

# Surface Stability of Epitaxial Elastic Films by the Casimir Force \*

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We investigate the morphological stability of epitaxial thin elastic films on a substrate by the Casimir force between the film surface and a flat plate. Critical undulation wavelengths are derived for two different limit conditions. Consideration of the Casimir force in both limit cases decreases the critical wavelength of the surface perturbation.

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In 1948, Casimir<sup>[1]</sup> predicted an attractive force between two parallel neutral plates in vacuum to be

$$F(H) = -\frac{\pi^2}{240} \frac{\hbar c}{H^4} S, \quad (1)$$

where  $H$  is the separation of the two plates,  $\hbar$  is the Planck constant,  $S \gg H^2$  is the area and  $c$  is the speed of light in vacuum. The Casimir energy per unit area between two parallel plates is

$$E(H) = -\frac{\pi^2 \hbar c}{720 H^3}. \quad (2)$$

It is noted that Eqs. (1) and (2) hold for two parallel plates of infinite conductivity. A coefficient smaller than unity should be introduced in Eqs. (1) or (2) for other dielectric cases. An important feature of the Casimir effect is that, even though it is quantum in nature, it predicts a force between macroscopic bodies.<sup>[2]</sup> This makes the Casimir effect a possible candidate for applications in both microelectromechanical systems (MEMS) and nanoelectromechanical systems (NEMS). The Casimir force actuation for NEMS has been achieved recently (see, for example, Ref. [3]). The Casimir force dominates over other forces at distances of a few nanometres. Thus, movable components in MEMS devices fabricated at distances less than 100 nm between each other often stick together due to the strong Casimir force. The Casimir force fundamentally influences the performance and yield of MEMS and NEMS devices.<sup>[2]</sup>

Buks and Roukes<sup>[4]</sup> measured the adhesion energy of gold using a micromachined cantilever. The adhesion is caused by the Casimir force with a gap of a few micrometres between the cantilever and the substrate. The Casimir force for small separation is reduced to the nonretarded van der Waals force<sup>[5]</sup> with interac-

tion energy per unit area

$$E = -\frac{A}{12\pi H^2}, \quad (3)$$

where  $A$  is the Hamaker constant (with typical values  $(0.4 \sim 4) \times 10^{-19}$  J). For the case of Au, it has been found<sup>[4]</sup> that Eq. (3) is a good approximation for the gap  $H < 2$  nm and the Hamaker constant is given by  $A = 4.4 \times 10^{-19}$  J.

Serry *et al.*<sup>[6]</sup> studied the criterion of adhesion of a microfabricated rectangular membrane strip with a parallel fixed surface by the Casimir force. Numerical simulation in Ref. [6] has shown for those systems what the adhesion-free stable equilibrium state exhibits.

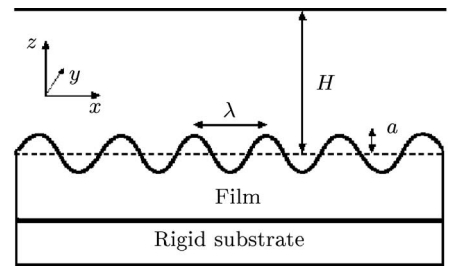


Fig. 1. Configuration of the interaction system.

The stability of epitaxial thin films by various effects has aroused great interest in the research community<sup>[7-11]</sup> due to their practical importance. To study the influence of the Casimir force in MEMS/NEMS structures, we consider the interaction of the Casimir force between a flat plate and a thin elastic epitaxial film of nominal thickness  $h_0$  on a flat substrate (illustrated in Fig. 1). The nominal gap between the film surface and the flat plate is  $H$ . The undulated film surface can be represented as a cosine

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function with wavelength  $\lambda$  and amplitude  $a$  by

$$h(x) = h_0 + a \cos\left(\frac{2\pi x}{\lambda}\right). \quad (4)$$

Such a surface profile conserves mass, since

$$\frac{1}{\lambda} \int_0^\lambda h(x) dx = h_0.$$

The total surface energy change in one period of undulation due to the film surface perturbation is<sup>[7,8]</sup>

$$\Delta E_{\text{surface}} = \pi^2 \frac{\gamma_0 a^2}{\lambda}, \quad (5)$$

where  $\gamma_0$  is the surface energy density of the thin elastic film. We suppose that the intrinsic stress of the film is  $\sigma$ . The elastic energy variation per period of the perturbation is given by<sup>[7,8]</sup>

$$\Delta E_{\text{elastic}} = -\pi \frac{\sigma^2 a^2}{E}. \quad (6)$$

From Eqs. (5) and (6) one knows that the elastic strain energy decreases but the surface energy increases when the flat film is undulated into a wavy shape. If the stability of the epitaxial film on a substrate is determined primarily by the competition between surface energy and elastic strain energy, then the critical wavelength is

$$\lambda_{\text{cr}} = \pi \frac{E\gamma_0}{\sigma^2}. \quad (7)$$

The film surface will be stable if the actual wavelength of undulation is less than such a critical value.<sup>[7]</sup>

To incorporate the Casimir force on the surface stability of the epitaxial film, the Casimir energy variation can be denoted by<sup>[12]</sup>

$$\begin{aligned} \Delta E_{\text{Casimir}} = & -\frac{\hbar c}{H^3} \left\{ \frac{\pi^2}{720} + \frac{a^2}{H^2} \left[ G_{\text{TM}}\left(\frac{H}{\lambda}\right) \right. \right. \\ & \left. \left. + G_{\text{TE}}\left(\frac{H}{\lambda}\right) \right] \right\}, \end{aligned} \quad (8)$$

where the electromagnetic (EM) field between the plates is described by the superposition of two independent scalar fields: one is the transverse magnetic (TM) wave, and the other is the transverse electric (TE) wave. Two different conditions can be distinguished for Eq. (8). One is the limit  $\lambda \gg H$ . The variation of the Casimir energy due to the film surface undulation is given by<sup>[12]</sup>

$$\Delta E_{\text{Casimir}} = -\frac{\pi^2 \hbar c a^2}{240 H^5}. \quad (9)$$

Therefore, from the minimum of free energy

$$\Delta E = \Delta E_{\text{surface}} + \Delta E_{\text{elastic}} + \Delta E_{\text{Casimir}} = 0,$$

we have

$$\frac{\pi^2 \gamma_0 a^2}{\lambda} - \frac{\pi \sigma^2 a^2}{E} - \frac{\pi^2 \hbar c a^2}{240 H^5} = 0, \quad (10)$$

which yields the critical wavelength for surface stability

$$\lambda_{\text{cr}} = \pi \frac{E\gamma_0}{\sigma^2} \left( 1 + \frac{\pi \hbar c E}{240 \sigma^2 H^5} \right)^{-1}. \quad (11)$$

The Casimir force will be negligible if

$$\frac{\pi \hbar c E}{240 \sigma^2 H^5} \ll 1, \quad (12)$$

which predicts that the gap between the surface of the thin film and the flat plate should satisfy

$$H \gg \left( \frac{\pi \hbar c E}{240 \sigma^2} \right)^{1/5}. \quad (13)$$

When inequalities (12) or (13) are met, Eq. (11) can be reduced to Eq. (7), which is the known classical result.<sup>[7]</sup>

On the other hand, in the opposite limit  $\lambda \ll H$ , the Casimir energy change due to the film surface undulation is given by

$$\Delta E_{\text{Casimir}} = -\frac{\pi^3 \hbar c a^2}{360 \lambda H^4}. \quad (14)$$

Therefore, from the requirement of minimum free energy of the system we have

$$\frac{\pi^2 \gamma_0 a^2}{\lambda} - \frac{\pi \sigma^2 a^2}{E} - \frac{\pi^3 \hbar c a^2}{360 \lambda H^4} = 0, \quad (15)$$

which yields the critical wavelength for stable surface

$$\lambda_{\text{cr}} = \pi \frac{E\gamma_0}{\sigma^2} \left( 1 - \frac{\pi^2 \hbar c}{360 \gamma_0 H^4} \right). \quad (16)$$

The positive wavelength in Eq. (16) requires

$$H > \left( \frac{\pi^2 \hbar c}{360 \gamma_0} \right)^{1/4}. \quad (17)$$

If inequality (17) is not met, undulation with any arbitrary wavelength will be unstable. It is also interesting that Eq. (16) is reduced to the known classical result in Eq. (7) provided that

$$H \gg \left( \frac{\pi^2 \hbar c}{360 \gamma_0} \right)^{1/4}. \quad (18)$$

In other words, the Casimir effect can be neglected if condition (18) is met. Equations (11) and (16) show that the consideration of the Casimir force decreases the critical wavelength of the surface undulation.

In conclusion, we have reported on the theoretical study of the influence of the Casimir force on the morphological stability of epitaxial thin elastic films.

It is noted that both the Casimir effect and the elastic fields are the driving effects for surface undulation, while the surface energy serves as the restraining effect. Critical undulation wavelengths for two limiting cases are derived from the minimum of the free energy of the system. The two newly derived results can be reduced to the known classical one if the Casimir force is neglected. When the two parallel plates are sufficiently close, the Casimir force may have an important contribution to the surface evolution. The consideration of the Casimir force in the two foregoing limiting cases decreases the critical wavelength of the surface perturbation.

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