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A fiber web is modeled as a three-dimensional random cylindrical fiber network. Nonlinear behavior of fluid flowing through the fiber network is numerically simulated by using the lattice Boltzmann (LB) method. A nonlinear relationship between the friction factor and the modified Reynolds number is clearly observed and analyzed by using the Fochheimer equation, which includes the quadratic term of velocity. We obtain a transition from linear to nonlinear region when the Reynolds numbers are sufficiently high, reflecting the inertial effect of the flows. The simulated permeability of such fiber network has relatively good agreement with the experimental results and finite element simulations.

Keywords Fiber network; Flow; Fochheimer equation; Three-dimensional lattice Boltzmann method

Introduction

The permeability of porous media is of interest in many areas, such as paper industries, petroleum industries, environmental studies, biological processes, and physiological systems. Permeability as a parameter for understanding the migration of fluid into porous media has been studied theoretically and experimentally for many years. The fundamental aspect that has brought the most interest is the relation of the applied pressure gradient and the resulting flow rate. For the flow at near zero Reynolds number, the pressure gradient and the flow rate have a linear relation, known as Darcy's law. For small but nonzero Reynolds numbers, the pressure gradient is a nonlinear function of the flow rate. The experimentation that proved this nonlinear relation was carried out by Forchheimer (1930), who indicated that there exists a quadratic term of flow rate when the Reynolds number is sufficiently high. Modeling and simulating this nonlinear relation and the inertial effect of porous materials have brought more attention to this area.

The inertial effect in periodic and random arrays has been the focus of a large number of studies. Koch and Ladd (Koch and Ladd, 1997; Hill et al., 2001) simulated moderate Reynolds number flows through periodic and random arrays

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of aligned cylinders and spheres in two dimensions. The study showed that the inertial term made the transition from linear to quadratic in the random arrays. The inertial effect became smaller at the volume fraction approaching close packing. Two-dimensional simulation considering inertia was also studied by Andrade et al. (1999). The porous medium was created by using square plaquettes as obstacles for fluid flow. They showed the departure from Darcy's law in flow through high porosity percolation structures and that at sufficiently high Reynolds numbers inertia became relevant. The Forchheimer equation was proved to be valid for low and also a limited range of high Reynolds numbers. Clauge and Phillips (1997) investigated the hydrodynamics permeability in a three-dimensional array. The disordered fibrous media was modeled as non-overlapped cylindrical fibers for pure collagen and proteoglycan fibers. They obtained a nonlinear region at very diluted fiber volume fraction.

Because the permeability in porous media is directly related to the pore geometry, complex structures of porous media have brought more difficulty for modeling and simulation. In most studies, the porous media are modeled either in two dimensions with random arranged cylinders or spheres (Martin et al., 1998; Lee and Yang, 1997; Ghadder, 1995) or as a three-dimensional structure using ordered cubic, body-centered cubic, or face-centered cubic lattices (Hingdon and Ford, 1996). Koch and Hill (2001) reviewed recent research of inertial effects on porous media. The microstructure was found to be more important at finite Reynolds numbers than at zero Reynolds number. To enhance the understanding of inertial effect, the simulation must be considered in three dimensions. In numerical simulation, the lattice Boltzmann (LB) method has been employed to investigate flows in complex geometries, especially in three-dimensional modeling and simulation. Succi et al. (1989) and Cancelliere et al. (1990) used the LB method to simulate the flow through porous media at pore scale and studied the microscopic behaviors of the flow. Koponen et al. (1998) employed the LB method in a three-dimensional simulation of flow through a fiber web, the fibers being laid in either x direction or y direction. They applied a gravitational body force to the fluid to simulate creeping flow. Although the LB method is employed to study flow in three-dimensional porous media, most of the simulations are focused on creeping flow, which eliminates the inertial effect. Recently, studies have began to focus on the nonlinear relation in porous media using the LB method. For example, Inamuro et al. (1999) applied the LB method to simulate the isothermal flows in three-dimensional sphere packed porous media at single porosity. The calculated pressure drops fit with the Ergun equation for high Reynolds numbers.

Despite the numerous simulation studies, it seems that the simulation of flow through three-dimensional random fibrous porous media with medium porosity has not been well simulated with consideration of the quadratic term of velocity by the lattice Boltzmann method. In this article, we use the lattice Boltzmann method to model and simulate fluid flows through a random fiber network at medium porosities. The fiber network is modeled with equal-sized and randomly distributed cylinders in three dimensions. Fibers can be overlapped. This geometry is believed to be close to that of many fibrous materials, such as paper, filters, and textile. The correlation of pressure drop versus velocity is studied to further prove the existence of a quadratic term of velocity in three-dimensional fibrous materials. The effect of inertia is focused on at a porosity range of 48% to 72%. A nonlinear behavior between the friction factor and the modified Reynolds number is clearly observed. The simulated permeability is compared with the experimental results (Lindsay and Brady, 1993).

Permeability of Porous Media

A single-phase fluid flowing through microscopically disordered porous media at low Reynolds numbers is described by Darcy's law (Bear, 1972). The superficial flow rate $\langle u \rangle$ of a viscous fluid through a porous medium of length L is proportional to the applied pressure difference ΔP and inversely proportional to the dynamic viscosity μ:

$$\langle u \rangle = \frac{k}{\mu}\frac{\Delta P}{L} \tag{1}$$

At low Reynolds numbers where the flow is laminar, viscous forces are predominantly linear. Darcy's law is valid. The symbol k is the permeability with the unit of length squared. However, as Reynolds number increases, the inertial force has to be considered, which describes the transition from viscous force predominated by creeping flow to another inertial force governed laminar region, which gradually passes to turbulent flow.

In order to always satisfy Darcy's law in the creeping flow region and to correctly capture the influence of inertia at high Reynolds numbers, the well-known Forchheimer equation (Perry, 1984) is used. This equation consists of a linear term of the viscous component and a power term of the inertial component:

$$-\frac{\Delta P}{L} = \alpha\mu\langle u \rangle + \beta\rho\langle u \rangle^2 \tag{2}$$

where α is the viscous coefficient, β is the inertial coefficient; they are both resistance coefficients describing the physical properties of the porous material. At low Reynolds numbers, the quadratic term of velocity is close to zero, and therefore can be ignored, which turns the Forchheimer equation to Darcy's law. The symbol α^{-1} is defined as the permeability of porous media.

The Forchheimer equation can be modified as friction factor and Reynolds number correlation (Andrade et al., 1999):

$$f = \frac{1}{\mathrm{Re}'} + 1 \tag{3}$$

where

$$f = -\frac{\Delta P}{L\beta\rho\langle u \rangle^2} \tag{4}$$

$$\mathrm{Re}' = \frac{\beta\rho\langle u \rangle}{\alpha\mu} \tag{5}$$

The formula can be used for calculating the friction factor of porous media with various geometries and porosities. The universal factors give a good comparison of different porous materials and flow conditions. We use the Forchheimer equation to analyze the numerical results in this study.

The Lattice Boltzmann Method

The lattice Bolzmann method has been successfully applied for simulating the interaction between fluid and solid particles. The kinetic nature of the lattice Boltzmann method enables it to simulate complex geometry such as fluid flow in porous media (Ladd, 1994; Ladd and Verberg, 2001; Qi, 1999; Ding and Aidun, 2000; Qian, 1990; Qian et al., 1992; Chen and Doolen, 1998; Guo et al., 2002; He and Luo, 1997a,b).

In the lattice Boltzmann (LB) method, fluid particles reside on the lattice nodes and move to their nearest neighbors along the links with unit spacing in each unit time step. The lattice Boltzmann equation with Bhatanaga-Gross-Krook (BGK) single relaxation time is given by

$$f_\sigma(\vec{x}_i + \vec{e}_\sigma, t+1) = f_\sigma(\vec{x}_i, t) - \frac{1}{\tau}[f_\sigma(\vec{x}_i, t) - f_\sigma^{eq}(\vec{x}_i, t)] \tag{6}$$

where $f_\sigma(\vec{x}, t)$ is the fluid particle distribution function for the particles with velocity $\vec{e}_\sigma$ at position x and time t, $f_\sigma^{eq}(\vec{x}, t)$ is the equilibrium distribution function, and τ is the single relaxation time.

The simulations described in this article were performed by using the D3Q15 model. It possesses a rest particle state, six links with the nearest neighbors, and eight links with the next nearest neighbors. Periodic boundary conditions in the flow direction with bounce back on the solid nodes were used. The equilibrium distribution function $f_\sigma^{eq}(\vec{x}, t)$ is taken as

$$f_\sigma^{eq}(\vec{x}, t) = \omega_\sigma \rho_f \left\{ 1 + 3(\vec{e}_\sigma \cdot \vec{u}) + \frac{9}{2}(\vec{e}_\sigma \cdot \vec{u})^2 - \frac{3}{2}(\vec{e}_\sigma \cdot \vec{e}_\sigma) \right\} \tag{7}$$

where ρ_f is the density of the fluid, $\vec{u}$ is the velocity, $\sigma = 1$ represents the particles moving to the nearest neighbors, $\sigma = 2$ represents the particles moving to the second nearest neighbors, and $\sigma = 0$ represents the particles resting at the nodes. The weight coefficient ω_σ depends on the discrete velocity set $\vec{e}_\sigma$ and the dimensions of space. In the D3Q15 model, the discrete velocity set is

$$\vec{e}_\sigma = \begin{cases} (0,0,0), & & & \sigma = 0 \\ (\pm 1, 0, 0) & (0, \pm 1, 0) & (0, 0, \pm 1), & \sigma = 1 \\ (\pm 1, \pm 1, \pm 1), & & & \sigma = 2 \end{cases} \tag{8}$$

and the weight coefficient is

$$\omega_\sigma = \begin{cases} \frac{2}{9}, & \sigma = 0 \\ \frac{1}{9}, & \sigma = 1 \\ \frac{1}{72}, & \sigma = 2 \end{cases} \tag{9}$$

The mass density ρ_f and the momentum density $\rho_f \vec{u}$ are given by

$$\rho_f = \sum_\sigma f_\sigma, \quad \rho_f \vec{u} = \sum_\sigma f_\sigma \vec{e}_\sigma \tag{10}$$

In a widely used class of models, the kinematic viscosity ν related to the relaxation time τ for convergence is given by:

$$\nu = \frac{1}{3}\left(\tau - \frac{1}{2}\right) \tag{11}$$

In order to drive the flow, a pressure difference is imposed between the two faces normal to the axis of the superficial flow by applying a uniform body force to the fluid. The LB method is modified to account for the applied external body force, which adds a fixed amount of momentum on the fluid points at every time step (Ladd and Verberg, 2001; Guo et al., 2002):

$$f_\sigma(\vec{x}+\vec{e}_\sigma\Delta t, t+\Delta t) = f_\sigma(\vec{x},t) - \frac{1}{\tau}[f_\sigma(x,t) - f_\sigma^{eq}(x,t)] + F_\sigma \tag{12}$$

The forcing term in the present work is written as

$$\vec{F}_\sigma = \rho_f\omega_\sigma[3(\vec{e}_\sigma \cdot \vec{G})] \tag{13}$$

where ω_σ is determined by Equation (9) and $\vec{G}$ is a pressure gradient parameter. For a spatially uniform force, the higher order variation can be neglected (Ladd and Verberg, 2001) and is not considered here.

Simulation Results and Discussions

Cylindrical fibers are used to simulate the random network structure of fibrous media. The structure is generated by randomly placing every fiber into the simulation box. With this growth method, the orientation of each fiber is random on the x-y plane. In z direction, the fibers are randomly laid with an angle less than ± 15 degrees. If a fiber meets the nodes occupied by the other fibers, these fibers occupy the same nodes. The porosity is calculated by dividing the number of nodes occupied by the fibers by the total number of lattice nodes. Fiber in this simulation is 25 μm in diameterand 1 mm in length. The fibrous web is simulated at 0.1 mm in thickness or the z direction. Three different grid resolutions have been tested ($64 \times 64 \times 64$, $128 \times 128 \times 64$, $160 \times 160 \times 80$). The maximum errors are less than 4.9%, therefore the size effect can be ignored. To reduce computational load, the data reported are based on the simulation box with lattices at $128 \times 128 \times 64$. The geometry is illustrated in Figure 1.

Notice that the porosity depends on fiber length, the diameter of cylindrical fiber, and the orientation of fibers. It is evident that this structure is close to the fibrous web, e.g., paper handsheet. In this work, to achieve a porosity of 72% requires 18 fibers in random arrangement and 29 fibers for a porosity of 63%.

Fluid flows through the fiber network in z direction in order to simulate the transversal permeability. The x and y directions of the simulation box are periodical. The non-slip boundary condition is used at the fluid and fiber interfaces. The flow is induced by applying a body force on fluid particles. For a given porosity, the geometry of the fiber network is the same. There is no change of fiber positions and orientations for every different velocity. As pressure gradient increases, the velocity of flow increases.

At a certain porosity, the simulation data fit to Equation (2), and the coefficients α and β are estimated thereafter. The modified Reynolds number Re' and the friction factor f are calculated by using Equations (3) and (4).

The simulated pressure gradient versus velocity is plotted in Figure 2. As shown in the figure, curves with quadratic term of velocity fit the simulation data very well.

The curves obtained by using the LB method captured the expected tendency and the important transitions. The curve-fitting parameters α and β used in Equations (3) and (4) are given in Table I.

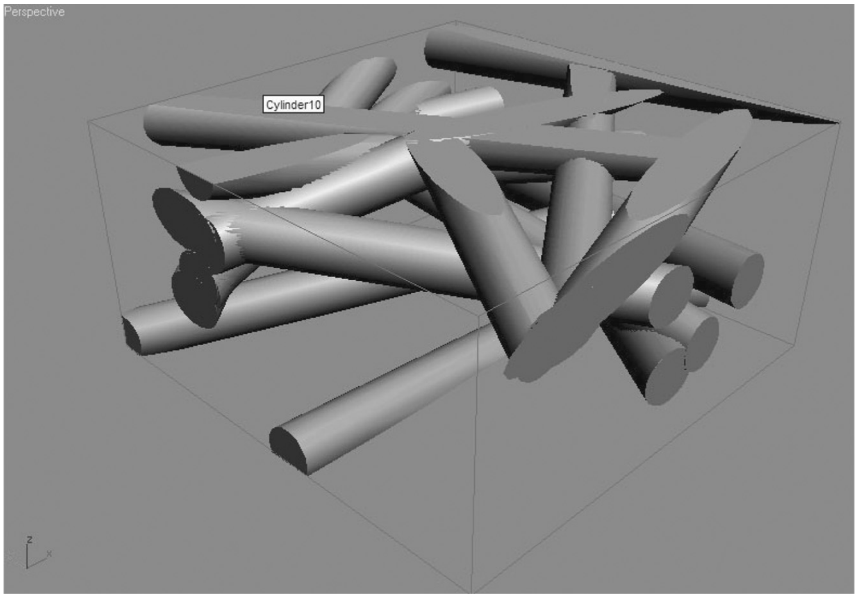

Figure 1. The geometry of simulated fiber network with 18 fibers.

As shown in the simulation results, pressure gradients versus superficial velocity curves are nonlinear after the superficial velocity reaches higher than 25 cm/s at 72% and 63% porosities. The curve of 48% porosity is more linear in that range, which

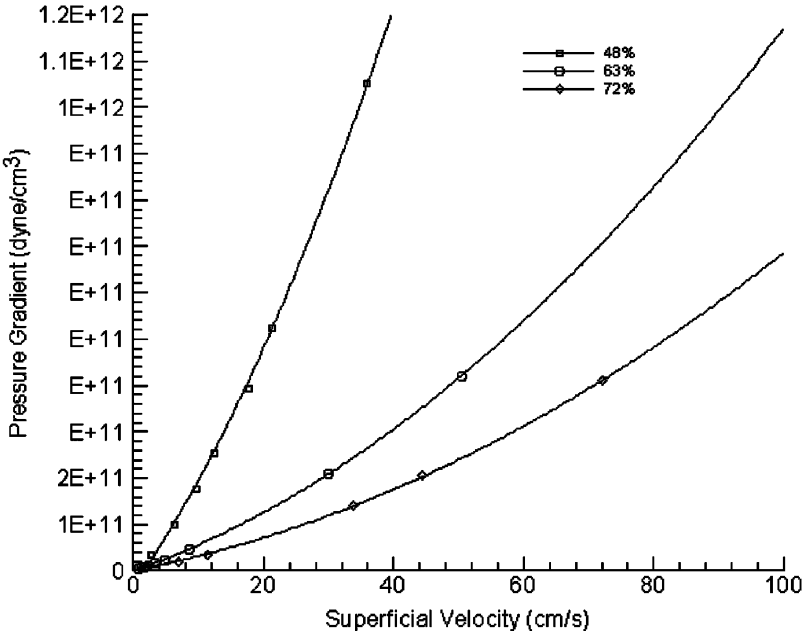

Figure 2. Quadratic curves fit the simulation data of pressure gradient versus superficial velocity.

Table I. Curve-fit parameters obtained from Equations (3) and (4) for random cylindrical fiber network

Porosity (%)	$\alpha\mu$	$\beta\rho$
48	2.028 E10	2.740 E8
63	4.926 E9	6.782 E7
72	2.680 E9	4.176 E7

proved that inertial force has less effect in the fiber network with low porosity (Koch and Ladd, 1997).

By plotting the modified friction factor f versus modified Reynolds number Re', we observed the transition zone from linear to nonlinear in terms of modified f and Re'. The curves shown in Figure 3 agree with experimental data (Bear, 1972). It is clear that the linear-to-nonlinear transition starts at Re' around 10^{-1}, which is also agrees well with the results of Andrade et al. (1999).

Forchheimer equation, dashed line is the fit to Darcy' law at low Re'.

The calculated permeability α^{-1} for a random fiber network is compared with the experimental results of transversal permeability for hardwood. The comparison is given in Figure 4. The simulation results showed good agreement with those obtained by Lindsay and Brady (1993) at porosity over 60%. A slight discrepancy may reflect the geometrical difference between the real paper and our fiber network model.

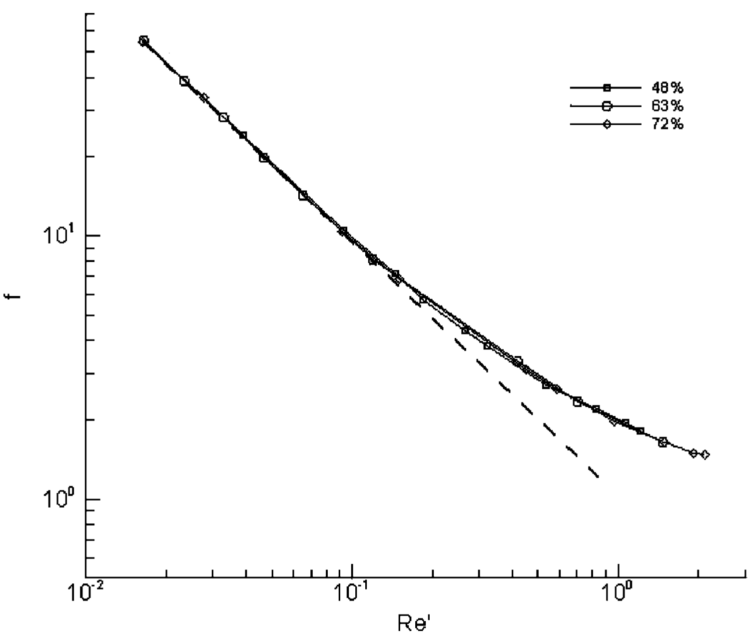

Figure 3. Modified friction factor and Reynolds number show the transaction zone from linear to nonlinear of random fiber network. Solid lines are the fit to the Forchheimer equation, dashed line is the fit to Darcy's law at low Re'.

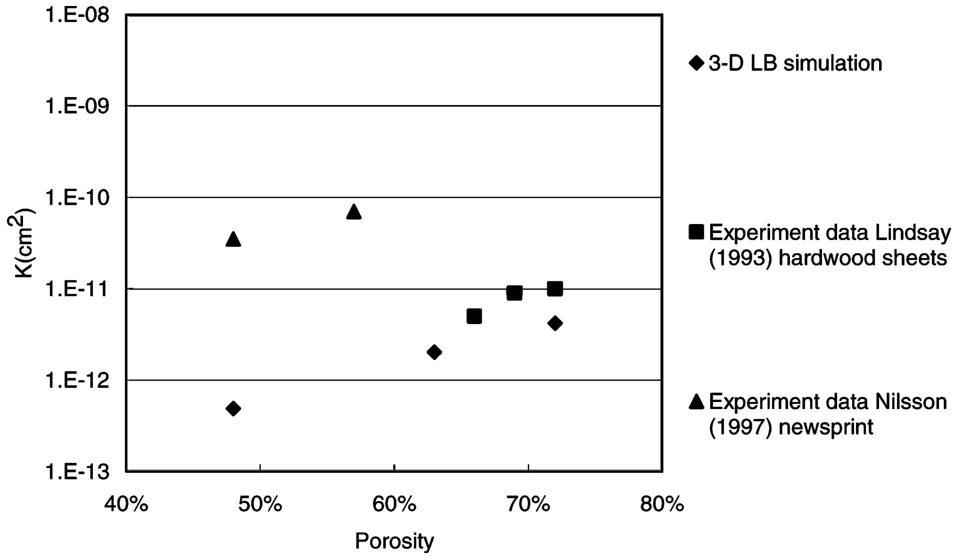

Figure 4. Comparison of permeabilities of three-dimensional LB simulation and experimental data.

The further conclusion to be drawn from the data in Figure 4 is that the permeability calculated with the quadratic velocity term has reasonable accuracy in predicting the permeability of a paper fiber network. The fiber network model we built estimated well the fluid transportation of hardwood sheets.

Conclusions

In this article, we report the numerical study of the nonlinear behavior of fluid flowing through a three-dimensional random fiber network in the porosity range from 48% to 72% by the LB method. We found that the curves of pressure gradient versus superficial velocity are nonlinear after the superficial velocity reaches higher than 25 cm/s at 72% and 63% porosities. The curve at 48% porosity is more linear. It is shown that the inertial effect is important at relatively high Reynolds numbers. The relation between the modified Reynolds number and the friction factor are in excellent agreement with the Forchheimer equation. The results of permeability in the fiber network have good agreement with the experimental data.

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