On the Definition of Coefficient of Strain-rate Sensitivity

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Abstract: An attempt is made to elaborate the definition of the coefficient of strain-rate sensitivity, which is a fundamental concept for macroscopic dynamic plasticity and dislocation dynamics. Since a solid subjected to large dynamic loading usually undergoes dynamic response with finite deformation, therefore, the definition of this coefficient is also put forward to the case of large dynamic plastic deformation. The analogy between internal friction increment and overstress due to strain-rate sensitivity is clarified.

Key words: plastic strain rate; strain-rate sensitivity; overstress; dislocation dynamics; finite deformation; internal friction

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摘　要：应变率敏感系数是宏观塑性动力学和位错动力学的一个基本概念，本文试图对其进行了重新解释。由于强动载荷作用下一般固体要发生有限变形的动力响应，还将应变率敏感系数推广到塑性动力大变形情形。本文也指出了由于应变率敏感性所引起的内耗增量和过应力的类比性。

关键词：塑性应变率；应变率敏感；过应力；位错动力学；有限变形；内耗

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Experimental evidence shows that plastic deformation of metals and their alloys is fundamentally rate (time) dependent. The major difference between dynamic plastic and quasi-static plastic behavior of materials is that both inertia and strain-rate effects are no longer neglected for large intense loading[1~4]. The strain-rate effect is of importance for the dynamic plastic analysis of strain-rate sensitive (or rate-dependent) materials such as mild steel, titanium alloy, OFHC copper and so forth. When these materials are subjected to external intensive loadings, their dynamic yield stresses are much larger than their static ones[2~3]; the difference between the dynamic stress and the corresponding stress point on the quasi-static surface is called overstress. Most of the theories that describe the transient and steady-state behavior of metallic alloys make the inelastic strain rate a function of the over (effective) stress[5].

As a matter of fact, the rate-dependent phenomenon is historically related to material internal friction. The rate dependence of tin was investigated by Ludwik[6] and he found that the maximum tensile strength was dependent on the strain rate. From these experimental observations he deduced that the current internal friction R must be composed of a base internal friction R_0 plus a contribution which depends on the tensile velocity v such that[5,6]

$$R = R_0 + kv^{1/n} \tag{1}$$

where k and n are material constants. Actually, Eq.(1) is also the relationship between internal friction and strain rate since [1]

$$\dot{\varepsilon} = v / l_0 \tag{2}$$

where l_0 is the initial length of the specimen. Then, the overstress corresponds to the difference of material internal frictions. As discussed in Ref.[1] and illustrated in Eq.(2), the strain-rate dependence has an intrinsic length scale effect.

Dynamic fracture of most materials is also rate dependent[7]. It is generally agreed by experiments that the dynamic fracture toughness of materials

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under small-scale yielding (SSY) decreases with the increasing strain rate; the fracture in this case is one of cleavage in nature. On the other hand, the dynamic fracture toughness of materials under large-scale yielding (LSY) increases with the increasing strain rate, resulting in a fibrous fracture. There is a linear relationship between strain rate $\dot{\varepsilon}$ and stress intensity factor rate $\dot{K}_I$ for linear elastic material fracture, *i.e.*,

$$\dot{\varepsilon} = k\frac{\sigma_y}{EK_I}\dot{K}_I \quad (3)$$

where E and σ_y are Young's modulus and quasi-static yield stress, respectively; K_I is the mode I stress intensity factor; k is a material constant reflecting intensification for strain singularity[7]. For nonlinear elastic material fracture, Eq.(3) is replaced by

$$\dot{\varepsilon} = c\frac{\dot{J}}{J} \quad (4)$$

where c is a parameter having different values in plastic zone or in process zone; J and $\dot{J}$ are J-integral and integral rate.

1 Definition of Coefficient of Strain-rate Sensitivity

As a characterization of the parameter for material strain-rate sensitivity in plastic dynamics, the coefficient (or exponent) of strain-rate sensitivity is commonly defined as[8,9]

$$\lambda = \frac{\partial\sigma}{\partial(\ln\dot{\varepsilon}^{\mathrm{p}})} \quad (5a)$$

where σ is the dynamic Cauchy stress (or true stress), and $\dot{\varepsilon}^{\mathrm{p}}$ is the dynamic plastic strain rate. For dynamic shear deformation with shear strain rate $\dot{\gamma}$ and shear stress τ, one has under constant temperature

$$\lambda = \left(\frac{\partial\tau}{\partial\ln\dot{\gamma}}\right)_T = -\left(\frac{1}{\kappa T}\frac{\partial U}{\partial\tau}\right)^{-1} \quad (5b)$$

where U is the activation energy; κ is the Boltzmann's constant. At a constant strain rate, $\lambda = (\partial\tau/\partial(\ln\dot{\gamma}))_{\dot{\gamma}}$ increases for the increasing stress or strain levels[9] over the strain rate region higher than $10^{-4}\ \mathrm{s}^{-1}$. It should be noted that Eq.(5a) and Eq.(5b) are for uniaxial deformation. For multi-axial plastic deformation, on the other hand, one should use the equivalent stress and equivalent plastic strain rate in Eqs.(5a) and (5b), *i.e.*,

$$\sigma_{\mathrm{equ}} = \sqrt{3\sigma_{ij}\sigma_{ij}/2}\ ,\ \dot{\varepsilon}^{\mathrm{p}}_{\mathrm{equ}} = \sqrt{2\dot{\varepsilon}^{\mathrm{p}}_{ij}\dot{\varepsilon}^{\mathrm{p}}_{ij}/3} \quad (5c)$$

In dislocation dynamics, the corresponding coefficient of strain-rate sensitivity is usually defined as[10]

$$\lambda = \frac{\partial\tau_{\mathrm{eff}}}{\partial(\ln v)} \quad (6a)$$

or[11]

$$\lambda = \frac{\partial\tau_{\mathrm{eff}}}{\partial(\ln\dot{\gamma})} \quad (6b)$$

where v and $\dot{\gamma}$ are dislocation velocity and shear strain rate, respectively. The effective shear stress driving the dislocation is the difference between external applied shear stress τ_{ext} and the back stress τ_{back}, *i.e.*,

$$\tau_{\mathrm{eff}} = \tau_{\mathrm{ext}} - \tau_{\mathrm{back}} \quad (7)$$

The dislocation velocity and the strain rate can be linked by the famous Orowan equation

$$\dot{\gamma} = \frac{1}{2}\rho_{\mathrm{m}}vb \quad (8)$$

where ρ_{m} and b are mobile dislocation density and Burgers vector of dislocation, respectively. It is seen that Eqs.(6a) and (6b) are physically equivalent. If forest dislocations are the prevailing sources of long-range interactions, the back stress τ_{back} is given by the famous Taylor formula

$$\tau_{\mathrm{back}} = \alpha\mu b\sqrt{\rho_{\mathrm{f}}} \quad (9)$$

where ρ_{f} is the forest dislocation density, μ is the elastic shear modulus, and α is a numerical coefficient. If, as another example, the back stress is mainly due to dislocation dipoles with a density ρ_{d} and a mean dipole width y_{d}, one has

$$\tau_{\mathrm{back}} = \alpha'\mu b y_{\mathrm{d}}\rho_{\mathrm{d}} \quad (10)$$

It is especially noted that the definitions in Eqs.(5a), (6a) and (6b) are dimensionally invalid, since $[\dot{\varepsilon}^{\mathrm{p}}] = [\dot{\gamma}] = \boldsymbol{T}^{-1}$, and $[v] = \boldsymbol{LT}^{-1}$. Here $[\cdots]$ means the dimension of a physical quantity. What is more, the coefficient of strain-rate sensitivity defined by Eqs.(5a)~(6b) has the dimension of stress, namely MPa (for comparatively small strain-rate sensitivity f.c.c. metals, $\lambda \approx 0.03 \sim 0.3$ MPa[11]; for strongly rate sensitive b.c.c. metals, $\lambda \approx 1 \sim 10$ MPa), which is hard to understand from its original definition. As a result, it is worthy to discuss the classical definition of the coefficient of strain-rate sensitivity in detail.

2 Reconsideration of Coefficient of Strain-rate Sensitivity

Eq.(5a) is equivalent to

$$\lambda = \lim_{\dot{\varepsilon}_2^p \to \dot{\varepsilon}_1^p} \frac{\Delta \sigma}{\ln \dot{\varepsilon}_2^p - \ln \dot{\varepsilon}_1^p} \tag{11}$$

which is actually

$$\lambda = \lim_{\dot{\varepsilon}_2^p \to \dot{\varepsilon}_1^p} \frac{\sigma(\dot{\varepsilon}_2^p) - \sigma(\dot{\varepsilon}_1^p)}{\ln(\dot{\varepsilon}_2^p / \dot{\varepsilon}_1^p)} \tag{12}$$

It is noted that Eq.(12) is dimensional valid. The dimension of the coefficient defined by Eq.(14) is that of stress. Eq.(12) can be practically adapted as

$$\lambda = \frac{\sigma(\dot{\varepsilon}_2^p) - \sigma(\dot{\varepsilon}_1^p)}{\ln(\dot{\varepsilon}_2^p / \dot{\varepsilon}_1^p)} \tag{13}$$

Here $\sigma(\dot{\varepsilon}_2^p) - \sigma(\dot{\varepsilon}_1^p)$ is the overstress. Both Eqs.(6a) and (6b) can be rewritten in the same manner.

It is noted that strain-rate sensitivity is also important for creep and superplasticity of metals. The corresponding coefficient of strain-rate sensitivity in creep is usually defined as[12,13]

$$m = \frac{\partial(\ln \sigma)}{\partial(\ln \dot{\varepsilon})} \tag{14}$$

which can be understood as

$$m = \frac{\ln(\sigma_2 / \sigma_1)}{\ln(\dot{\varepsilon}_2 / \dot{\varepsilon}_1)} \tag{15}$$

Superplasticity happens when[13] $0.3 \le m \le 1$. m is obviously a dimensionless number.

3 Finite Deformation

When subjected to large dynamic loading, a solid usually undergoes dynamic response with finite deformation. Therefore, it is worthy to extend the definition of the coefficient of strain-rate sensitivity to the case of finite deformation.

3.1 The second P-K stress tensor and Green strain tensor

For finite deformation, Eq.(12) can be rewritten as

$$\lambda = \frac{T(\dot{E}_2^p) - T(\dot{E}_1^p)}{\ln(\dot{E}_2^p / \dot{E}_1^p)} \tag{16}$$

where T is the second Piola-Kirchoff stress, and E is the Green strain. T and E are work conjugate pair. T is related to Cauchy (true) stress through [14]

$$\boldsymbol{T} = J\boldsymbol{F}^{-1} \cdot \boldsymbol{\sigma} \cdot \boldsymbol{F}^{-\mathrm{T}} \tag{17}$$

where $\boldsymbol{F}$ is the deformation gradient tensor, $J = \det(F_{ij})$. The plastic Green strain rate tensor is

$$\dot{\boldsymbol{E}}^p = \frac{1}{2}\left(\boldsymbol{F}^{p^\mathrm{T}} \cdot \dot{\boldsymbol{F}}^p + \dot{\boldsymbol{F}}^{p^\mathrm{T}} \cdot \boldsymbol{F}^p\right) \tag{18}$$

The plastic deformation gradient tensor $\boldsymbol{F}^p$ and the elastic deformation gradient tensor $\boldsymbol{F}^e$ is related to the deformation gradient $\boldsymbol{F}$ by the famous E.H. Lee's multiplicative decomposition theorem (1969)

$$\boldsymbol{F} = \boldsymbol{F}^e \cdot \boldsymbol{F}^p \tag{19}$$

And $\dot{(\ )}$ means the material time derivative. Lee's decomposition is not unique; Clifton[15] discussed the equivalence of Eq.(19) and the inverse decomposition $F = \bar{F}^p \cdot \bar{F}^e$, which is found to be slightly more convenient than Eq.(19) for analysis of one-dimensional wave propagation in elastic/viscoplastic solids.

3.2 Kirchoff stress tensor and plastic deformation rate tensor

Since the first Piola-Kirchoff stress tensor $\boldsymbol{\tau}$ and deformation rate tensor $\boldsymbol{d}$ are work conjugate pair, then the coefficient of strain-rate sensitivity can be defined as

$$\lambda = \frac{\tau(\dot{d}_2^p) - \tau(\dot{d}_1^p)}{\ln(\dot{d}_2^p / \dot{d}_1^p)} \tag{20}$$

The first P-K stress tensor is related to Cauchy stress tensor through

$$\tau = J\sigma \tag{21}$$

and the plastic deformation rate tensor is

$$\boldsymbol{d}^p = \frac{1}{2}\left(\boldsymbol{F}^e \cdot \bar{\boldsymbol{L}}^p \cdot \overset{-1}{\boldsymbol{F}^e} + \overset{-\mathrm{T}}{\boldsymbol{F}^e} \cdot \bar{\boldsymbol{L}}^{p^\mathrm{T}} \cdot \boldsymbol{F}^{e^\mathrm{T}}\right) \tag{22}$$

where

$$\bar{\boldsymbol{L}}^p = \dot{\boldsymbol{F}}^p \cdot \overset{-1}{\boldsymbol{F}^p} \tag{23}$$

is the plastic velocity gradient in the unloading configuration.

4 Discussion and Summary

The advantage of Eq. (13) over Eq. (5a) is two-fold: (1) It is dimensional valid, and (2) it is convenient for experimental application. The dimension of the coefficient of strain-rate sensitivity is that of stress, and this feature can be reflected by another expression

$$\lambda = \frac{\partial\sigma}{\partial(\ln\dot{\varepsilon})} = \dot{\varepsilon}\frac{\partial\sigma}{\partial\dot{\varepsilon}} \tag{24}$$

Johnson and Cook presented a dynamic constit utive equation in 1983 as follows[16]

$$\sigma = (A + B\varepsilon^n)(1 + C\ln\dot{\varepsilon}^*)(1 - T^{*m}) \tag{25}$$

where A, B, C, n, and m are yield strength, work hardening coefficient, work hardening exponent, strain rate sensitivity and thermal coefficient; ε is the equivalent plastic strain; $\dot{\varepsilon}^* = \dot{\varepsilon}/\dot{\varepsilon}_0$ is the dimensionless plastic strain rate taking $\dot{\varepsilon}_0 = 1\,\mathrm{s}^{-1}$; and $T^* = (T - T_r)/(T_m - T_r)$ is the homologous temperature, with T being the immediate absolute temperature of the deformed specimen, T_r and T_m being the room temperature and the melting point, respectively.

If the dimensionless temperature T^* and the equivalent plastic strain ε are kept invariable, the coefficient of strain-rate sensitivity is

$$\lambda = \frac{\partial\sigma}{\partial(\ln\dot{\varepsilon})}\Big|_{T,\varepsilon} = C(A + B\varepsilon^n)(1 - T^{*m})\Big|_{T,\varepsilon} \tag{26}$$

The dynamic behavior of most metallic materials can be modeled by the Cowper-Symonds constitutive equation as follows

$$\sigma_0'/\sigma_0 = 1 + (\dot{\varepsilon}/D)^{1/q} \tag{27}$$

where σ_0' and σ_0 denote, respectively, the dynamic and static yield stress at a uniaxial plastic strain rate $\dot{\varepsilon}$; D and q are constants for a particular material. The coefficient of strain-rate sensitivity for Cowper-Symonds dynamic constitutive equation is found to be

$$\lambda = \frac{\sigma_0}{q}\left(\frac{\dot{\varepsilon}}{D}\right)^{1/q} \tag{28}$$

It is noted from a comparison between Eqs.(26) and (28) that the coefficient of strain-rate sensitivity for Johnson-Cook constitutive equation does not depend on the strain rate for given strain and temperature as Cowper-Symonds equation. Some constants in Eq.(27) or Eq.(28) for several typical materials are listed in Table 1.

Table 1 Coefficients in Cowper-Symonds constitutive equation for some typical materials

Material	D/s^{-1}	q
Mild steel	40.4	5
Aluminum alloy	6500	4
α-Titanium(Ti 50A)	120	9
Stainless steel	100	10

It is interesting to note that there is an analogy between internal friction and overstress. This can be clearly seen by comparing Eqs.(1) and (27), *i.e.*,

$$R - R_0 = k(l_0\dot{\varepsilon})^{1/n} \tag{29a}$$

$$\sigma - \sigma_0 = \sigma_0\left(\frac{\dot{\varepsilon}}{D}\right)^{1/q} \tag{29b}$$

which holds for uniaxial tension. Eq.(2) has been used for derivation of Eq.(29a). From Eqs.(29a) and (29b) one has

$$R - R_0 \sim (\sigma - \sigma_0)^{q/n} \tag{30}$$

Eq.(29a) means that there exists a direct relationship between strain rate and internal friction, and internal friction increment due to strain-rate sensitivity is analogous to overstress.

There exist other kinds of definitions for strain-rate sensitivity. For instance, Hsu and Clifton[17] defined a characteristic time parameter for consideration of influence of rate-dependent plastic flow on plastic wave propagation

$$t_c = \frac{\partial\sigma/\partial\dot{\varepsilon}^p}{\partial\sigma/\partial\varepsilon} \tag{31}$$

The above parameter is a measure of the relative importance of strain-rate sensitivity and strain-hardening in determining changes in flow stress due to changes in strain-rate and strain. Large values of t_c suggest that rate dependence of the flow stress will influence wave profiles for longer time after impact, so that for large t_c it is more likely that a rate-dependent theory will be required to explain observed profiles.

To summarize, this paper attempts to elaborate the definition of the coefficient (exponent) of strain-rate sensitivity, since the classic one is dimensional invalid. This definition is also extended to the case of finite deformation. This paper also clarifies the analogy between internal friction increment and overstress due to strain-rate sensitivity.

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