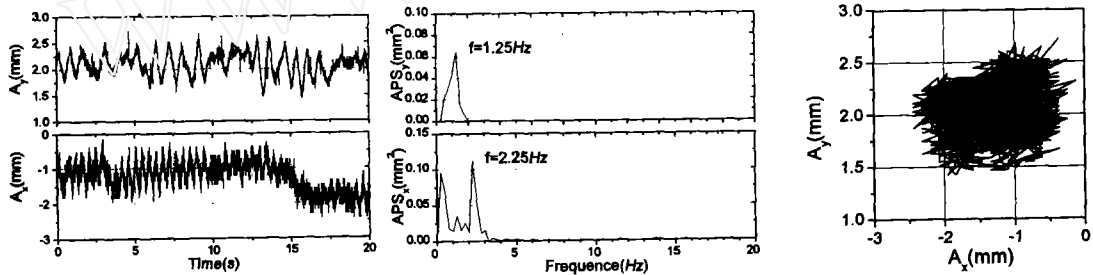


Fig. 5. V23M vibration curves, auto-power spectrum and x - y curves ($T = 24$ min).

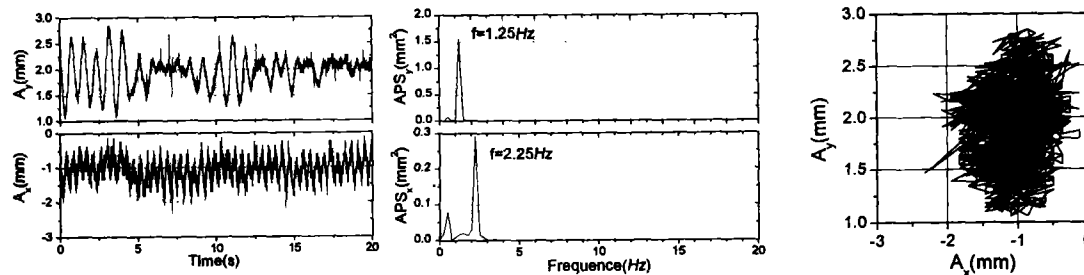


Fig. 6. V23E vibration curves, auto-power spectrum and x - y curves ($T = 40$ min).

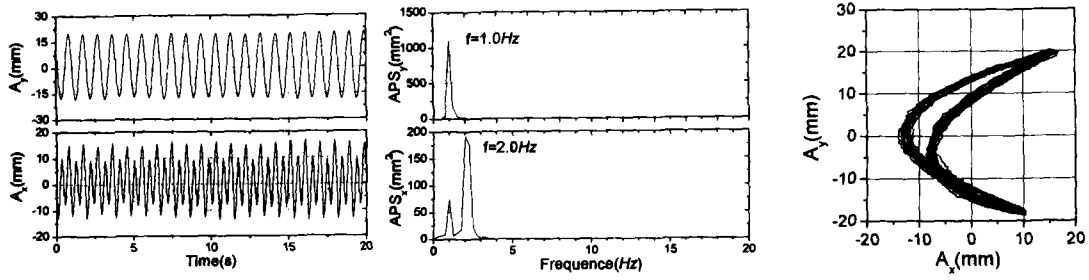


Fig. 7. V23A vibration curves, auto-power spectrum and x - y curves ($T = 46$ min).

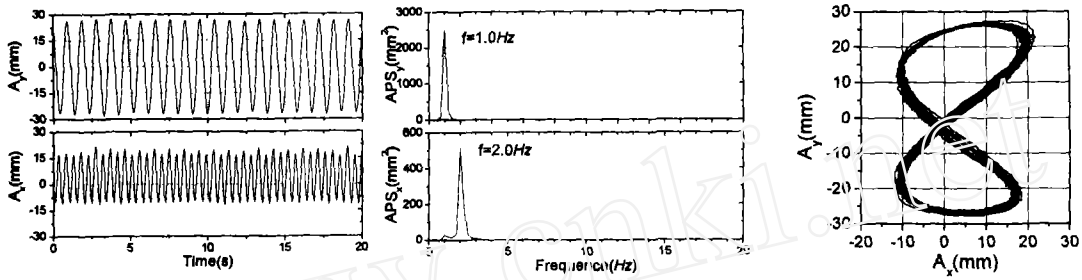


Fig. 8. V23X vibration curves, auto-power spectrum and x - y curves ($T = 51$ min).

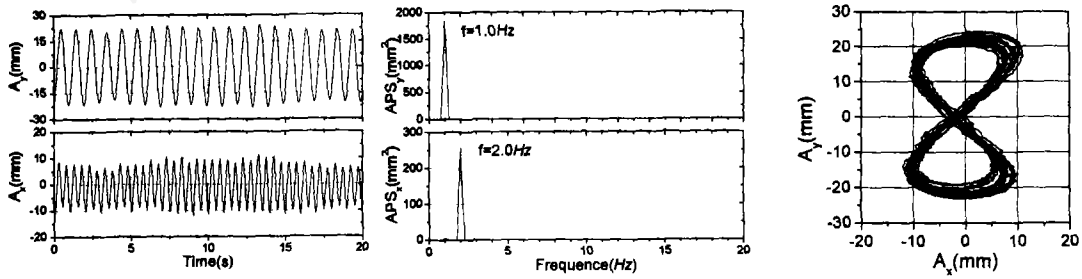


Fig. 9. V23N vibration curves, auto-power spectrum and x - y curves ($T = 70$ min).

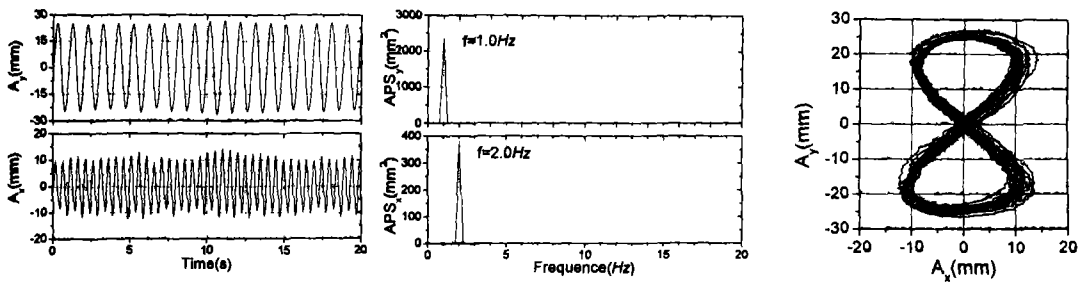


Fig. 10. V23O vibration curves, auto-power spectrum and x - y curves ($T = 106$ min).

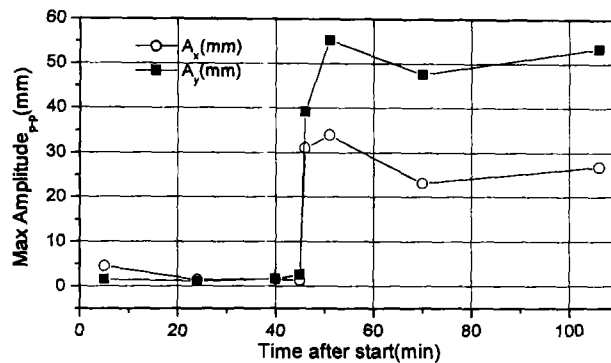


Fig. 11. V23 maximum horizontal and vertical amplitude (peak-peak).

times of the horizontal amplitude. Moreover, the vibration frequencies fall into the range of the natural frequencies of the system.

In this test, both static and dynamic scours have been investigated at seven different velocities. The results are listed in Table 2. The test data show that for the scour under a flow velocity lower than 21.5 cm/s the lock-in vibration can not occur and the vibration does not have significant influence on the scour. That is to say, at the same flow velocity, there is almost no discrepancy between the hole depth induced by static scour and that by dynamic scour. For tests at 4 different velocities ranging from 21.5 cm/s to 27.1 cm/s, lock-in vibration takes place, intensifying the dynamic scouring with the depth 1.5 times that of the static scour. In this case, the maximum dynamic depth can be 40 mm. After the flow velocity reaches 35.7 cm/s, the condition for lock-in vibration is destroyed, and dynamic scouring disappears although the size of scour is large.

6. Conclusions

The tests on current-induced vibration of pipelines and measurements of vibration and scouring of the pipeline-current-sand foundation system are conducted, on the basis of which the interaction of pipeline vibration and scour in current conditions of different flow velocities is simulated and investigated in detail. The test data reveal some new facts that are useful to understanding current-induced vibration of pipelines and corresponding scour.

— The test model must include the movements in both horizontal and vertical directions so as to understand completely current-induced vibration of pipelines.

— During the lock-in vibration, the pipeline vibrates at its natural frequency. While the flow velocity increases, the vibration amplitude increases accordingly. But after vibration reaches a stable state, the increase of the amplitude is very small, and the state will last until the lock-in vibration ends. The reduced velocity V_r corresponding to lock-in vibration ranges from 4 to 8.

— The current-induced vibration of the pipeline has a significant effect on scour on a sandy bottom. At the same velocity, the depth of dynamic scour is about 1.5 times that of static scour.

— In dynamic scouring, if the scour depth is smaller than 0.4 times of the pipe diameter, the interaction between the pipe, current and sand grains is very complicated and irregular.

— The vibration of the pipe speeds up the development of scour after large amplitude vibra-

tion appears.

—It can be concluded from the test investigation that the shape of the scour hole has significant effect on the flow pattern, and also, the flow pattern will affect the behavior of the pipe. For instance, the shape edge of the downstream slope will drastically affect the flow pattern and the dynamic behavior of the pipe.

—After the scour hole reaches the depth of more than 35~40 mm, a dynamic balance between sand grain rising and settling will be reached. The pipe vibrates along an 8-shaped locus, and the maximum peak amplitudes of the pipeline in the vertical and horizontal directions are 0.7 times and 0.35 times of the pipe diameter respectively.

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