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Prediction of structural dynamic plastic shear failure by Johnson's damage number

Ya-Pu Zhao

Abstract It is proved that Johnson's damage number is the sole similarity parameter for dynamic plastic shear failure of structures loaded impulsively, therefore, dynamic plastic shear failure can be understood when damage number reaches a critical value. It is suggested that the damage number be generally used to predict the dynamic plastic shear failure of structures under various kinds of dynamic loads (impulsive loading, rectangular pressure pulse, exponential pressure pulse, etc.,). One of the advantages for using the damage number to predict such kind of failure is that it is conveniently used for dissimilar material modeling.

Vorhersage des dynamisch-plastischen Scherversagens von Konstruktionen durch die Johnsonsche Schadenszahl

Zusammenfassung Es hat sich erwiesen, daß allein unter Verwendung der sog. Johnsonschen Schadenszahl des dynamisch- plastische Scherversagen von stoßartig belasteten Konstruktionen beschreiben werden kann, sofern ein bestimmter Wert dieser Kennzahl erreicht worden ist. Die generelle Verwendung dieser Kennzahl zur Vorhersage des dynamisch-plastischen Scherversagens von Konstruktionen, die einer dynamischen Belastung (stoßartige Belastung, Druckimpulse, steigende Druckbelastung u.s.w.) ausgesetzt sind, wird daher vorgeschlagen. Diese Kennzahl wird bereits erfolgreich zur Beschreibung unterschiedlicher Materialverhaltensweisen eingesetzt.

List of symbols

Dn damage number
H thickness of structures
I impulse of rectangular pressure pulse
I_e effective impulse of applied load
$H(t)$ Heaviside function
k, k' material constant
p_0 magnitude of rectangular pulse
S sliding displacement
t_f time when plastic deformation ends
t_y time when plastic deformation begins
T_0 time
V_0 initial velocity
ρ material density
σ_0 yield stress of material
τ duration of rectangular pressure pulse

Subscripts

m model
p prototype

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1 Introduction

Both experimental and theoretical studies involving dynamic failure analysis of beams and plates show that transverse shear failure is a fundamental mode of structural failure under large intensive loading [1–9]. Menkes and Opat [1] observed that transverse shear at the support was one of the three basic failure modes for impulsively loaded fully clamped strain-rate-insensitive aluminium alloy beam (Fig. 1a), these three basic failure modes illustrated in Fig. 1b are:

- excessive permanent transverse deflection (Mode I);
- tensile tearing failure at the supports (Mode II);
- transverse shear failure (Mode III).

Similar to Menkes and Opat's experiment, Teeling-Smith and Nurick [2] observed the similar three major failure modes for impulsively loaded circular plate. As with the circular plate, Olson, Nurick and Fagnan [3] also observed the three similar major failure modes for blast loaded square plate. Liu and Jones [4] conducted an experimental investigation into the dynamic plastic response and failure of strain-rate-sensitive mild-steel beams due to concentrated impact loads, they found that the beams were dominated by shear failures. Jouri and Jones [5] have found experimentally that the transverse shear severance of beams in double shear loading occurs at a shear displacement which is much smaller than the beam thickness. On the basis of Menkes and Opat's experiment [1], Jones [6] carried out an approximate theoretical study of this problem for predicting the onset of these three failure modes. Duffey [7] found that hard-point shear failure in cylindrical shells is adequately predicted by the theory for shear failure in beams, by studying two loading cases (i.e. rectangular pressure pulse and exponential pressure pulse). Zhao et al. found that consideration of the influence of rotatory inertia and the presence of crack increases the initial kinetic energy required to cause dynamic plastic shear failure [8, 9].

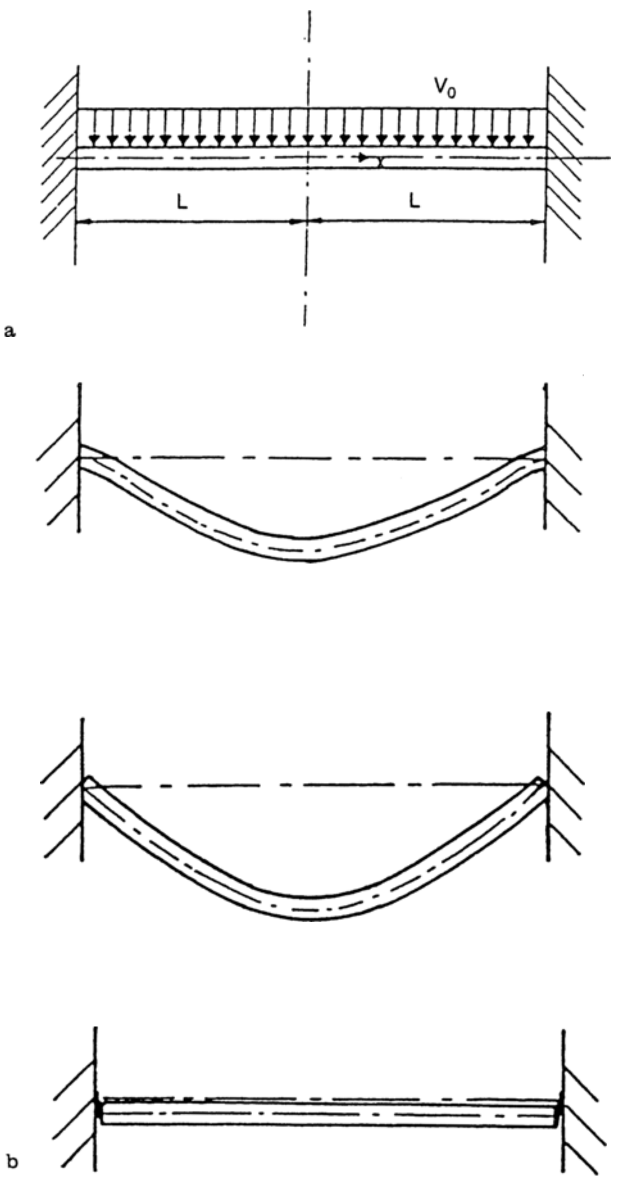

Fig. 1. a Impulsively loaded fully clamped beam, **b** Three typical failure modes

There has been an analogy between failure of solid and turbulence [10, 11]. Turbulence occurs when Reynolds number Re, a similarity parameter in fluid dynamics, reaches a critical value. This would be instructive for the study of dynamic plastic shear failure of structures. The objective of the present paper is to apply the universal dimensional analysis to the structural dynamic plastic shear failure analysis, and derive the general dimensionless condition for this kind of failure.

2 Similarity consideration of dynamic plastic shear failure of structures

It is believed that Hopkinson's "cube-root" scaling in 1915 [12] proposed to the British Ordnance Board to be the first statements on the applicability of the similarity concept to the assessment of structural response to dynamic loads [13]. Since then, similarity methods have been widely used in impact dynamics. Generally speaking, there are two kinds of modeling [14]: the first one is called replica model, which is geometrically similar in all aspects to the prototype and employs identically the same materials at similar locations; Another one is called dissimilar material model, which is geometrically similar to a prototype but made of different material.

For a piece of material subjected to impulsive loading, or impinged by an initial impulsive velocity V_0, Johnson's damage number is defined by [15]

$$\mathrm{Dn} = \rho V_0^2/\sigma_0 \ , \tag{1}$$

where ρ and σ_0 are density and yield stress of the material, respectively. Johnson's damage number is a basic dimensionless similarity parameter in impact dynamics. Actually, the damage number can be obtained by making dimensionless the motion equation. As an example, the well-known motion equation of the material for one dimensional problem is

$$\frac{\partial \sigma}{\partial x} = \rho \frac{\partial V}{\partial t} \ , \tag{2}$$

where σ and V denote stress and particle velocity, respectively. To make dimensionless (2), we introduce the dimensionless variables as follows

$$\Sigma = \sigma/\sigma_0, \quad \tau = t/T, \quad X = x/(V_0 T) \ , \quad \overline{V} = V/V_0 \ , \tag{3}$$

where T is a characteristic time. By using transformation (3), we can render (2) into a dimensionless one as

$$\frac{\partial \Sigma}{\partial X} = \frac{\rho V_0^2}{\sigma_0} \frac{\partial \overline{V}}{\partial \tau} \ . \tag{4}$$

It is evident from (4) that the damage number is a dominant dimensionless parameter for the dynamic plastic response of material. Damage number can be understood as a measure of the order of strain imposed in the region where severe plastic deformation occurs, it can also be considered the ratio of inertia force of the loading (ρV_0^2) to the resistance ability of the dynamically loaded material (σ_0), so damage number can be regarded as a measure of the fluid-like (hydrodynamic) behavior imposed in the region of severe plastic deformation during high velocity impact [16].

Consider a fully clamped rigid, perfectly plastic beam (illustrated in Fig. 1a) with thickness H and unit width subjected to uniformly distributed impulsive loading V_0. The relative sliding displacement $[S]$ at the hard point can be expressed by the following general function

$$[S] = F(\rho, V_0, \sigma_0, H) \ , \tag{5}$$

where ρ and σ_0 are the mass density and yield stress, respectively. It is noted that the length of the beam does not inter (1) for dynamic plastic shear failure at the supports. By using Buckingham's π-theorem, we can render (5) into a dimensionless functional relationship as follows

$$[S]/H = f(\rho V_0^2/\sigma_0) \ . \tag{6}$$

Eq. (6) means that dynamic plastic shear failure under impulsive loading can be predicted by damage number

Dn. In other words, dynamic plastic shear failure can be considered when damage number reaches a critical value.

For complete shear failure at the hard points $[S]/H = 1$, then from (6) we have the shear failure condition as

$$\mathrm{Dn} = \mathrm{const} \tag{7}$$

In fact, Jones [6] obtained the condition for complete shear failure as

$$V_0 = \frac{2}{3}\sqrt{2\frac{\sigma_0}{\rho}} \tag{8}$$

Obviously, (8) can be rewritten as

$$\mathrm{Dn} = 8/9 \tag{9}$$

Actually, shear failure occurs when [5]

$$[S]/H = k \ , \tag{10}$$

where $0 < k \leq 1$, k is a material constant to be determined by experiment, complete severance occurs when $k = 1$, but transverse shear failure is likely to develop for a smaller value of k for beams [5]. The same situation is likely to occur for plates; there has been no experimental investigation concerning the determination of material constant k as far as the present author is aware. The value of k may be larger for ductile materials, and smaller for brittle materials [8]. Generally, shear failure at the hard points occurs when

$$\mathrm{Dn} = \frac{8}{9}k' \ , \tag{11}$$

where k' can also be considered a material constant. Besides impulsive loading, rectangular pressure pulse is another idealized dynamic loading. Rectangular pressure pulse can be described by

$$p(t) = p_0[H(t) - H(t-\tau)] \ , \tag{12}$$

where p_0 is the magnitude of the dynamic pulse, τ the duration of the pulse, $H(t)$ the Heaviside function. The impulse of such kind of pulse is

$$I = p_0\tau \ . \tag{13}$$

For such pulse, the damage number equivalent to the case of impulsive loading is given by

$$\mathrm{Dn} = \frac{I^2}{\rho\sigma_0 H^2} \ . \tag{14}$$

The general functional relationship for the relative sliding displacement at the hard points is

$$[S] = F(\rho, p_0, \tau, \sigma_0, H) \ . \tag{15}$$

By using Buckingham's π-theorem we have the following simplified relationship as

$$[S]/H = f(\mathrm{Dn}, \sigma_0/p_0) \ . \tag{16}$$

The damage number in (16) is expressed by (14). For complete shear failure $[S]/H = 1$, (16) can be rewritten as

$$\mathrm{Dn} = \Phi(\sigma_0/p_0) \ . \tag{17}$$

As a matter of fact, the analytical solution of complete shear failure at the hard points presents [7]

$$\frac{I^2}{8H^2\rho\sigma_0} = \frac{1}{9(1 - 4\sigma_0/9p_0)} \ . \tag{18}$$

It is required that $p_0 > \frac{4}{9}\sigma_0$ for shear failure to occur. Similarly, the generally condition for shear failure in the case of rectangular pressure pulse is

$$\mathrm{Dn} = \frac{8k'}{9(1 - 4\sigma_0/9p_0)} \ . \tag{19}$$

It is easy to note that (19) can be reduced to (11) when $\sigma_0/p_0 \to 0$ and $\tau \to 0$.

For dynamic loading of general shape, the damage number analogous to the case of impulsive loading is

$$\mathrm{Dn} = \frac{I_e^2}{\rho\sigma_0 H^2} \ , \tag{20}$$

where $I_e = \int_{t_y}^{t_f} p(t)\mathrm{d}t$ is the effective impulse of the applied loading $p(t)$, t_y and t_f are the times when plastic deformation begins and ends. Duffey [7] also studied the dynamic plastic shear failure of a fully clamped beam subjected to exponential pressure pulse

$$p(t) = p_0 \exp(-t/T_0) \tag{21}$$

The effective impulse of the exponential pressure pulse is then

$$I_e = p_0 T_0[1 - \exp(-t_f/T_0)] \ . \tag{22}$$

Duffey obtained the complete hard-point shear failure condition as

$$\frac{\rho H^2}{\sigma_0 T_0} = \left(\frac{p_0}{\sigma_0} - \frac{4}{9}\right)t_f - \frac{2}{9}\frac{t_f^2}{T_0} \ . \tag{23}$$

It is also required that $p_0 > \frac{4}{9}\sigma_0$ for shear failure to occur in case of exponential pulse expressed in (21). (19) can be rearranged into the following by the present paper as

$$\mathrm{Dn} = \frac{8}{9}\frac{1}{\frac{p_0 T_0}{\sigma_0 t_f}\left(\frac{9}{2} - 2\frac{\sigma_0}{p_0}\right) - 1} \ , \tag{24}$$

where the damage number Dn is expressed in (20) and the relation

$$t_f = \frac{9}{4}\frac{p_0}{\sigma_0}T_0[1 - \exp(-t_f/T_0)]$$

given by Duffey in [7] has been used. (24) means that complete shear failure condition for dynamic loading of general shape can be also considered when damage number reaches a critical value, it is easy to see that (24) is analogous to (19) and (11). Similarly, the general failure condition for (20) is

$$\mathrm{Dn} = \frac{8}{9}\frac{k'}{\frac{p_0 T_0}{\sigma_0 t_f}\left(\frac{9}{2} - 2\frac{\sigma_0}{p_0}\right) - 1} \ . \tag{25}$$

3
Discussion

If the dynamic loading of the experiment by Menkes and Opat [1] is considered approximately a rectangular

Table 1. Value of Dn in (14) of reference [1]

Beam thickness		Impulse		Dn
(in)	(mm)	(ktaps)	(Ns)	(eqn 14)
0.187	4.75	40	4000	0.92
0.250	6.35	48	4800	0.74
0.375	9.53	65	6500	0.60

For $\sigma_0 \approx 285$ MPa; $\rho = 2700$ kg/m^3

pressure pulse, we can calculate the values of the Dn in (14) for three different impulses, which are given in Table 1. It is shown in Table 1 that the larger the impulse, the smaller the value of Dn for the occurrence of shear failure. This is simply because that the higher the impulse, the more accurate the approximation of the rigid, perfectly plastic model and the more localized the deformation. Actually, any real dynamic loading has a rise time to reach its peak value, when the rise time could not been ignored, then (20) should be used, this can be one explanation when the first value of Dn in Table 1 is slightly larger than 8/9.

From first glimpse it changes nothing from (8) to (9) or from (18) to (19), nevertheless, from the point of view of similitude, (9) and (19) are more reasonable. For dissimilar modeling, (11) gives

$$\frac{\rho_m V_{0m}^2}{\rho_p V_{0p}^2} \frac{\sigma_{0p}}{\sigma_{0m}} = \frac{k'_m}{k'_p} \quad , \tag{26}$$

where subscripts m and p refer to model and prototype, respectively. (26) gives the law that modeling should abide by. Especially, if constitutive similarity is met, then we have $k'_m = k'_p$, in such case (26) reduces to

$$\frac{\rho_m V_{0m}^2}{\rho_p V_{0p}^2} \frac{\sigma_{0p}}{\sigma_{0m}} = 1 \quad . \tag{27}$$

Constitutive similarity means that model and prototype materials have homologous constitutive properties and homologous stress-strain curves [14].

From (19) we have the following similitude conditions of replica modeling as

$$\begin{cases} \left[p_0^2\tau^2/(\rho\sigma_0 H^2)\right]_m = \left[p_0^2\tau^2/(\rho\sigma_0 H^2)\right]_p \\ (\sigma_0/p_0)_m = (\sigma_0/p_0)_p \\ k'_m = k'_p \quad . \end{cases} \tag{28}$$

(28) can be further reduced to the following form

$$\begin{cases} \left(\dfrac{p_0\tau^2}{\rho H^2}\right)_m = \left(\dfrac{p_0\tau^2}{\rho H^2}\right)_p \\ k'_m = k'_p \quad . \end{cases} \tag{29}$$

4 Conclusion

To summarize, this paper suggests that Johnson's damage number be used to predict the dynamic plastic shear failure of structures subjected to large intensive loading. Dynamic plastic shear failure can be considered when damage number reaches a critical value. Similitude conditions are derived for replica and dissimilar modeling for structures subjected to impulsive loading as well as rectangular pressure pulse.

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