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A nonlinear threshold model applied to spallation analysis

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Abstract

Spallation in heterogeneous media is a complex, dynamic process. Generally speaking, the spallation process is relevant to multiple scales and the diversity and coupling of physics at different scales present two fundamental difficulties for spallation modeling and simulation. More importantly, these difficulties can be greatly enhanced by the disordered heterogeneity on multi-scales. In this paper, a driven nonlinear threshold model for damage evolution in heterogeneous materials is presented and a trans-scale formulation of damage evolution is obtained. The damage evolution in spallation is analyzed with the formulation. Scaling of the formulation reveals that some dimensionless numbers govern the whole process of deformation and damage evolution. The effects of heterogeneity in terms of Weibull modulus on damage evolution in spallation process are also investigated.

Keywords: Driven nonlinear threshold model; Relaxation time of damage; Spallation; Deborah number

1. Introduction

The failure of heterogeneous media under loading is a typical multiscale coupling process in far-from equilibrium systems. Let us take spallation as an example. Firstly, the spallation process incorporates a wide range of spatial and temporal scales. The dynamics on various scales differs from each other [1,2]. Secondly, spallation is a process far from equilibrium. The dynamics at various scales can be excited simultaneously, and there may be strong or sensitive coupling among them. In addition, the dynamics involved in the process are often nonlinear in both time and stress [2,3]. Thirdly, at mesoscopic scales, most engineering materials are disordered and heterogeneous. During spallation process, the effects of some disordered structures at mesoscopic scales can be amplified significantly, and become important at macroscopic scale, which makes the development of predictive models for spallation analysis particularly challenging [4].

There have been various efforts to formulate this multi-scale process, such as the integral criterion, continuum measure of spallation, microstatistical fracture mechanics, etc. [1,2,5,6]. These studies provide progressively helpful means to reveal the essence of spalling. In recent 10 years, some new and informative studies relevant to spallation are made to have deeper understanding of the process [7–14]. All these work intended to link the macroscopic spallation

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and the microdamage evolution inside materials. However, there is still an acute lack of quantitative/predictive models based on the dynamics of spallation [15].

In principle, the problem of spallation can be represented by a statistical approach linking microscopic scale and macroscopic scale. However, it is difficult to represent non-equilibrium statistical evolution in a statistical approach linking microscopic and macroscopic scales due to the huge span of the scale. In addition, there are not simple direct connections between microscopic and macroscopic features in the process. Furthermore, a noticeable feature in the problem is the richness of structures and processes at mesoscopic scale. These mesoscale structures, such as grains, microcracks, etc. play significant role in the problem. Hence, a rational approach is to develop a statistical approach linking mesoscopic and macroscopic scales. Such a theory is called statistical mesoscopic damage mechanics. Statistical mesoscopic damage mechanics can be constructed based on various mesoscopic representations, e.g., mesoscopic damage representation, and mesoscopic unit representation. In terms of the mesoscopic damage representation, one can properly deal with the two phases of damage accumulation, i.e., globally stable accumulation of microdamage and damage localization [2]. In this paper, we present a model based on mesoscopic unit representation, which is called driven nonlinear threshold model [16].

2. Driven nonlinear threshold model

We consider a macroscopic representative volume element (RVE) (at $\mathbf{x}$) comprised of a great number of interacting, nonlinear, mesoscopic units, that is, a driven nonlinear threshold model [17]. The heterogeneity of the mesoscopic units can be characterized by their broken threshold. The mesoscopic units are assumed to be statistically identical, and their threshold σ_c follows a statistical distribution function $\varphi(\sigma_c, t, \mathbf{x})$.

The RVE is subjected to nominal driving force $\sigma_0(t, \mathbf{x})$, which is adopted as macroscopic variable. In the RVE, a unit will have probability to break as the real driving force (true stress) $\sigma(t, \mathbf{x})$ on it becomes higher than its threshold. When a unit breaks, it will be excluded from the distribution function. Hence, we introduce a time-dependent distribution function of intact units $\varphi(\sigma_c, t, \mathbf{x})$ with initial condition

$$\varphi(\sigma_c, t = 0, \mathbf{x}) = h(\sigma_c), \tag{1}$$

where $h(\sigma_c)$ is normalized as

$$\int_0^\infty h(\sigma_c)\mathrm{d}\sigma_c = 1. \tag{2}$$

In Eq. (2), σ_c is non-dimensionalized and normalized by a parameter σ^*, the characteristic value of σ_c.

We adopt local mean field approximation (LMFA), i.e., all intact units in the RVE support identical driving force (true stress) $\sigma(t, \mathbf{x})$. That is, the real driving force (true stress) $\sigma(t, \mathbf{x})$ applied on intact units is determined by

$$\sigma(t, \mathbf{x}) = \frac{\sigma_0(t, \mathbf{x})}{1 - D(t, \mathbf{x})}, \tag{3}$$

where $D(t, \mathbf{x})$ is continuum damage of the RVE at time t and is defined as

$$D(t, \mathbf{x}) = 1 - \int_0^\infty \varphi(\sigma_c, t, \mathbf{x})\mathrm{d}\sigma_c. \tag{4}$$

The evolution of distribution function $\varphi(\sigma_c, t, \mathbf{x})$ is suggested to follow an equation based on relaxation time model:

$$\frac{\partial\varphi(\sigma_c, t, \mathbf{x})}{\partial t} = -\frac{\varphi(\sigma_c, t, \mathbf{x})}{\tau}, \tag{5}$$

where τ is the characteristic relaxation time of damage. Generally speaking, τ is a function of the true driving force $\sigma(t, \mathbf{x})$ and the threshold σ_c of mesoscopic units, $\tau = \tau(\sigma_c, \sigma(t, \mathbf{x}))$.

Integrating Eq. (5) and substituting the definition of continuum damage (Eq. (4)) to the obtained equation, we obtain the evolution equation of continuum damage:

$$\begin{aligned}\frac{\mathrm{d}D(t, \mathbf{x})}{\mathrm{d}t} = f &= -\int_0^\infty \frac{\partial\varphi(\sigma_c, t, \mathbf{x})}{\partial t}\mathrm{d}\sigma_c \\ &= \int_0^\infty \frac{\varphi(\sigma_c, t, \mathbf{x})}{\tau(\sigma_c, \sigma(t, \mathbf{x}))}\mathrm{d}\sigma_c,\end{aligned} \tag{6}$$

where f is the dynamic function of damage (DFD), the agent linking mesoscopic microdamage relaxation and macroscopic damage evolution.

Similar to [2], in order to establish a closed, complete formulation, Eq. (6) should be associated with traditional, macroscopic equations of continuum, momentum, and energy, as well as constitutive relationship. This is a formulation with intrinsic trans-scale closure. However, it is worth noticing that in the constitutive relationship, the effects of microdamage should be taken into account as a reduction in the elastic modulus:

$$E(\mathbf{x}, t) = E_0(\mathbf{x})(1 - D(\mathbf{x}, t)), \tag{7}$$

where E_0 is the elastic modulus of intact media, E the effective elastic modulus of damaged media. In addition, the stress appeared in traditional macroscopic equations is nominal stress denoted by $\sigma_0(\mathbf{x},t)$, while the stress in mesoscopic dynamics equation (Eq. (4)) is the true driving stress $\sigma(\mathbf{x},t)$.

With the abovementioned formulation, we numerically investigated the process of spallation and analyzed the effects of microdamage relaxation time and Weibull modulus on the propagation of damage.

3. Numerical analysis of spallation

Consider a problem of damage evolution owing to the normal impact of a flying plate of thickness L with velocity v_{f} striking on a target plate, i.e., spallation. For simplicity, we assume that the impactor-plate system deforms in uniaxial strain. For the time-dependent damage process, an associated equations of continuum, momentum and damage evolution should be formed. In one-dimensional strain state, these are

$$\frac{\partial \varepsilon}{\partial T} = \frac{\partial v}{\partial X} \tag{8}$$

$$\rho_0 \frac{\partial v}{\partial T} = \frac{\partial \sigma}{\partial X} \tag{9}$$

$$\frac{\partial D(T,X)}{\partial T} - \frac{v}{1+\varepsilon}\frac{\partial D(T,X)}{\partial X} = f = \int_0^\infty \frac{\varphi(\sigma_{\mathrm{c}},t,\mathbf{x})}{\tau(\sigma_{\mathrm{c}},\sigma(t,\mathbf{x})}) \mathrm{d}\sigma_{\mathrm{c}}, \tag{10}$$

where ρ_0 is density of intact material and v the velocity of RVE. It is noticeable that the damage evolution equation Eq. (6) is written in Eulerian coordinate system (t, x), while Eq. (10) in Lagrangian coordinate system (T, X). The transformation between the two systems is $\frac{\partial}{\partial t} + v\frac{\partial v}{\partial X} = \frac{\partial}{\partial T}$ and $\frac{\partial}{\partial x} = \frac{1}{1+\varepsilon}\frac{\partial}{\partial X}$.

For simplicity, the Al alloy is assumed to be an elastic material in the simulation. The constitutive equation is:

$$\mathrm{d}\sigma = E_0(1-D)\mathrm{d}\varepsilon - E_0\varepsilon\mathrm{d}D, \tag{11}$$

where E_0 is the elastic modulus of the sample in uniaxial strain state and the acoustic speed in intact material is $a, \rho_0 a^2 = E_0$.

Similar to Weibull's statistical strength theory [18], we suppose that the initial distribution of threshold $h(\sigma_{\mathrm{c}})$ follows Weibull distribution:

$$h(\sigma_{\mathrm{c}}) = m\frac{(\sigma_{\mathrm{c}}-\sigma^*)^{m-1}}{(\sigma^*)^m}\exp\left[-\left(\frac{\sigma_{\mathrm{c}}-\sigma^*}{\sigma^*}\right)^m\right], \tag{12}$$

where m is the Weibull modulus and σ^* the characteristic value of σ_{c}. Fig. 1 shows typical distributions expressed by Eq. (12). The smaller Weibull modulus (m), the broader the distribution becomes, and the material is more heterogeneous. On the other hand, larger m value represents a homogeneous material in which the stress threshold is almost constant.

There are various ways to determine the relaxation time of damage τ. For example, we may assume that

$$\tau = \begin{cases} \infty, & \text{as } \sigma(t,\mathbf{x}) < \sigma_{\mathrm{c}}, \\ \tau_{\mathrm{D}}\left(\frac{\sigma(t,\mathbf{x})}{\sigma_{\mathrm{c}}}\right)^{-q}, & \text{as } \sigma_{\mathrm{M}} \geqslant \sigma(t,\mathbf{x}) \geqslant \sigma_{\mathrm{c}}, \\ 0, & \text{as } \sigma(t,\mathbf{x}) > \sigma_{\mathrm{M}}. \end{cases} \tag{13}$$

Model defined as Eq. (13) implies that the damage relaxation can be characterized by three time scales:

(1) for low driving force, $\sigma < \sigma_{\mathrm{c}}$, the damage appears as a very slow relaxation process, and we simply assume $\tau \to \infty$;
(2) for intermediate driving force, $\sigma_{\mathrm{c}} \leqslant \sigma < \sigma_{\mathrm{M}}$, the damage relaxation can be described by a finite relaxation time, which depends on $\sigma(t,\mathbf{x})/\sigma_{\mathrm{c}}$ and q is a positive parameter. τ_{D} is the characteristic relaxation time;
(3) for very high driving force, $\sigma \geqslant \sigma_{\mathrm{M}}$, the damage evolution becomes a very fast relaxation process, and the relaxation time is nearly zero, which is corresponding to the catastrophic rupture. In addition, it also means a cutoff of distribution function $h(\sigma_{\mathrm{c}})$ at $\sigma_{\mathrm{c}} = \sigma_{\mathrm{M}}$.

We can further simplify the model by assuming the exponent in the second equality of Eq. (13) to

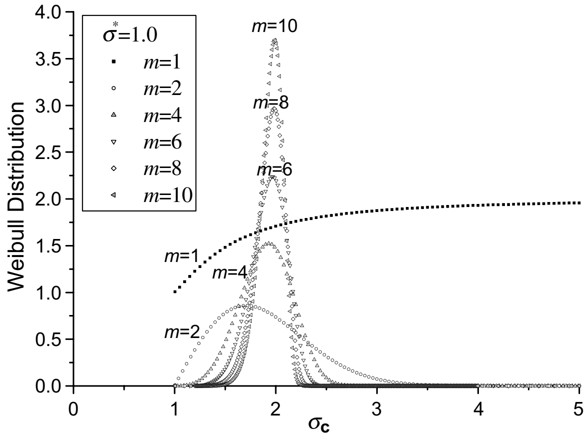

Fig. 1. Distributions expressed as Eq. (13).

be zero, that is, if the true driving force satisfy $\sigma_M \geqslant \sigma(t, \mathbf{x}) \geqslant \sigma_c$, the relaxation time of microdamage is constant as τ_D.

Due to the trans-scale nature of spallation, it is helpful to non-dimensionalize the variables in Eqs. (8)–(11). Then the dimensionless equations are:

$$\frac{\partial \bar{\varepsilon}}{\partial \overline{T}} = M \frac{\partial \bar{v}}{\partial \overline{X}} \tag{14}$$

$$\frac{\partial \bar{v}}{\partial \overline{T}} = S \frac{\partial \bar{\sigma}}{\partial \overline{X}} \tag{15}$$

$$\frac{\partial D}{\partial \overline{T}} + M\bar{v}\frac{\partial D}{\partial \overline{X}} = \bar{f} = \int_0^\infty \frac{\varphi a}{\tau L} \mathrm{d}\sigma_c \tag{16}$$

$$\mathrm{d}\bar{\sigma} = (1 - D)d\bar{\varepsilon} - \bar{\varepsilon}\mathrm{d}D. \tag{17}$$

In Eqs. (14)–(17), the non-dimensionalized variables are: $\overline{T} = Ta/L$, $\overline{X} = X/L$, $\bar{v} = v/v_f$, $\bar{\varepsilon} = \varepsilon/\varepsilon^*$, $\bar{\sigma} = \sigma/\sigma^*$ and $\sigma^* = E_0\varepsilon^*$; and the dimensionless numbers are:

$$\text{Mach number: } M = \frac{v_f}{a\varepsilon^*}, \tag{18}$$

$$\text{Damage number: } S = \frac{\sigma^*}{\rho_0 a v_f}. \tag{19}$$

The ratio of characteristic microdamage relaxation time τ_D over the lasting time of imposed stress pulse forms another dimensionless number, Deborah number De^*:

$$De^* = \frac{\tau_D}{L/a}. \tag{20}$$

The effects of De^* and m on the damage evolution in the target plate are studied.

4. Results and discussion

All other parameters (M, S, m) fixed, we change the Deborah number De^* in the simulation. The effect of Deborah number De^* on the evolution of damage in the target plate is shown in Fig. 2. In these calculations, Mach number M, Damage number S and Weibull modulus m remain unchanged for all curves in Fig. 2. Obviously, the maximum damage in the target plate increases with decreasing De^*. Therefore, the decrease of De^* speeds up the process of microdamage evolution.

Physically speaking, there are two kinetic processes involved in spallation. They are: the macroscopic impact loading, and microdamage relaxation process. As shown in Eq. (20), De^* is the characteristic time ratio of the two processes. Therefore, De^* represents the competition and coupling between the microdamage relaxation process and the macroscopic impulse loading process. For a given impact loading time, smaller De^* means shorter microdamage relaxation time, that is, the microdamage evolves

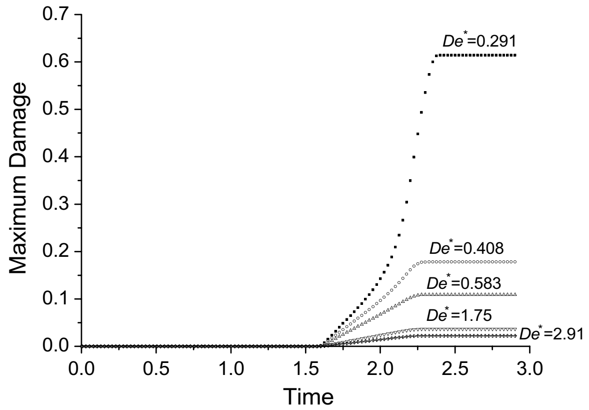

Fig. 2. Effects of De^* on damage evolution. ($M = 3.30$, $S = 0.303$, $m = 5.0$).

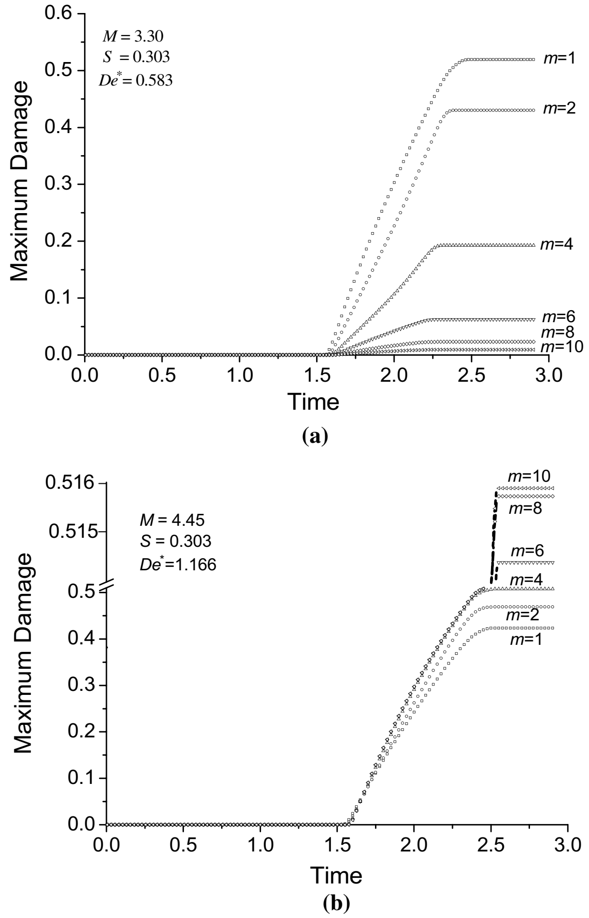

Fig. 3. Effects of m on damage evolution. (a) $M = 3.30$, $S = 0.303$, $De^* = 0.583$ and (b) $M = 4.45$, $S = 0.303$, $De^* = 1.166$.

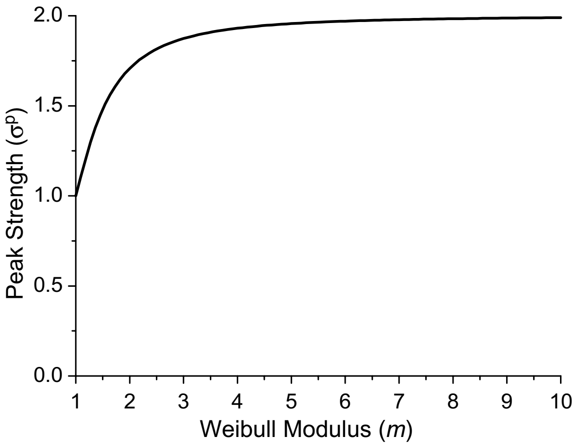

Fig. 4. Variation of σ^p with m.

faster. Therefore, smaller De^* results in more damage in the target plate.

Fixing all other parameters, we studied the effects of the Deborah number De^* on the damage evolution by changing the Weibull modulus as $m = 1, 2, 4, 6, 8, 10$. However, the effects of Weibull modulus m is not so straightforward as that of De^*. Fig. 3(a) and (b) show the effects of m on the maximum damage in target plate at two different stresses. In Fig. 3(a), the increase of m leads to smaller damage. While in Fig. 3(b), the maximum damage increase with an increase in m.

Then, what caused the opposite effects of m on damage evolution?

Fig. 4 illustrates the variation of the threshold with peak density σ^p with the Weibull modulus m. Obviously, when m varies from 1 to 10, σ^p only varies slightly. The stress in Fig. 3(a) is below the mean. In these cases, a smaller m implies a broader threshold distribution, and physically represents a more heterogeneous material. Therefore, there are more mesoscopic units failing at low stress levels, and ultimately induces a higher macroscopic damage, as shown in Fig. 3(a). However, for stress above σ^p (Fig. 3(b)), in cases with a smaller m, there are more mesoscopic units survive at high stress levels, and therefore, induces smaller damage in the target plate, as shown in Fig. 2.

5. Summary

A driven nonlinear threshold model for damage evolution was presented and a trans-scale formulation of damage evolution is obtained. With the presented model, the spallation in an Al plate is studied numerically. The study shows that Mach number M, damage number S, Deborah numbers De^* and Weibull modulus m govern the damage evolution process in the target plate. The decrease of De^* accelerates damage evolution in the target plate. At low stress levels, smaller m induces higher damage, while at high stress levels, higher m results in more damage in the target plate.

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