

Microdamage evolution, energy dissipation and their trans-scale effects on macroscopic failure

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Abstract

The process of damage evolution concerns various scales, from micro- to macroscopic. How to characterize the trans-scale nature of the process is on the challenging frontiers of solid mechanics. In this paper, a closed trans-scale formulation of damage evolution based on statistical microdamage mechanics is presented. As a case study, the damage evolution in spallation is analyzed with the formulation. Scaling of the formulation reveals that the following dimensionless numbers: reduced Mach number M , damage number S , stress wave Fourier number Ψ , intrinsic Deborah number D^* , and the imposed Deborah number De^* , govern the whole process of deformation and damage evolution. The evaluation of Ψ and the estimation of temperature increase show that the energy equation can be ignored as the first approximation in the case of spallation. Hence, apart from the two conventional macroscopic parameters: the reduced Mach number M and damage number S , the damage evolution in spallation is mainly governed by two microdamage-relevant parameters: the Deborah numbers D^* and De^* . Higher nucleation and growth rates of microdamage accelerate damage evolution, and result in higher damage in the target plate. In addition, the mere variation in nucleation rate does not change the spatial distribution of damage or form localized rupture, while the increase of microdamage growth rate localizes the damage distribution in the target plate, which can be characterized by the imposed Deborah number De^* .

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1. Introduction

When a medium with micrometer structure is subjected to strong stress pulse with microsecond duration, a number of micrometer cracks may nucleate and grow, and finally the medium fails.

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onstrates that we have to go into mesoscopic kinetics of microcracks to find out the mechanism governing the μs - and μm -damage evolution.

Many efforts have been made to explore the mesoscopic kinetics of microcracks and to find out the mechanism coupling meso- and macroscopic damage evolution (Curran et al., 1987; Bai et al., 1992; Grady and Kipp, 1993; Han et al., 1997; Lemanska et al., 1997; Voyiadjis et al., 2002; Zhou and Clifton, 1997). However, as McDowell (1997) pointed out, “rigorous treatment of non-uniformly distributed defects requires tools not yet fully developed in continuum damage mechanics”. “Weighing the influence of distributed damage at the microscale on the collective macroscale stiffness and evolution of damage is a challenge as well.”

Therefore, for the multi-scale and rate-dependent process of damage evolution, how to characterize its trans-scale nature is a key point. Barenblatt made a very significant statement addressing this in his closing plenary lecture at the 18th ICTAM (Barenblatt, 1992). He stated that in the mathematical models describing the governing influence of the microstructural variations on the macroscopic behavior of bodies, the macroscopic equations of mechanics and the kinetic equations of the microstructural transformations should form a unified set that should be solved simultaneously. This inevitably appeals to statistical considerations of microdamage ensemble.

In this paper, a closed trans-scale formulation of damage evolution based on statistical microdamage mechanics is presented. In addition, scaling in the formulation is analyzed. The scaling shows that several dimensionless numbers control deformation and damage evolution in the process of spallation. The effects of these dimensionless parameters are discussed.

2. Trans-scale formulation of damage evolution

Our trans-scale formulation of damage evolution is based on statistical microdamage mechanics. In statistical microdamage mechanics, we firstly need to define a mesoscale between micro- and macroscale according to the features of damage evolution. The microdamage on mesoscale is

the ensemble of our interest. We define microdamage nucleation as the appearance of mesoscopic voids or cracks resulted from sub-mesosopic process (Curran et al., 1987). Once nucleated, the microdamage may grow and coalesce leading to eventual failure. Hence, the main issues in damage evolution are the mesoscopic kinetics of microdamage nucleation, growth and coalesce, as well as the statistical effects of them.

In order to investigate the mesoscopic kinetics of microdamage, experimental measurements of microdamage evolution in spallation process in an Al alloy were performed. Bai et al. (1992) and Han et al. (1997) described the experiments in detail. The experiments show that microdamage is formed at the scale of inhomogeneities, e.g., the second phase particulates in the alloy. Hence, the scale of the second phase particulates is chosen as the mesoscale, which is on the order of micrometer. In addition, the nucleation size of microdamage is usually the dimensions of the particulates or grains. Furthermore, the total number of the microdamage on the surface of a specimen is in the range of 10^2 – 10^4 mm^{-2} . The observations also reveal that the nucleation rate of microdamage is governed by the applied stress and the size distribution of inhomogeneities. In addition, the growth rate is controlled by the applied stress, the current size (the instantaneous size of microdamage at the very time of observation) and the nucleation size of microdamage. Furthermore, since coalescence lasts very short time before failure, it will be neglected in the following formulation. Based on the experimental observations, the nucleation rate density and growth rate of microdamage can be approximately fitted to the following functions (Bai et al., 1992; Han et al., 1997):

$$n_N(c_0, \sigma_s, \theta) = n_N^*(\theta) g_1(\bar{\sigma}_s) p(\bar{c}_0) \quad (2)$$

and

$$V(c, c_0, \sigma_s, \theta) = V^*(\theta) g_2(\bar{\sigma}_s) (\bar{c} - \bar{c}_0)^v \quad (3)$$

where

$$\bar{\sigma}_s = \frac{\sigma_s}{\sigma^*}, \quad g_1(\bar{\sigma}_s) = \begin{cases} \bar{\sigma}_s - 1 & \text{if } \bar{\sigma}_s \geq 1 \\ 0 & \text{if } \bar{\sigma}_s < 1 \end{cases}, \quad g_2(\bar{\sigma}_s) = [g_1(\bar{\sigma}_s)]^\mu \quad (4)$$

$$\bar{c} = \frac{c}{c^*}, \quad \bar{c}_0 = \frac{c_0}{c^*}, \quad p(\bar{c}_0) = \bar{c}_0^{k-1} \exp(-\bar{c}_0^k) \quad (5)$$

where $n_N^*(\theta)$ is the characteristic nucleation rate density and V^* the characteristic growth rate of microdamage respectively, and the values of parameters in Eqs. (2)–(5) can be fitted from experimental data (Bai et al., 1992; Han et al., 1997).

The observations of microdamage evolution (Curran et al., 1987) suggest a statistical description of number density of microdamage $n(t, \mathbf{x}, c)$, where \mathbf{x} and t are the macroscopic position and time, respectively. According to statistical microdamage mechanics, the evolution of microdamage number density in phase space $\{c, \mathbf{x}\}$ is governed by the following equation (Xia et al., 1995; Bai et al., 2002):

$$\frac{\partial n}{\partial t} + \frac{\partial(nA)}{\partial c} + \frac{\partial(n\mathbf{v})}{\partial \mathbf{x}} = n_N \quad (6)$$

where \mathbf{v} is particle velocity vector and A is the average growth rate of microdamages with current size c

$$A = \frac{\int_0^c V(c, c_0, \sigma_s, \theta) n_0(t, \mathbf{x}, c, c_0) dc_0}{\int_0^c n_0(t, \mathbf{x}, c, c_0) dc_0} \quad (7)$$

and $n_0(t, \mathbf{x}, c, c_0)$ is the number density of microdamage when both current size c and nucleation size c_0 are taken into account, $n(t, \mathbf{x}, c) = \int_0^c n_0(t, \mathbf{x}, c, c_0) dc_0$.

We define continuum damage D as:

$$D(t, \mathbf{x}) = \int_0^\infty n(t, \mathbf{x}, c) \tau dc \quad (8)$$

where τ is the failure volume of an individual microdamage with size c . The failure volume of a microdamage is defined as the volume which can no longer be loaded because of microdamage nucleation and growth. For example, for a microcrack with length c , the failure volume can be assumed as a sphere with diameter c , then $\tau \approx \pi c^3/6$.

Multiplying Eq. (6) by the microdamage failure volume τ and integrating the resulted equation with respect to microdamage size c , from zero to infinity, yield the continuum damage field equation (Xia et al., 1995; Bai et al., 2002):

$$\frac{\partial D}{\partial t} + \frac{\partial(D\mathbf{v})}{\partial \mathbf{x}} = f \quad (9)$$

$$f = \int_0^\infty n_N(c_0, \sigma_s, \theta) \tau dc_0 + \int_0^\infty n(t, \mathbf{x}, c) A(c, \sigma_s, \theta) \tau' dc \quad (10)$$

where $\tau' = d\tau/dc$. The function f is the dynamic function of damage (DFD), the agent bridging microdamage and continuum damage evolution. To establish a complete formulation, Eq. (9) should be associated with traditional, macroscopic equations of continuum, momentum, and energy, constitutive relation, the relationship between nominal stress and true stress $\sigma_s = \frac{\sigma}{1-D}$, as well as the damage number density evolution equation (Eq. (6)).

To illustrate the framework, we use it to investigate the damage evolution in spallation. Let us examine a one-dimensional strain state in Lagrangian coordinate (T, Y) . In such a state, all displacement and velocity components and spatial derivatives are zero, except the displacement component u and the particle velocity v in the y direction, as well as the derivative operator $\partial/\partial y$. The transformation from Eulerian (t, y) to Lagrangian (T, Y) coordinates is $\frac{\partial}{\partial T} = \frac{\partial}{\partial t} + v \frac{\partial}{\partial y}$ and $\frac{\partial}{\partial Y} = (1 + \varepsilon) \frac{\partial}{\partial y}$. With the transformation, an associated system of continuum, momentum, energy and damage evolution equations are obtained as follows:

Continuum equation:

$$\frac{\partial \varepsilon}{\partial T} = \frac{\partial v}{\partial Y} \quad (11)$$

Momentum equation:

$$\rho_0 \frac{\partial v}{\partial T} = \frac{\partial \sigma}{\partial Y} \quad (12)$$

Damage evolution equation:

$$\frac{\partial D}{\partial T} + \frac{D}{1 + \varepsilon} \frac{\partial v}{\partial Y} = f \quad (13)$$

Energy equation:

$$\rho_0 \frac{\partial e}{\partial T} = \sigma \frac{\partial \varepsilon}{\partial T} - \frac{\partial h}{\partial Y} + \rho_0 q \quad (14)$$

Constitutive equation:

$$\sigma_s = \sigma(\varepsilon_s, \theta) \quad (15)$$

Relationships between σ_s and σ , ε_s and ε :

$$\sigma_s = \frac{\sigma}{1-D}, \quad \varepsilon_s = \varepsilon \quad (16)$$

Eqs. (11)–(16), (6), and (10) form a unified, closed trans-scale formulation of damage evolution.

Let us consider a special case in which stress and temperature are treated as parameters. In this case, the basic solution of microdamage number density n has been obtained as (Ke et al., 1990; Han et al., 1997)

$$n = \begin{cases} \int_0^c \frac{n_N(c_0; \sigma_s, \theta)}{V(c, c_0; \sigma_s, \theta)} dc_0 & c < c_{f,0} \\ \int_{c_{of}}^c \frac{n_N(c_0; \sigma_s, \theta)}{V(c, c_0; \sigma_s, \theta)} dc_0 & c \geq c_{f,0} \end{cases} \quad (17)$$

where c_{of} and $c_{f,0}$ are determined by $T = \int_{c_{of}}^c \frac{dc'}{V(c', c_0; \sigma_s, \theta)}$ and $T = \int_0^{c_{f,0}} \frac{dc'}{V(c', 0; \sigma_s, \theta)}$, respectively.

Substitution of Eq. (17) into the integral (Eq. (10)) leads to a trans-scale DFD without microdamage number density but still with mesoscopic kinetics. The trans-scale DFD is directly expressed by two mesoscopic kinetic laws of nucleation rate density and growth rate of microdamages (Bai et al., 1998, 2000, 2002):

$$f = \int_0^\infty n_N(c_0; \sigma_s, \theta) \tau dc + \int_0^\infty (\tau_f - \tau_0) n_N(c_0; \sigma_s, \theta) dc_0 \quad (18)$$

where $\tau_f = \pi c_f^3/6$, and c_f is defined as $T = \int_{c_0}^{c_f} \frac{dc'}{V(c', c_0; \sigma_s, \theta)}$. For nucleation and growth laws like Eqs. (2) and (3), the damage evolution equation becomes

$$\begin{aligned} \frac{\partial D}{\partial T} + \frac{D}{1+\varepsilon} \frac{\partial v}{\partial Y} \\ = f = \frac{\pi n_N^* c^{*4}}{6} p_3 g_1 \\ \times \left(1 + \frac{\int_0^\infty \left(\left(1 + \left(\frac{(1-\nu)^* V^* t}{c^*} g_2 \right)^{\frac{1}{1-\nu}} \frac{1}{c_0} \right)^3 - 1 \right) \bar{c}_0^3 p d\bar{c}_0}{p_3} \right) \end{aligned} \quad (19)$$

where p_3 is the third order moment of $p(\bar{c}_0)$, $p_3 = \int_0^\infty p(\bar{c}_0) \bar{c}_0^3 d\bar{c}_0$. Eq. (19) together with Eqs. (11), (12), and (14)–(16) forms a closed formulation of

damage evolution, combining traditional continuum mechanics and mesoscopic kinetics closely and explicitly.

It is worth noticing that the closed DFD in Eq. (19) is obtained based on the assumption of stress and temperature being parameters. Otherwise, it is quite difficult to obtain explicit, exact expression of DFD. In fact, since $D \ll 1$ in the process of spallation (we will show it later), the difference between the nominal stress and true stress in the case is negligibly small, based on Eq. (16). In addition, the loading stress in spallation tests is usually kept as a rectangular stress pulse with short front, hence the nominal stress in spallation tests is generally treated as a parameter. Finally, in the present calculation stress is actually treated as a variable, which to some extent reflects the coupling effect of varying stress on microdamage evolution. Furthermore, the most significant point about Eq. (19) is that the DFD consists of two terms, the nucleation term and combination term of nucleation and growth. This is in accord with the concept of simple and compound damage proposed by Davison and Stevens (1972) and Davison et al. (1977). Therefore, the derived DFD demonstrates that the microscopic bases of simple and compound damage are the microdamage nucleation and growth, respectively.

To specify the energy equation (Eq. (14)), we confine our discussion to damageable materials exhibiting elastic or elastoplastic behavior. In addition, we approximately hypothesize that the internal energy consists of mechanical part and thermal part. The mechanical part corresponds to the elastic deformation energy stored in the material w^e , the energy blocked in dislocations w^d , and the surface energy of microcracks w^c . The mechanical part can be expressed in terms of the elastic strain ε^e , plastic strain ε^p , macroscopic damage D , and the crack surface area Σ . The thermal part w^t comes from ordinary external heating and dissipation associated with irreversible bulk plastic flow and damage evolution. It is also assumed that all dissipated energy is converted to heat and causes the temperature increase. Hence, the internal energy increment can be written as:

$$\begin{aligned}
d(\rho_0 e) &= dw^t + dw^e + dw^d + dw^c \\
&= \rho_0 C d\theta + d \left[\frac{1}{2} E(1-D)(\varepsilon^e)^2 \right] \\
&\quad + dw^d(\varepsilon^p) + d(\gamma^e \Sigma) \\
&= \rho_0 C d\theta + \left[\sigma d\varepsilon^e - \frac{1}{2} E(\varepsilon^e)^2 dD \right] \\
&\quad + \frac{dw^d}{d\varepsilon^p} d\varepsilon^p + \gamma^e d\Sigma(D)
\end{aligned} \quad (20)$$

In the concerned case, there is no external heat source. In addition, the heat flux can be expressed by Fourier's law in terms of the temperature θ as:

$$h = -\lambda \frac{\partial \theta}{\partial Y} \quad (21)$$

Neglecting the point source of heat and substituting Eqs. (20) and (21) into Eq. (14) leads to:

$$\begin{aligned}
\rho_0 C \frac{\partial \theta}{\partial T} + \left[\sigma \frac{\partial \varepsilon^e}{\partial T} - \frac{1}{2} E(\varepsilon^e)^2 \frac{\partial D}{\partial T} \right] + \frac{dw^d}{d\varepsilon^p} \frac{\partial \varepsilon^p}{\partial T} \\
+ \gamma^e \frac{d\Sigma(D)}{dD} \frac{\partial D}{\partial T} = \sigma \frac{\partial \varepsilon}{\partial T} + \lambda \frac{\partial^2 \theta}{\partial Y^2}
\end{aligned} \quad (22)$$

Since $\varepsilon = \varepsilon^e + \varepsilon^p$, Eq. (22) can be rewritten as:

$$\begin{aligned}
\rho_0 C \frac{\partial \theta}{\partial T} = \left[\sigma - \frac{dw^d}{d\varepsilon^p} \right] \frac{\partial \varepsilon^p}{\partial T} \\
+ \left[\frac{1}{2} E(\varepsilon^e)^2 - \gamma^e \frac{d\Sigma(D)}{dD} \right] \frac{\partial D}{\partial T} + \lambda \frac{\partial^2 \theta}{\partial Y^2}
\end{aligned} \quad (23)$$

According to Taylor and Quinney (1934), the amount of plastic work dissipated as heat can be in excess of 90%. Therefore, it is generally accepted that the energy blocked in dislocations is negligible compared to the plastic work dissipation. In addition, for ductile materials like Al alloy, the elastic surface energy of microcracks is very small portion of the elastic energy reduction due to damage evolution. Hence, Eq. (23) can be simplified as:

$$\rho_0 C \frac{\partial \theta}{\partial T} = \sigma \frac{\partial \varepsilon^p}{\partial T} + \frac{1}{2} E(\varepsilon^e)^2 \frac{\partial D}{\partial T} + \lambda \frac{\partial^2 \theta}{\partial Y^2} \quad (24)$$

Eq. (24) is the simplified energy equation for damaged, elastoplastic materials. It is a remarkable fact that the three terms on the right side of Eq. (24) are directly related to the temperature increment due to plastic work dissipation, damage evolution and heat diffusion, respectively.

3. Scaling of the trans-scale formulation of damage evolution

The trans-scale equations in Section 2 have provided a fundamental tool to explore damage evolution process. However, till now, the investigation into the essence of the process is far from complete. How to deal with the various length and time scales on meso- and macroscopic levels is the biggest obstacle hindering the exploration.

The existence of different length and time scales indicates that different mechanisms are involved in the process of damage evolution. The mechanisms include wave propagation, work hardening effect, softening effect caused by temperature increase or damage evolution, heat diffusion, microdamage nucleation, microdamage growth and so on. It is impractical and unnecessary to consider all mechanisms thoroughly in the study. The most reasonable way is to evaluate the relative importance of these possible mechanisms, and to adopt a properly simplified model based on the evaluation. So, to unveil the most predominant mechanisms governing damage evolution is the essence of scaling.

In order to understand the procedure of scaling, in this section, we consider the problem of damage evolution owing to the impact of a flying plate with thickness L and velocity v_f on a target plate, i.e. spallation.

There are many dependent variables involved in the problem, e.g. particle velocity (v), strain (ε), stress (σ), temperature (θ), damage (D), and the number density of microdamage (n) etc. These variables are linked by the trans-scale equations presented in last section. In view of the fact that these variables have different dimensions as well as different magnitudes, we non-dimensionalize and normalize all the terms in the equations. The corresponding scaled equations of mass, momentum, damage evolution, and energy are:

$$\frac{\partial \bar{\varepsilon}}{\partial \bar{T}} = M \frac{\partial \bar{v}}{\partial \bar{X}} \quad (25)$$

$$\frac{\partial \bar{v}}{\partial \bar{T}} = S \frac{\partial \bar{\sigma}}{\partial \bar{X}} \quad (26)$$

$$\frac{\partial \bar{D}}{\partial \bar{T}} + \frac{M \varepsilon^* \bar{D}}{1 + \varepsilon \varepsilon^*} \frac{\partial \bar{v}}{\partial \bar{X}} = \frac{\pi p_3 g_1}{6} \frac{1}{De^*} \left(1 + \frac{\int_0^\infty \left(\left(1 + \left((1 - \nu) g_2 \frac{\bar{T}}{De^*} \right)^{\frac{1}{1-\nu}} \frac{1}{c_0} \right)^3 - 1 \right) \bar{c}_0^3 p d\bar{c}_0}{p_3} \right) \quad (27)$$

$$\frac{\partial \bar{\theta}}{\partial \bar{T}} = \bar{\sigma} \frac{\partial \bar{v}^p}{\partial \bar{T}} + \frac{1}{2} (\bar{\varepsilon}^e)^2 D^* \frac{\partial \bar{D}}{\partial \bar{T}} + \Psi \frac{\partial^2 \bar{\theta}}{\partial \bar{Y}^2} \quad (28)$$

In Eqs. (25)–(28), the dimensionless variables are:

Independent variables:

$$\bar{T} = \frac{aT}{L}, \quad \bar{Y} = \frac{Y}{L} \quad (29)$$

Dependent variables:

$$\bar{v} = \frac{v}{v_f}, \quad \bar{\sigma} = \frac{\sigma}{\sigma^*}, \quad \bar{\theta} = \frac{\rho_0 C (\theta - \theta_0)}{\sigma^* \varepsilon^*}, \quad \bar{D} = \frac{D}{D^*}, \quad \bar{\varepsilon} = \frac{\varepsilon}{\varepsilon^*}, \quad \bar{\varepsilon}^e = \frac{\varepsilon^e}{\varepsilon^*}, \quad \bar{\varepsilon}^p = \frac{\varepsilon^p}{\varepsilon^*} \quad (30)$$

The dimensionless numbers are defined as follows:

Reduced Mach number:

$$M = \frac{v_f}{a \varepsilon^*} \quad (31)$$

Damage number:

$$S = \frac{\sigma^*}{\rho_0 a v_f} \quad (32)$$

Intrinsic Deborah number:

$$D^* = \frac{n_N^* c^{*5}}{V^*} \quad (33)$$

Imposed Deborah numbers:

$$De^* = \frac{a c^*}{LV^*} \quad (34)$$

Stress wave Fourier number:

$$\Psi = \frac{\lambda}{\rho_0 CLa} \quad (35)$$

It is worth recalling that all dimensionless variables are in $O(1)$. So in principle, the magnitudes of the terms in the equations can be estimated according to the dimensionless numbers before them. That is to say, the five dimensionless numbers indicate the relative importance of the mech-

anisms involved in spallation process. Roughly speaking, the five numbers can be cataloged into three groups. The reduced Mach number and damage number are the representation of macroscopic material properties and imposed loading. The macroscopic stress wave Fourier number, concerns the energy aspect of the phenomenon. Another group, consisting of two Deborah numbers, is closely related to mesoscopic kinetics of microdamage evolution. Some essential effects of these numbers on spallation are discussed as follows:

(1) The reduced Mach number M is directly related to the ratio of the inertial force to the applied load, while the damage number S is defined as the ratio of the threshold stress to the amplitude of stress pulse. In addition, M and S can be correlated by

$$M = \frac{\sigma^*}{E \varepsilon^*} S^{-1} \quad (36)$$

For the aluminum alloy employed in our spallation tests (Bai et al., 1992), $\sigma^* \sim 450$ MPa, $\rho_0 \sim 2700$ kg/m³, $v_f \sim 200$ m/s, $a \sim 5000$ m/s, $\varepsilon^* \sim 0.005$, then $M \sim 7$ and $S \sim 0.167$. Therefore, in the spallation analysis, the dynamic analysis must be adopted, and plasticity and damage evolution cannot be ignored.

(2) The intrinsic Deborah number D^* characterizes the damage rate ratio of two intrinsic processes: nucleation over growth. In addition, as the measure to normalize damage, D^* implies a certain characteristic damage. Actually, D^* is a proper indicator of macroscopic critical damage to localization (Bai et al., 2002). For the aluminum alloy, $c^* \sim 4 \times 10^{-6}$ m, $V^* \sim 6$ m/s, and $n_N^* \sim 5 \times 10^4$ mm⁻³ μm^{-1} μs^{-1} . Then, $D^* \sim 0.009$. Therefore, in spallation test, the critical damage to localization should be about $(10^{-3} - 10^{-2})$. In particular, D^* is also an indicator of energy dissipation owing to microdamage. From Eq. (28), since $D^* \ll 1$, the dissipation is very small in comparison with plastic work.

(3) The imposed Deborah number De^* is a unique trans-scale dimensionless parameter, because the acoustic speed a and the sample size L are macroscopic parameters whereas microdamage size c^* and microdamage growth rate V^* are mesoscopic

ones. This is very different from all other dimensionless parameters. Also, De^* refers to the ratio of microdamage growth time scale over the macroscopically imposed time scale. Hence, it represents the competition and coupling between the macroscopically imposed wave loading and the intrinsic microdamage growth. In current study, $De^* < 1$, which means that microdamage has enough time to grow during the macroscopic wave loading. Thus, the microdamage growth must be considered in spallation analysis. In addition, since damage localization is directly related to microdamage growth (Wang et al., 2002), De^* is a key factor governing damage localization process.

Of course, the ratio of the above two Deborah numbers gives another imposed Deborah number $De = \frac{a}{L n_N^* c^{*4}}$. Similar to De^* , De refers to the ratio of microdamage nucleation time scale over the macroscopically imposed time scale. However, among the three Deborah numbers, only two of them are independent of each other, for instance $D^* = De^*/De$.

(4) Stress wave Fourier number Ψ can be rewritten as:

$$\Psi = \frac{\lambda}{\rho_0 C L a} = \frac{\lambda / \rho_0 C}{L a} = \frac{k / a}{L} = \left(\frac{l_\kappa}{L} \right)^2 \quad (37)$$

Thus, Ψ represents the ratio of heat diffusion region over sample size. In this study, $\lambda = 238 \text{ W/m K}$, $\rho_0 \sim 2700 \text{ kg/m}^3$, $C \sim 903 \text{ J/kg K}$, $a \sim 5000 \text{ m/s}$. So, $l_\kappa / L \sim 10^{-3}$ and $\Psi \sim 10^{-6} \ll 1$. This means that the heat diffusion zone is much smaller compared to the sample size. It follows that the process of spallation in specimen can be treated as an adiabatic one.

Under adiabatic assumption, Eq. (28) indicates that the temperature increase is mostly contributed by the plastic work in volume and microdamage dissipation. Although exact data of plastic work dissipation and microdamage dissipation are not available, the temperature increment can be estimated as follows. For the tested aluminum alloy, $\sigma^* \sim 450 \text{ MPa}$ and $\varepsilon^* \sim 0.005$, so, the characteristic temperature increase $\Delta\theta^* \sim \frac{\sigma^* \varepsilon^*}{\rho_0 C} \sim 1 \text{ K}$. In general spallation tests, $\varepsilon^p \sim 0.008$, $\varepsilon^c \sim 0.016$, $\sigma \sim 1400 \text{ MPa}$, hence, $\Delta\theta = \bar{\sigma} \Delta \bar{\varepsilon}^p + \frac{1}{2} (\bar{\varepsilon}^c)^2 D^* \Delta \bar{D} \sim 4.7$. Therefore, the temperature increase due to bulk

plastic work and microdamage dissipation may be about several degrees only. This rise of temperature does not affect material properties effectively. Hence, as the first approximation, energy dissipation can be ignored in the study of spallation.

4. Numerical analysis of spallation

The scaling in Section 3 suggests that as the first approximation, the energy equation can be neglected in the formulation of spallation process. Therefore, only four dimensionless parameters (reduced Mach number M , damage number S , intrinsic Deborah number D^* and imposed Deborah number De^*) govern the damage evolution process in the target plate. Now that the effects of the macroscopic numbers M and S on damage evolution have been well documented in literature, the emphasis in this paper is put on the effects of two mesokinetics related numbers, D^* and De^* .

We perform numerical simulations of spallation in terms of the associated system of the equations of mass, momentum, microdamage evolution, Eqs. (25)–(27), as well as the equation of constitutive relationship, with the initial and boundary conditions corresponding to our plate impact tests. In the simulations, we fix the macroscopic numbers M and S according to the experimental condition but adopt variable Deborah numbers to examine the evolution of maximum damage and the distribution of damage in the target plate, aiming at understanding the effects of mesoscopic kinetics of microdamage on the eventual rupture in spallation.

Fig. 1 illustrates the effect of the intrinsic Deborah number D^* on the evolution of maximum damage in the target plate. In this figure, the reduced Mach number M , Damage number S and the imposed Deborah number De^* remain constant for all curves. Accordingly, the increase of the intrinsic Deborah number D^* implies the increase of the nucleation rate of microdamage only, as shown in Eq. (33). Therefore, increasing D^* results in higher damage in the target plate. This is in agreement with numerical results (Fig. 1), which shows that the maximum damage in the target plate increases with D^* , owing to higher nucle-

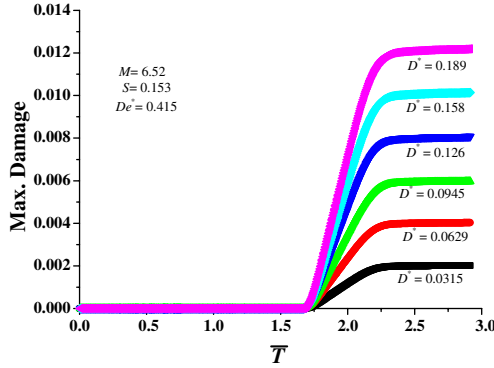


Fig. 1. Effects of D^* on the evolution of maximum damage in target plate ($M = 6.52$, $S = 0.153$, $De^* = 0.415$).

ation rate of microdamage. Obviously, the result is rational but trivial.

Furthermore, we compared the spatial distributions of damage obtained from the above cases with different D^* but the same M , S , and De^* . Fig. 2 demonstrates the distribution of normalized cumulative damage at a given time. For each case in Fig. 2, the damage is normalized by the maximum damage D_{\max} in the target plate. Fig. 2 clearly shows that all curves coincide with each other and have a plateau although D_{\max} differs in these cases. These two features mean that the mere variation in nucleation rate of microdamage cannot change the spatial distribution of damage or form localized rupture. Actually, this is not unreasonable, if one notice that all maximum damage in Fig. 1 are less than their corresponding D^* , i.e. the indicator of critical damage to localization.

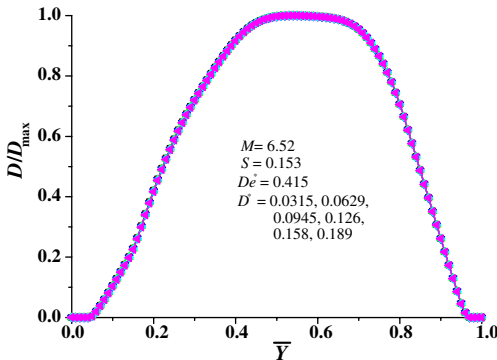


Fig. 2. Effects of D^* on damage localization ($M = 6.52$, $S = 0.153$, $De^* = 0.415$, $\bar{T} = 2.91$).

Now we turn to the effects of imposed Deborah number De^* on damage evolution. Actually, the imposed Deborah number De^* , as defined in Eq. (34), characterizes the competition and coupling between macroscopic loading and microdamage growth. Smaller De^* but the same M , S , and De physically means that microdamage grows faster. Hence, decreasing De^* may accelerate the damage evolution and result in higher damage in the target plate. The numerical results (Fig. 3) do demonstrate that the maximum damage increases with decreasing De^* .

Our further studies reveal that De^* affects damage localization significantly. Fig. 4 shows the normalized cumulative damage distributions at a fixed time. Similar to Fig. 2, the damage is normalized by the maximum damage in the target plate. In marked contrast to the distribution of damage in Fig. 2, damage distributes very unevenly in the plate (Fig. 4). In particular, when the imposed Deborah number De^* decreases, the damage gets more localized in the plate. Since De^* decrease is directly related to the increase of microdamage growth rate, Fig. 4 illustrates that damage localization is caused by microdamage growth and damage is prone to localize in materials with higher microdamage growth rate. To further explain damage localization, we calculate the intrinsic Deborah number D^* corresponding to each curve in Fig. 3. One can notice that only for the case of $De^* = 0.138$, the maximum damage is about 0.02, comparable with its corresponding $D^* = 0.0209$.

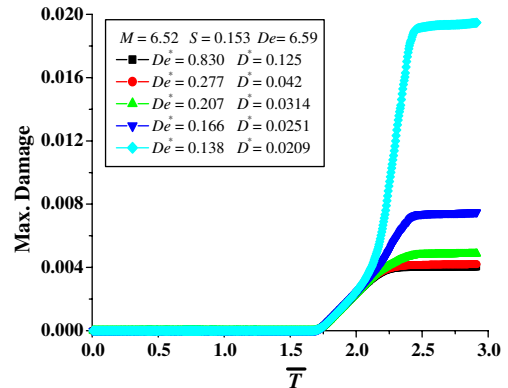


Fig. 3. Effect of De^* on the maximum damage evolution in the target plate ($M = 6.52$, $S = 0.153$, $De = 6.59$).

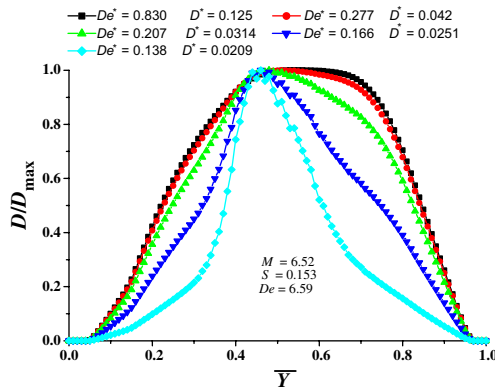


Fig. 4. Effects of De^* on damage localization ($M = 6.52$, $S = 0.153$, $De = 6.59$, $\bar{T} = 2.91$).

This is why damage localization becomes clear in this case.

5. Concluding remarks

For trans-scale formulation of damage evolution, there are several length scales and time scales on meso- and macroscopic levels. The connection between the two levels is usually based on non-linear coupling of stress field and the population of microdamage. Hence, the associated system of macroscopic continuum, momentum and energy equations as well as the equation of microdamage population evolution is proposed and discussed.

Several dimensionless numbers, such as the reduced Mach number M , damage number S , stress wave Fourier number Ψ , intrinsic Deborah number D^* , and imposed Deborah number De^* govern spallation.

In respect of energy dissipation in spallation, the stress wave Fourier number Ψ indicates that the heat diffusion region is very small compared to the sample size, hence the process is nearly adiabatic. The temperature increase resulted from the dissipative energy is estimated for the tested metal and have minor effects on spallation. So, in the concerned spallation process, as the first approximation, the energy equation can be ignored in the analysis.

Apart from the reduced Mach number M and damage number S , the damage evolution in spall-

ation is mainly governed by the intrinsic Deborah number D^* and the imposed Deborah number De^* . The intrinsic Deborah number D^* is a characteristic representation of competition and coupling between microdamage nucleation and growth, whilst the imposed Deborah number De^* characterizes the competition and coupling between the imposed wave loading and the intrinsic microdamage growth.

Increasing nucleation or growth rate of microdamage accelerates damage evolution, and results in higher damage in the target plate. In addition, the mere variation in nucleation rate does not change the spatial distribution of damage or form localized rupture. However, the increase of microdamage growth rate localizes the damage distribution in the target plate, which can be characterized by the imposed Deborah number De^* .

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