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A Probabilistic Model for Fatigue Crack Propagation Analysis

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Abstract — A simple probabilistic model for predicting crack growth behavior under random loading is presented. In the model, the parameters c and m in the Paris-Erdogan Equation are taken as random variables, and their stochastic characteristic values are obtained through fatigue crack propagation tests on an offshore structural steel under constant amplitude loading. Furthermore, by using the Monte Carlo simulation technique, the fatigue crack propagation life to reach a given crack length is predicted. The tests are conducted to verify the applicability of the theoretical prediction of the fatigue crack propagation.

Key words: *probabilistic model*; *fatigue crack propagation*; *random loading*; *Monte Carlo simulation*; *offshore structural steel E36 – Z35*

1. Introduction

The failure of engineering structures is well known to be the fatigue of the components due to cyclic stress and the analysis of fatigue crack propagation is one of the most important tasks in the design and life prediction of fatigue-critical structures. The Paris-Erdogan law (Paris and Erdogan, 1963) has been applied to the propagation of fatigue cracks in the following from:

$$\frac{\mathrm{d}a}{\mathrm{d}N} = c(\Delta K)^m \tag{1}$$

in which da/dN is the crack growth, ΔK the stress intensity factor range, and both c and m the test constants defined by material parameters, environmental temperatures and loading frequencies as well (Duan, 1999a; 1999b). The traditional analytical method in engineering fracture mechanics usually assumes that crack size, stress level, material properties and crack growth rate, etc. are all deterministic values which will lead to conservative results. However, many test results and field data of crack propagation (Virkler *et al.*, 1979) presented a consideriable variability even in well-controlled laboratory conditions. This has been addressed by randomizing the material parameters in the Paris-Erdogen law (Kozin and Bogdanoff, 1983; Yang *et al.*, 1983; Lin and Yang, 1983; Ghonem and Dore, 1987; Ding *et al.*, 1999). For example Virkler *et al.* (1979) chose c from an appropriate distribution in such a way that each increment of crack growth was uncorrelated with the next. At the

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other extreme, Yang *et al*. (1983) considered the case in which the values of c for each successive cycle are highly correlated. Between these extremes, Lin and Yang (1983) modeled the fatigue as a diffuse Markov process whose critical parameters are determined from an experimental data base under constant amplitude loading conditions. A major problem for such analysis is the difficulty of obtaining sufficient data, and fatigue experiments are usually time consuming with the test data inevitably showing certain degrees of deviation even in a well controlled laboratory. To overcome these difficulties, numerical simulation of fatigue crack propagation data has become a very important and efficient tool in the study of fatigue-related problems (Itagaki and Shinozuka, 1972; Cheng, 1988). This is especially true in the case where the fatigue damage is resulted from random loading. Due to the random nature of the loads, although a lot of experiments have been performed for the past three decades (Kitagawa *et al*., 1978; Alawi and Shaban, 1989), all the data support the above conclusion.

The purpose of this paper is to present a probabilistic model for predicting fatigue crack growth life under random loading. Crack growth parameters c and m in the Paris-Erdogan law are considered as random variables, and their stochastic characteristic values can usually be obtained from a few constant-amplitude fatigue tests. The fatigue crack growth life is predicted by using the Monte Carlo simulation technique. The predicted results are in close agreement with those of crack growth experiments on offshore structural steel E36-Z35.

2. Description of Response by Random Loading

Random loading can be described by a power spectrum $w(f)$. A particularly common form of response for lightly damped structures takes narrow band spectrum. This occurs when a structure is excited over a wide range of frequency but responds at a well-defined single frequency. In this case, the signal of stress response takes Gaussian distribution with a root mean square of σ and a zero mean value, and the peaks k of stress response have a probability density function known as the Rayleigh distribution (Huang and Hancock, 1989):

$$p(k) = \frac{k}{\sigma^2} \exp\left(\frac{-k^2}{2\sigma^2}\right). \tag{2}$$

The probability of finding a peak between k and $k + \mathrm{d}k$ is then $p(k)\mathrm{d}k$, and for a stationary process, this is independent of time. However, in fatigue it is the range, s, that is of interest, in a statistical sense

$$s = 2k. \tag{3}$$

The probability density function of the ranges $p(s)$ is then obtained using the transformation

$$p(s)\mathrm{d}s = p(k)\mathrm{d}k; \tag{4}$$

$$p(s) = \frac{s}{4\sigma^2}\exp\left(\frac{-s^2}{8\sigma^2}\right). \tag{5}$$

As the bandwidth of the loading increases, the form of the signal changes to broad band. In this signal, the cycles are not clearly defined. However, several methods of cycle counting have been proposed including rain-flow counting method and range method (Dowling, 1972). These cycle counting algorithms allow broad band signals to be related to narrow band or simple sinusoidal loading. Unfortunately, rigorous analytic expressions for the distribution of rainflow cycles in broad band random signals have yet to be derived. But from experimental measurements of analogue signals, it has been suggested that the cumulative distribution of cycles should be expressed by a Weibull curve fitting function of the form

$$p\left(\frac{s}{\sigma}\right) = 1 - \exp\left[-\left(\frac{s}{\gamma\sigma}\right)^n\right] \tag{6}$$

where γ and n depend upon the bandwidth parameter ε

$$n = 2 - \varepsilon^2 \qquad (\varepsilon \leq 2/3)\ ; \tag{7}$$

$$\gamma = \sqrt{2}(2 - \varepsilon^2) \qquad (\varepsilon \leq 2/3)\ . \tag{8}$$

The Rayleigh distribution characteristics of narrow band random loading are recovered by substituting $\varepsilon = 0$ when $n = 2$ and $\gamma = 2\sqrt{2}$.

3. Crack Propagation Under Random Loading

During each cycle of the fatigue loading, the crack is assumed to propagate according to the current stress intensity range ΔK such that the increment of the crack growth $\mathrm{d}a$ is independent of the previous stress cycling and there is no interaction effect between cycles. The stress intensity factor can be written in the form

$$\Delta K = fs\sqrt{\pi a} \tag{9}$$

where f is a function of the geometry of the component (Rooke and Cartwright, 1985). The crack growth rate then becomes

$$\frac{\mathrm{d}a}{\mathrm{d}N} = c[fs\sqrt{\pi a}]^m . \tag{10}$$

The increments of crack length due to the individual stress cycle S_1, S_2, ..., S_m are

$$\left.\begin{aligned} \Delta a_1 &= c[s_1\sqrt{\pi a}]^m f^m(a) \\ \Delta a_2 &= c[s_2\sqrt{\pi a}]^m f^m(a) \\ &\ \vdots \qquad \vdots \\ \Delta a_m &= c[s_m\sqrt{\pi a}]^m f^m(a) \end{aligned}\right\} \tag{11}$$

The average crack growth rate $E[\mathrm{d}a/\mathrm{d}N]$ then becomes

$$E\left[\frac{\mathrm{d}a}{\mathrm{d}N}\right] = c\left[f\sqrt{\pi a}\right]^m \int_0^\infty s^m p(s)\mathrm{d}s \tag{12}$$

In non-dimensional manner, Eq. (10) takes the form

$$\frac{\mathrm{d}\left(\frac{a}{a_0}\right)}{\mathrm{d}N} = \lambda f^m \pi^{m/2}\left(\frac{s}{\sigma}\right)^m\left(\frac{a}{a_0}\right)^{m/2} \tag{13}$$

where λ is defined as

$$\lambda = c\sigma^m a_0^{(m-2)/2} \tag{14}$$

and a_0 is the initial crack length. For narrow band random loading, the mean value of $(s/\sigma)^m$ denoted by μ_s, is

$$\mu_s = E\left[\left(\frac{s}{\sigma}\right)^m\right] = 8^{m/2}\Gamma\left(\frac{m}{2}+1\right) \tag{15}$$

where $\Gamma(\ )$ is the gamma function.

The corresponding standard deviation of $(s/\sigma)^m$ is

$$\sigma_s = 8^{m/2}\left[\Gamma(m+1) - \Gamma^2\left(\frac{m}{2}+1\right)\right]^{1/2}. \tag{16}$$

For a broad band loading, the distribution of stress cycles obtained by the rain-flow algorithm can be approximated by Eq. (6), and the corresponding definitions of μ_s and σ_s are as follows:

$$\mu_s = \gamma^m\Gamma\left(\frac{m}{n}+1\right); \tag{17}$$

$$\sigma_s = \gamma^m\left[\Gamma\left(\frac{2m}{n}+1\right) - \Gamma^2\left(\frac{m}{n}+1\right)\right]^{1/2}. \tag{18}$$

Therefore, the fatigue crack growth rate under random loading can be expressed in the general form:

$$\frac{\mathrm{d}\left(\frac{a}{a_0}\right)}{\mathrm{d}N} = \lambda f^m(a)\pi^{m/2}\left(\frac{a}{a_0}\right)^{m/2}\mu_s; \tag{19}$$

or

$$\frac{\mathrm{d}a}{\mathrm{d}N} = c\sigma^m f^m(a)\pi^{m/2}a^{m/2}\mu_s. \tag{20}$$

Assuming that a_0 is the initial crack size and ΔN the cycle step length, integrating Eq. (20) yields

$$\left.\begin{aligned}
a_1 &= \int_0^{\Delta N} c\sigma^m \pi^{m/2} f^m(a) a^{m/2} \mu_s \mathrm{d}N + a_0 = F_a^{(1)} \Delta N + a_0 \\
N_1 &= \Delta N \\
a_2 &= \int_{N_1}^{N_1+\Delta N} c\sigma^m \pi^{m/2} f^m(a_1) a_1 m^{m/2} \mu_s \mathrm{d}N + a_1 = F_a^{(2)} \Delta N + a_1 \\
N_2 &= N_1 + \Delta N \\
&\vdots \qquad \vdots \\
a_n &= \int_{N_{n-1}}^{N_{n-1}+\Delta N} c\sigma^m \pi^{m/2} f^m(a_{n-1}) a_{n-1}{}^{m/2} \mu_s \mathrm{d}N + a_{n-1} = F_a^{(n)} \Delta N + a_{n-1} \\
N_n &= N_{n-1} + \Delta N
\end{aligned}\right\} \tag{21}$$

where a_n adds up to a given crack length, and N_n is the appropriate crack growth life.

4. Materials and Test Specimens

The material used in these tests is E36-Z35 steel prescribed for key structures of offshore platforms. The mechanical properties of this material are given in Table 1, while the chemical composition is shown in Table 2.

Table 1 Mechanical properties of steel E36-Z35

σ_s (MPa)	σ_b (MPa)	δ (%)	E (MPa)
42	57	34	206.2

Table 2 Chemical composition of steel E36-Z35

C	Si	Mn	P	S	Cu	Al	Nb
0.16	0.33	1.34	0.10	0.01	0.02	0.49	0.35

The specimens are three-point bending ones with dimensions of 28 mm in thickness, 85 mm in width and 370 mm in length. All specimens are pre-cracked to 8 mm in initial length under constant-amplitude loading at room temperature. To three-point bending specimen, $f(a)$ takes the following form:

$$f(a) = \frac{1}{B\sqrt{w}}\left[7.51 + 3\left(\frac{a}{w} - 0.5\right)^2\right] \frac{\sqrt{\tan\left(\frac{\pi a}{2w}\right)}}{\cos\left(\frac{\pi a}{2w}\right)} \tag{22}$$

where B and w are the thickness and width of the specimen respectively.

5. Analysis of Test Results

The tests are carried out on a servohydraulic MTS machine. The test frequency is 10 Hz. The carck length during fatigue tests at room temperature can be measured directly by a microscope. Two kinds of effective stress amplitude distributions are investigated. One has a probability density function known as the Gaussian distribution with the mean value of 11.10 kN and the variance of 0.55 kN. The other has the Weibull probability density function with a shape parameter of 1 and scale parameter of 11.85 kN. The experimental results are shown in Figs. 1(a) and 1(b), respectively. In Eq. (21), c and m are considered as random variables, whose means and variances are given in Table 3. The Monte Carlo simulation technique is applied to the process of the crack growth a vs N data. The simulation results are also listed in Figs. 1(a) and 1(b), respectively. It is evident that the results from the Monte Carlo simulation agree closely with those of the crack propagation experiments on steel E36-Z35, which presents the accuracy and efficiency of the developed probabilistic model for predicting fatigue crack growth life.

6. Conclusion

A probabilistic model is presented to predict fatigue crack growth life under random loading by considering material inhomogenety. Crack growth parameters c and m are taken as random variables defined by specified probability density functions. By using the Monte Carlo simulation, the crack growth life to reach a given crack length is predicted. The predicted results are in close agreement with the test data on steel E36-Z35. This probabilistic model shall be applied to predict fatigue crack growth life of a structural component subjected to random loads.

Table 3 The stochastic characteristic values of crack growth parameters c and m for steel E36-Z35

Characteristic values	$\log c$	m
Mean value	− 13.93	2.2309
Variance	0.0997	0.335
Distribution	Normal	Normal

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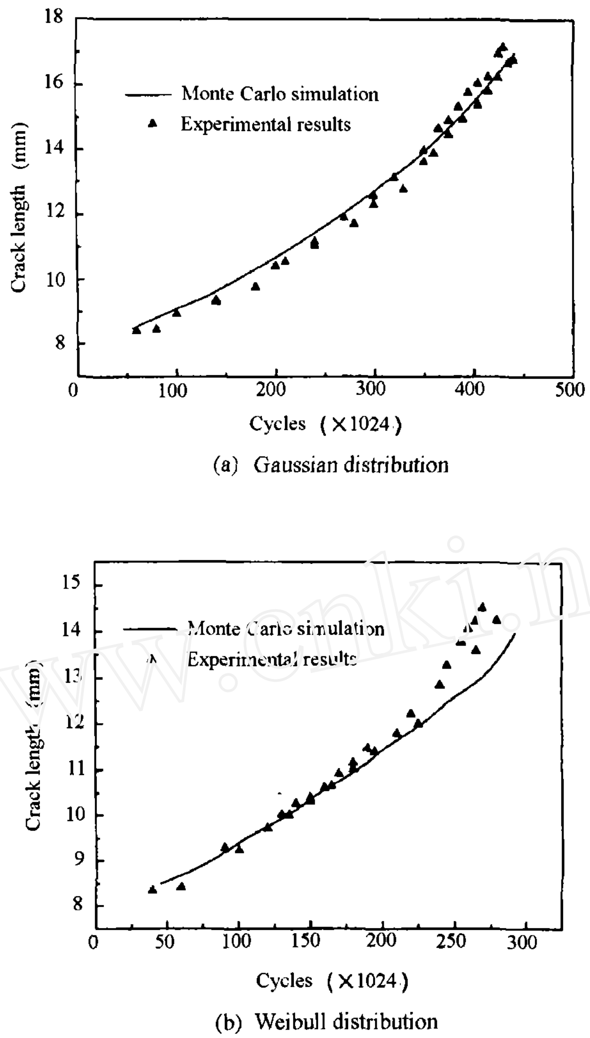

Fig. 1. Comparison of the simulation results with the test ones.

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