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Evolutions of Compaction Bands of Saturated Soils

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Abstract The development of compaction bands in saturated soils, which is coupling-rate, inertial and pore-pressure-dependent, under axisymmetric loading was discussed, using a simple model and a matching technique at the moving boundary of a band. It is shown that the development of compaction bands is dominated by the coupling-rate and pore-pressure effects of material. The soil strength makes the band shrinking, whilst pore pressure diffusion makes the band expand. Numerical simulations were carried out in this paper.

Key words axisymmetric loading, saturated soil, matching technique, compaction band.
MSC 2000 74F20

1 Introduction

Mollema and Antonelli[1] recently identified localized deformation structures in high porosity sandstone, which they referred to as "compaction bands". These bands consist of thin tabular zones of pure compressional deformation that were characterized by a significant reduction in porosity (from 20-25% to a few percent). Owing to reduced permeability, these compaction bands could trap hydrocarbons and act as barriers to fluid flow in otherwise more permeable rock. Olsson[2] discussed these field observations and experimental evidences of compaction bands associated with artificial shear cracks and stressed boreholes. In addition, he conducted axisymmetric compression tests on Castlegate sandstone. Several specimens exhibited zones of localized compaction perpendicular to the direction of maximum compressive stress, which was interpreted as compaction bands. To explain the origin of compaction bands, Olsson[2] recognized that the approach of Rudnicki and Rice[3], used to predict the occurrence of shear bands, could be applied. In particular, he noted that for a special combination of material parameters, which could be appropriate for compaction rock, the theory predicts a planar band perpendicular to the most compressive principal stress, as expected for a compaction band. The instability analysis gives usually the condition for the emergence of compaction bands. But there is no indication of the evolution of softening area.

Except for the brittle rocks, the compaction band may occur in dense saturated soils. In this paper, we will mainly analyze the dynamics of the compaction band of saturated soils and the factors governing the process by the approach of Lu *et al*[4].

2 Formulation of Problem

In this paper, a body of saturated soil subjected to a axisymmetry compressive stress is considered. The axisymmetry compressive stress is applied on the body based on a hydrostatic pressure. The instability is assumed to occur already, and our aim here is to study the evolution of the compaction band. The x axis is located in the vertical direction and the y axis is in the horizontal direction(Fig. 1). The origin is located at the center of the compact band.

Under axisymmetric compression conditions, volume equilibrium of soils and water according to the Darcy law and dilatant relation yields[5]:

$$\frac{\partial p}{\partial t}-\frac{E_r}{K}\frac{\partial^2 p}{\partial x^2}=C_1E_r\sigma_e\frac{\partial \varepsilon}{\partial t}, \tag{1}$$

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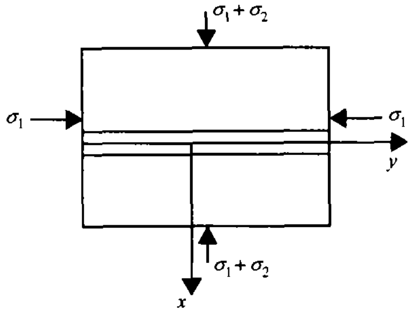

Fig. 1 The sketch of a body with potential compact band and indication of displacement and stresses

in which p is pore pressure and n is porosity; K is called drag coefficient here with the dimension $[M/L^3T]$, $K=\rho_w g/k$, where k is the Darcy permeability and ρ_w is the density of water, g is the Earth gravity acceleration; C_1 is a parameter and $C_1=C_1(p)$; E_r is unloading module; σ_e is the effective stress; ε is the strain caused by axial load.

The equations of motion implies

$$\rho\frac{\partial^2\varepsilon}{\partial t^2}-\frac{\partial^2\sigma_e}{\partial x^2}=\frac{\partial X}{\partial x}+\frac{\partial Y}{\partial y}, \tag{2}$$

where $\rho=(1-n)\rho_s$; n is the porosity; X, Y are the volume forces in the x and y directions respectively, and X is a constant (is equal to the gravity acceleration), Y equals zero. Therefore, Eq.(2) becomes

$$\rho\frac{\partial^2\varepsilon}{\partial t^2}-\frac{\partial^2\sigma_e}{\partial x^2}=0. \tag{3}$$

Now, the governing equations can be rewritten as

$$\begin{cases}\dfrac{\partial p}{\partial t}-\dfrac{E_r}{K}\dfrac{\partial^2 p}{\partial x^2}=C_1E_r\sigma_e\dfrac{\partial\varepsilon}{\partial t},\\[2mm] \rho\dfrac{\partial^2\varepsilon}{\partial t^2}=\dfrac{\partial^2\sigma_e}{\partial x^2},\end{cases} \tag{4}$$

where C_1 and E_r are both the functions of p.

Now give the dimensionless form of the control equations, and they may be simplified as

$$\begin{cases}\dfrac{E_r\dot{\varepsilon}_k\rho}{\sigma_{e0}K}\dfrac{\partial\dot{\bar{\varepsilon}}}{\partial t}=\dfrac{\partial^2\bar{\sigma}_e}{\partial x^2},\\[2mm] \dfrac{\partial\bar{p}}{\partial t}-\dfrac{\partial^2\bar{p}}{\partial x^2}=\dfrac{\sigma_e\dot{\bar{\varepsilon}}}{2},\end{cases} \tag{5}$$

where $\bar{p}=p/\sigma_{e0}$, $\dot{\bar{\varepsilon}}=\dot{\varepsilon}/\dot{\varepsilon}_k$, $\bar{\sigma}_e=\sigma_e/\sigma_{e0}$, $\bar{t}=t/t_k$, $\bar{y}=y/\delta_k$. Let $t_k=1/2C_1E_r\dot{\varepsilon}_k$ and $\delta_k^2=E_rt_k/K$. Because the dimensions of the parameters are as follows: $\sigma_{e0}\sim10^8$ Pa, $K\sim10^{10\text{-}12}$ kg/m^{3}s, $\dot{\varepsilon}_k\sim10^{-1\sim-2}$/s, $n\sim10^{-1}$, $\rho_s\sim10^3$ kg/m^3, $E_r\sim10^{6\sim7}$ Pa, the smallness of $E_r\dot{\varepsilon}_k\rho/\sigma_{e0}K$ reduces Eq.(5) to Eq.(6). From now on the over-bar used to indicate a dimensionless quantities will be omitted.

Therefore the approximate model for compaction bands is as follows:

$$\begin{cases}\dfrac{\partial^2\sigma_e}{\partial x^2}=0,\\[2mm] \dfrac{\partial p}{\partial t}-\dfrac{\partial^2 p}{\partial x^2}=\dfrac{\sigma_e\dot{\varepsilon}}{2},\end{cases} \tag{6}$$

$$y=0,\ \frac{\partial p}{\partial y}=0, \tag{6a}$$

$$y=\delta(t),\ p_{\delta_-}=p_{\delta_+} \tag{6b}$$

$$\frac{\partial p}{\partial y}\Big|_{\delta_-}=\frac{\partial p}{\partial y}\Big|_{\delta_+}, \tag{6c}$$

$$v|_{\delta_-}=RV(t), \tag{6d}$$

$$t=0,\ p=p_0(y), \tag{6e}$$

where $R=v_0/\delta_k\dot{\varepsilon}_k$; $p_0(y)$ and v_0 are the initial disturbances of pore pressure and velocity respectively; $V(t)$ is dimensionless boundary velocity at $V=1$ and $t=0$; δ_- and δ_+ express the inner and outer of the boundary of the compaction band respectively. Eqs. (6b) and (6c) mean that the pore pressure and its gradient at the boundary are continuous. Eq. (6d) gives the compaction velocity at the boundary of the compaction band. Eq.(6e) gives the pore initial pore pressure distribution.

Here, the constitutive equation is assumed to be the one for pore pressure dependent and strain rate dependent:

$$\sigma_e=\sigma_e(\dot{\varepsilon},p). \tag{7}$$

Outside the band, the material is assumed to remain rigid (which means the deformation outside the band may be neglected compared with that inside the band), no matter how high the pore pressure is. The governing equation here is the one for homogeneous diffusion with

$$p(0,x)=B, \tag{8}$$

where B is the assumed to be uniform initial pore pressure.

3 Analytical Solution

In this section, an analytical solution is obtained by making some approximations.

One of the simplifications is a linear version of the constitutive equation:

$$\sigma_e=\dot{\varepsilon}+1-p. \tag{9}$$

The variations in the above equation are dimensionless (the over-bar has been omitted after Eq.(5)). For an example, when $\sigma_e = 0, \dot{\varepsilon} = 0$, it means that $p = \sigma_{e0}$. For the saturated soils, p and σ_e cannot both equal zero. The linear softening approximates the behavior of a variety of soils between the statistic pore pressure and the effective stress, so it is easy to obtain the solutions. The minus before p in Eq.(9) denotes the softening effect on the soil and the positive sign before $\dot{\varepsilon}$ denotes the hardening effect on the soil in this model.

Substitution of Eq. (9) into Eq.(6) leads to an inhomogeneous equation in p. The solution to it then can be expressed as

$$p = \exp(H(t))\left[p_1 + \int_0^t e^{-H(\tau)}\sigma_e \frac{\sigma_e - 1}{2}d\tau\right], \qquad (10)$$

where $H(t) = \int_0^t \frac{\sigma_e}{2}d\eta$, η is a variable in integration, and because the strain is assumed to be rigid at the boundary of the band, we can obtain

$$\sigma_e(t) = 1 - p_\delta(t), \qquad (11)$$

in which $p_\delta(t)$ is the pore pressure at the boundary of the compaction band.

The pore pressure p_1 satisfies the equation

$$\frac{\partial p_1}{\partial t} - \frac{\partial^2 p_1}{\partial x^2} = \frac{\sigma_e p_1}{2}. \qquad (12)$$

To deal with the moving boundary $\delta(t)$, we consider the initial boundary conditions

$$x = 0, \ \frac{\partial p_1}{\partial x} = 0,$$

$$x = \delta_s, \frac{\partial p_1}{\partial x} = S(t)\exp(-H(t)), \qquad (14a)$$

$$t = 0, \ p_1 = p_0(x), \qquad (14b)$$

where δ_s is an imaginary fixed boundary chosen to be greater than $\delta(t)$, and $S(t)$ is an arbitrary function determined by matching the condition at $y = \delta(t)$, $i.e.$, Eqs. (6c) and (6d).

The solution p to the problem, but with initial boundary values (6c), can be expressed as the Fourier cosine series:

$$p = e^{H(t)}\frac{1}{\delta_s}\left\{C_0 + \int_0^t S(\eta)e^{-H(\eta)}d\eta - \delta_s\int_0^t \sigma_e \frac{1-\sigma_e}{2}e^{-H(t)}d\eta\right\} +$$
$$\frac{2}{\delta_s}\sum_1^\infty \exp(H(t) - \alpha_n^2 t)\cdot\left\{C_n + (-1)^n\int_0^t S(\eta)\exp(\alpha_n^2\eta - H(\eta))d\eta\right\}\cos\alpha_n x, \qquad (15)$$

where $\alpha_n = n\pi/\delta_s, 0 < x < \delta_s$, C_n are constants determined by the initial condition (6d) as

$$C_n = \int_0^{\delta_s} p_0(\xi)\cos(\alpha_n\xi)d\xi, \qquad (16)$$

where ξ is a variable in integration, and Eq.(6d) requires that

$$RV(t) = v|_{\delta_-} = \int_0^\delta \dot{\varepsilon}dx$$
$$= \frac{2}{\delta_s}\sum_1^\infty \exp(H(t) - \alpha_n^2 t)\left\{C_n + (-1)^n\int_0^t S(\eta)\exp(\alpha_n^2\eta - H(\eta))d\eta\right\}\frac{\delta_s}{n\pi}\left\{\sin\frac{\alpha_n\delta(t)}{\delta_s} - \frac{\alpha_n\delta(t)}{\delta_s}\cos\frac{\alpha_n\delta(t)}{\delta_s}\right\}.$$

The solution for the rigid material outside the band can be easily obtained as

$$p = \int_0^t [p^*(t-\eta) - B]\frac{x}{2\sqrt{\pi\eta^3}}\exp\left(-\frac{x^2}{4\pi}\right)d\eta. \qquad (18)$$

If it is assumed that B is constant and the edge effect is neglected, $p^*(t)$ here is the pore pressure at an imaginary boundary $y = 0$.

In all, there are four unknown functions: $p^*(t)$, $S(t)$, $\delta(t)$ and $\sigma_e(t)$. They can be determined by solving equations (12), (15), (17) and (18) simultaneously.

4 Mechanics of Compaction Band

Equation (15) shows that three factors control a non-uniform compaction field, namely $S(t)$, $H(t)$ and $\alpha_n^2 t$. The first one is related to the pore pressure diffusing out the band, the second represents the accumulative effect of the stress (the yield stress or the strength), and the third concerns the decaying mode of pore pressure diffusion within the band. The second and the third are both exponential ones, and so are much more important than the first. Even at the early stage of compaction band development, the pore pressure diffusion (accounted for by $S(t)$) to surrounding soil appears to be negligibly small, because $\partial p/\partial x|_{\delta_0}=0$, where $\delta_0=\delta(0)$. Therefore $H(t)$ and $\alpha_n^2 t$ are bound to be the governing factors in compaction band formation.

With the two assumptions that of $S(t)=0$ and $c_n=0(n\neq 1)$, which represent the most influential part of pore pressure diffusion and the simplier case, Eq. (15) becomes

$$\begin{cases}\sin(\alpha_1\delta(t))-L=\alpha_1\delta(t)\cos(\alpha_1\delta(t)),\\ L=\dfrac{\pi RV}{2C_1}\exp(\alpha_1^2 t-H(t)).\end{cases}\tag{19}$$

Assume that $\delta_0=\delta_s$ and a constant-velocity boundary condition can be introduced. It is clear from Eq. (19) that thrinkage of the compaction deformation field requires a decreasing value of L, *i.e.*,

$$\frac{\mathrm{d}}{\mathrm{d}t}(H(t)-\alpha_1^2 t)>0\tag{20}$$

or

$$\sigma_e(t)/2>(\pi/\delta(0))^2.\tag{21}$$

Therefore the shrinkage is due to the strength σ_e, whereas pore pressure diffusion tends to smooth the compaction.

For a material governed by the pore-pressure-independent constitutive relation $\sigma_e=\sigma_e(\dot{\varepsilon})$, the solution to (6) is

$$p=p_3(t,x)+\int_0^t\frac{\sigma_e\dot{\varepsilon}}{2}\mathrm{d}\eta,\tag{22}$$

$$\frac{\partial p_3}{\partial t}-\frac{\partial^2 p_3}{\partial x^2}=0.\tag{23}$$

Unlike the solution (11), this solution shows that the strength σ_e will not be incorporated into a non-uniform compaction field. This case corresponds to simple pore-pressure diffusion, whereas $\sigma_e=\sigma_e(p)$ would lead to a trivial solution. Therefore, one can conclude that the pore pressure diffusion makes the band expanding while the soil strength makes the band shrinking.

With decreasing $\sigma_e(t)$, which usually happens in the saturated soil under vibration load, a narrowing compaction band will change to a expanding one at a certain time, because the right-hand side of the inequality (21) is constant. This means that there is a stable phase deformation dominated by pore pressure diffusion.

Figure. 1, based on the inequality (21), shows that the width of compact band decreases with the increase of the effective stress.

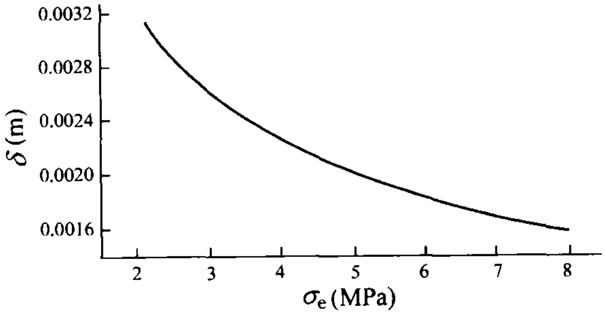

Fig.2 The changes of compaction band δ/δ_0 with compaction load σ_e/σ_{e0}

By introducing Eq. (11) and the condition $\dot{\varepsilon}|_{y=\delta}=0$ into Eq. (15), we may obtain

$$RV=\int_0^\delta(p-p_\delta(t))\mathrm{d}x.\tag{24}$$

Differentiation of (24) with respect to time t under the constant-velocity boundary condition leads to

$$\left.\frac{\partial p}{\partial x}\right|_\delta\delta(t)\frac{\partial\delta}{\partial t}=\frac{RV}{2}+\left.\frac{\partial p}{\partial x}\right|_\delta-\delta(t)\frac{\partial p_\delta}{\partial x}.\tag{28}$$

For $\partial p/\partial y\neq 0$ and $\delta(t)\neq 0$, Eq. (28) becomes an expression for compaction band development:

$$\frac{\partial\delta}{\partial t}=\left\{\frac{\sigma_e\ \bar{\dot{\varepsilon}}(t)}{2}+\frac{\bar{\partial}^2\bar{P}}{\partial x^2}-\frac{\partial p}{\partial t}\Big|_\delta\right\}\Big/\frac{\partial p}{\partial x}\Big|_\delta.\tag{29}$$

When $\dot{\varepsilon}>0$, within the compaction band, $\partial p/\partial y|_\delta$ must be negative. Moreover, $\partial p/\partial t|_\delta=\partial^2 p/\partial x^2|_\delta$. Then Eq. (19) becomes

$$\frac{\partial\delta}{\partial t}=\left(\frac{\sigma_e\ \bar{\dot{\varepsilon}}}{2}+\frac{\partial^2\bar{p}}{\partial x^2}-\frac{\partial^2 p}{\partial x^2}\Big|_\delta\right)\Big/\frac{\partial p}{\partial y}\Big|_\delta.\tag{30}$$

It is obvious that the term $\sigma_e\ \bar{\dot{\varepsilon}}/2$ is always positive and therefore governs compaction band contraction. However, there is usually a simple monotonic decreasing pore pressure distribution,

$$\frac{\partial^2\bar{p}}{\partial x^2}-\frac{\partial^2 p}{\partial x^2}\Big|_\delta<0,\tag{31}$$

from which it is seen that pore pressure diffusion in the compaction band tends to expand the band.

5 Conclusions

The analytical solution has been obtained in this paper to understand the mechanism of the compaction band of saturated soil. It shows the development of compaction bands is dominated by pore pressure diffusion and compressive strength. The first often makes compaction bands expanding while the second makes the bands shrinking.

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Appendix

The procedure to obtain the solution of Eq.(15)

$$\begin{cases}\dfrac{\partial^2\sigma_e}{\partial x^2}=0,\\ \dfrac{\partial p}{\partial t}-\dfrac{\partial^2 p}{\partial x^2}=\dfrac{\sigma_e\dot{\varepsilon}}{2}.\end{cases}\tag{32}$$

Substituting Eq.(9) into the second of last equation, and assuming that $p=p_2'(x,t)+p_2(t)$, we can obtain

$$\frac{\partial p_2}{\partial t}=\frac{\tau(\tau-1)}{2}+\frac{\tau p_2}{2},\tag{33}$$

$$\frac{\partial p_2'}{\partial t}-\frac{\partial^2 p_2'}{\partial x^2}=\frac{\tau p_2'}{2}.\tag{34}$$

Solving Eq.(33), we can obtain

$$p_2=\left(c+\int_0^t\frac{\tau(\tau-1)}{2}e^{-H(\eta)}d\eta\right)e^{H(t)}.\tag{35}$$

Then, we can assume the solution of Eq.(32) has the following form

$$p_2=\left(p_2'+\int_0^t\frac{\tau(\tau-1)}{2}e^{-H(\eta)}d\eta\right)e^{H(t)}.\tag{36}$$

Substituting Eq.(36) into Eq.(32), gives

$$\frac{\partial p_1}{\partial t}-\frac{\partial^2 p_1}{\partial x^2}=\frac{\tau p_1}{2}.\tag{37}$$

Solving the Eq.(37) under the following conditions, in which $p_1=p_2'e^H$:

$$\left.\frac{\partial p_1}{\partial x}\right|_{x=0},\ \left.\frac{\partial p_1}{\partial x}\right|_{x=\delta_s}=S(t)e^{-H},\tag{37.1}$$

$$p_1|_{t=0}=p_0(y).\tag{37.2}$$

We solve the above equation. First, we rewrite the above equation in the following form, assuming $p_1=p_2+p_3$:

$$\begin{cases}\dfrac{\partial p_1}{\partial t}-\dfrac{\partial^2 p_1}{\partial x^2}=\dfrac{\tau p_1}{2},\\ \left.\dfrac{\partial p_1}{\partial x}\right|_{x=0}=0,\ \left.\dfrac{\partial p_1}{\partial x}\right|_{x=\delta_s}=0,\\ p_1|_{t=0}=p_0(y);\end{cases}\tag{38}$$

$$\begin{cases}\dfrac{\partial p_1}{\partial t}-\dfrac{\partial^2 p_1}{\partial x^2}=\dfrac{\tau p_1}{2},\\ \left.\dfrac{\partial p_1}{\partial x}\right|_{x=0}=0,\ \left.\dfrac{\partial p_1}{\partial x}\right|_{x=\delta_s}=S(t)e^{-H},\\ p_1|_{t=0}=0.\end{cases}\tag{39}$$

Then, it is easy to obtain the solutions by the Fourier series method.

$$p_1=\sum c_n\cos\alpha_n y\exp(H(t)-\alpha_n^2 t)+\frac{1}{\delta_s}\int_0^t s(\eta)\exp(-H(\eta))d\eta,$$

in which

$$c_n=\frac{2}{\delta_s}\int_0^{\delta_s}p_0(\eta)\cos\frac{n\pi\eta}{\delta_s}d\eta.\tag{40}$$

Therefore, the solution of Eq.(32) is

$$\begin{aligned}p_1=&\exp(H(t))\frac{1}{\delta_s}\left\{c_0+\frac{1}{\delta_s}\int_0^t s(\eta)\exp(-H(\eta))d\eta-\delta_s\int_0^t\frac{1-\tau}{2}\tau(\eta)\exp(-H(\eta))d\eta\right\}+\\&\frac{2}{\delta_s}\sum_{n=1}^{\infty}\exp(H(t)-\alpha_n^2 t)\left\{c_n+(-1)^n\int_0^t s(\eta)\exp(\alpha_n^2 t-H(t))d\eta\right\}\cos\frac{n\pi x}{\delta_s}.\end{aligned}\tag{41}$$

The procedure to obtain the solution of Eq.(18)

Out of the shear band, $\dot{\gamma}=0$, therefore,

$$\frac{\partial p}{\partial t}-\frac{\partial^2 p}{\partial x^2}=0.\tag{42}$$

B.C. $p|_{x=0}=p^*(t)$ and $x\to+\infty$, p should be a finite value.

I.C. $p|_{x=0}=B$.

It is easy to obtain the solution by the Laplace transformation

$$p=\int_0^t[p^*(t-\eta)-B]\frac{x}{2\sqrt{\pi\eta^3}}\exp\left(-\frac{x^2}{4\pi}\right)d\eta.\tag{43}$$

(*Executive editor SHEN Mei-fang*)