Effects of reconstruction matrix and controller coefficients on performances of an adaptive optics system investigated by a numerical simulation

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ABSTRACT

Phase compensation effectiveness and long time period working stability are two most important performances of an adaptive optics (AO) system. Effects of reconstruction matrix and controller coefficients on these performances are investigated by means of a pure numerical simulation in this paper. A new modified reconstruction matrix is proposed and compared with other existing matrixes in practical application. It is shown that the recently proposed reconstruction matrix can produce better phase compensation results and much better long time period working stability in an AO system. Different reconstruction matrix can greatly affect the long time period working stability under certain conditions. Under some conditions, an AO system with a reconstruction matrix can work stably in a quite long time period, but a working instability can occur in a super-long time period. It is found that the controller coefficients have even greater effect on the long time period working stability in an AO system. The controller coefficient a_I=1.00 is a marginally stable (sub-stable) condition for an AO system. When a_I<1, the long time period working stability of an AO system is much improved.

KEY WORDS: Adaptive optics system, Numerical simulation, Wavefront reconstruction, Controller, Working stability.

1. INTRODUCTION

A turbulent atmosphere can deteriorate optical waves which propagate in the atmosphere. An adaptive optics (AO) system can be utilized to overcome, at least partially, the atmospheric deterioration of the optical wave. Thus, the AO system has been widely used in improving the astronomical observation of ground-based telescope and the laser beam propagation in the atmosphere.[1,2]

It is also well known that modeling and numerical simulation can provide important assistances to the design, examination, investigation and application of a complex integrated system like an AO system. The present authors have been pursuing modeling and numerical simulation of atmospheric propagation of optical waves and an AO system since 1988.[3-6] In Ref. [3], a comprehensive theoretical model has been developed and coded into a numerical algorithm. A numerical simulation of the atmospheric propagation of an optical wave in the turbulence and a numerical simulation of

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the operation of an AO system in a static state were combined. Simulating the operation of a real AO system, a laser beam from a beacon propagates through a turbulent media to obtain the distorted wavefront, then the distorted wavefront is detected, reconstructed and corrected by the AO system. Finally, a phase-compensated beam from the main laser propagates in the same but transported (because of the time delay in the AO system and the lateral wind and/or lateral movements of target and main laser) turbulent media again to reach the target. The direct wavefront gradient control method is used to reconstruct the wavefront phase and the long-exposure Strehl ratio is used to evaluate the performances of the AO system in that paper. In Ref. [4], a theoretical model of numerical simulation of an AO system by modal wavefront reconstruction (i.e. by means of the Zernike polynomials expansion) was presented and a corresponding computer program was compiled. A numerical simulation investigation of an AO system by means of the modal wavefront reconstruction was carried out in that paper. It is known that there are three important factors to affect the performances of an AO system. The first of them is the limited spatial bandwidth resulted from the limited numbers of the sub-apertures for wavefront detection and the actuators in the deformable mirror for wavefront correction. This factor was investigated in Refs. [3,4]. The second is effects of noise and detection error. The third is the limited temporal bandwidth resulted from the limited response speed for wavefront detection, reconstruction and correction. Refs. [5,6] focused on the effects of noise and detection error in an AO system and on the dynamic control process and frequency response characteristics in an AO system, respectively. Thus, a comprehensive modeling and numerical simulation of an AO system were completed in this way.

Essentially, the phase compensation effectiveness and the long time period working stability are two most important performances of an AO system. Furthermore, both the phase compensation effectiveness and the long time period working stability must be reasonably good at the same time in an AO system. When any one of them is poor, the AO system cannot work properly. In a certain sense, the long time period working stability is more important in an AO system. However, this factor is very rare to be investigated either experimentally or by means of a numerical simulation. It is found that the reconstruction matrix and the controller coefficients have great effects on these performances of an AO system. These effects are investigated by means of a pure numerical simulation in this paper for the first time within our knowledge.

This paper is arranged as follows. A new modified reconstruction matrix is proposed and compared with the existing matrixes in a practical AO system by a numerical simulation in Section 2. The long time period and super long time period working stabilities in an AO system by using different reconstruction matrixes are investigated in Section 3. It is found in Section 4 that the controller coefficient a_1 has even more important effluence on the super long time period working stability in an AO system. Some conclusions are presented in Section 5.

2. A NEW MODIFIED RECONSTRUCTION MATRIX AND ITS COMPARISON WITH OTHERS

In the modern AO systems, Hartmann-Shack sensor (HS) and deformable mirror (DM) are used to detect and correct the distorted wavefront, respectively.[1] Wavefront reconstruction calculation is carried out in real time to obtain the control voltages of the DM from slopes (or gradients) of the HS subapetures in an AO system.

The wave field of the beam which radiates from a beacon propagates through a turbulent medium and arrives a HS. The

HS is divided into several subapertures. The wave field of each subaperture propagates to reach the focal plane of the HS and the position of the optical center (centroid) can be determined. Taking the position of optical center of the focused plane wave field of the subaperture as 0, the average slopes of i-th subaperture in the x and y directions G_{xi} and G_{yi} (i=1, 2, …m, m is the subapeture number) can be calculated. The direct gradient control method is one of the simplest and quite useful wavefront reconstruction algorithms.[7] In this algorithm, it can be shown that

$$G_{xi} = \sum_{j=1}^{K} G_{vxij} V_j \tag{1}$$

$$G_{yi} = \sum_{j=1}^{K} G_{vyij} V_j \tag{2}$$

where

$$G_{vxij} = \frac{1}{s_i} \iint_{S_i} \frac{\partial\, R_j(x,y)}{\partial\, x} dxdy \tag{3}$$

$$G_{vyij} = \frac{1}{s_i} \iint_{S_i} \frac{\partial\, R_j(x,y)}{\partial\, y} dxdy \tag{4}$$

Here $\iint_{S_i}$ is the integral on i-th subaperture, S_i is the area of i-th subaperture. G_{vxij} and G_{vyij} can be considered as the average slopes on the i-th subaperture produced by a unit displacement of the j-th actuator. On the other hand, the surface shape of the DM that is the corrected wavefront can be expressed as

$$\psi_m(x,y) = \sum_{j=1}^{K} V_j R_j(x,y), \tag{5}$$

where K is the total number of actuators in the DM, V_j is the voltage of the j-th actuator, and $R_j(x,y)$ is the influence function of the j-th actuator which describes the effect of a unit displacement of the j-th actuator on the surface shape of the DM. In principle, the influence function of each actuator may be different and this effect can be included in the numerical simulation. For this study, it is assumed that all the actuators of the DM have same Gaussian influence function

$$R_j(x,y) = \frac{2\pi}{\lambda} e^{\ln b \left[(x-x_j)^2 + (y-y_j)^2 \right] / d^2}, \tag{6}$$

where b is the coupling factor between the adjacent actuators, x_j and y_j are the coordinates of the j-th actuator and d is the distance between the adjacent actuators.

In matrix form

$$G = G_v V, \tag{7}$$

where $G = [G_1, G_2, ..., G_{2m}]^T$ is the slope vector and $V = [V_1, V_2, ..., V_K]^T$ is the control voltage vector and G_v is the response matrix (see Eqs.(3) and (4)). The method of obtaining G_v is quite easy in an experiment by measuring the slope response on each subapeture while adding a unit voltage to the DM actuators one by one. This method can be also used in a numerical simulation in a quite similar way. In the direct gradient reconstruction algorithm, Eq.(7) can be solved using the method of least squares to obtain

$$V = G_v^+ G, \tag{8}$$

where G_v^+ is the generalized inverse matrix with least-square and minimum norm of matrix G_v. G_v^+ can be calculated by the singular value decomposition (SVD) method. G_v^+ is the control matrix of the direct gradient wavefront reconstruction. G_v is a $2m \times K$ matrix and G_v^+ is a $K \times 2m$ matrix.

The configuration of the DM actuators and the HS subapetures in a practical AO system is shown in Fig. 1. There are 32 actuators locating on the grids of 20 square subapetures. That is $m = 20$ and $K = 32$ in this system. The diameter of the telescope mirror is assumed to be 0.6 meter.

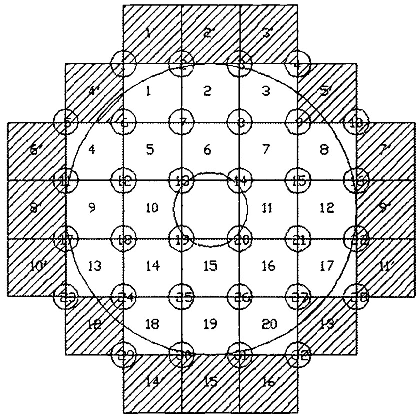

Fig. 1 Configuration of DM actuators and HS sensor subapetures in a 32-element AO system.

Generally, for the configuration of Fig. 1, G_v is a 40×32 matrix and G_v^+ is a 32×40 matrix. In order to ensure separation of the tip-tilt mirror control with the DM control and improve stability of the deformable mirror control, Xinyang Li and his colleagues proposed some restrain conditions for the actuator voltages in the DM as follows:[8]

$$\sum_{j=1}^{K} V_j x_j = 0, \quad \sum_{j=1}^{K} V_j y_j = 0, \quad \sum_{j=1}^{K} V_j = 0. \tag{9}$$

After a proper treatment, three control matrixes having small differences were obtained. They are called in this paper as Z-2, Z-3 and Z-4 matrix, respectively. Among them, the Z-4 matrix is thought to have the best performance. Thus, the Z-4 matrix is chosen to do a comparison in this paper.

We propose a new approach to produce a modified reconstruction matrix in this paper. When obtaining the response matrix G_v by measuring the slope response on each subapeture while adding a unit voltage to the DM actuators one by one in a numerical simulation, some additional blank subapetures are artificially supplemented to the original ones. In Fig. 1, these additional blank subapetures are shown as those with shadow. Their total number in present case is 16. In this way, a response matrix G_v' with 72×32 elements can be obtained. Then, by using the common SVD method a control matrix $G_v^{+'}$ with 32×72 elements can be deduced. Finally, all the columns in the $G_v^{+'}$ matrix corresponding to the blank subapetures are removed to obtain the final control matrix G_v^+ with 32×40 elements which is used in our subsequent numerical simulation.

A laser propagation scene and an AO system with dynamic control process[6] but without noise and detection effects[5] are chosen in this paper to compare the recently proposed reconstruction matrix with some other reconstruction matrixes such as the Z-4 matrix. A comparison is shown in Fig. 2. The Strehl ratio *STRCC* in Ref. 3 is used as an evaluation parameter. The *STRCC* is defined as a ratio of the optical energies within a circle around the centroid (optical center) with a radius of the first dark ring in the Airy pattern after propagations through the turbulent medium and through a vacuum. There are two kinds of *STRCC* which are used in this paper. One of them is called as instantaneous *STRCC*. Another of them is called as long-exposure *STRCC*. In calculating the instantaneous *STRCC*, the centroid and the Strehl ratio are determined from an optical pattern at target corresponding to each dynamic iteration. A large number of patterns at target corresponding to different time steps are accumulated to obtain a long-exposure pattern. In calculating the long-exposure *STRCC*, the centroid and the Strehl ratio are determined from the accumulated long-exposure pattern at target.

In the practical operation of an AO system hardware, a difference equation

$$U_n = a_1 U_{n-1} + a_2 U_{n-2} + a_3 U_{n-3} + \cdots + b_1 E_n + b_2 E_{n-1} + b_3 E_{n-2} + \cdots \tag{10}$$

connects the corrected wavefront at the *n*th time step U_n with the corrected wavefronts at the foregoing time steps U_{n-1}, U_{n-2}, ... and the residual wavefronts at the present time step and the foregoing time steps E_n, E_{n-1}, E_{n-2}, ... A different controller shows a different performance in terms of the difference of this connection, i.e. in terms of the different coefficients in Eq. (10). See Ref. 6 in more details.

It is shown in Fig. 2 that both the phase compensation effectiveness and long time period working stability by using our reconstruction matrix are obviously better than those by using the Z-4 matrix. Because the same turbulent phase screens are used for two numerical simulation computations, it can be seen from Fig. 2 that at a certain time step when the instantaneous *STRCC* by using Z-4 matrix goes lower, the instantaneous *STRCC* by using our reconstruction matrix goes lower as well. But generally the instantaneous *STRCC* by using our reconstruction matrix is higher than that by using Z-4 matrix at almost all the time steps.

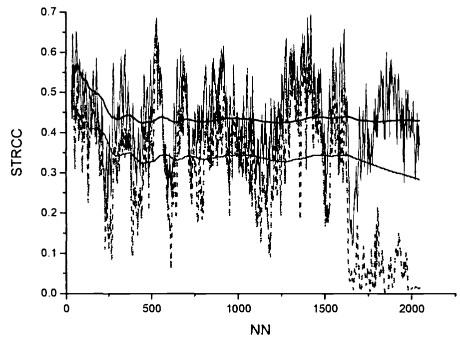

Fig. 2 Comparison of instantaneous and long-exposure *STRCCs* of long time period dynamic iteration by using our reconstruction matrix with those by using the Z-4 matrix. (1)

Wherein Solid line: results by using our reconstruction matrix; Dashed line: results by using the Z-4 matrix.

Calculation conditions: r_0=10 cm, *STRCC* (open loop)=0.1498; the frame rate in the AO system is 1500 Hz; the controller coefficients a_1=1.0, b_1=0.30, others=0; focusing factor in the numerical simulation is 12; the starting position of the dynamic iteration NW=0.

3. LONG TIME PERIOD WORKING STABILITY IN AO SYSTEM

3.1 Long time period working stability in an AO system

It can be seen from Fig. 2 that under a certain turbulence condition (for example, at the time step number NN around 1660 in Fig. 2) the AO system seems to undergo a strong disturbance and the instantaneous *STRCC* decreases abruptly. The difference between our reconstruction matrix and the Z-4 matrix is that our reconstruction matrix can "endure" this strong disturbance and the instantaneous *STRCC* can recover back to a normal value soon, while the Z-4 matrix cannot "endure" this strong disturbance and the instantaneous *STRCC* cannot recover back to a normal value again.

In order to do a deeper investigation, more phase compensation results of the AO system with the dynamic control process and without the dynamic control process (i.e. the AO system in the static state) by using different reconstruction matrixes at certain time steps are summarized in Table 1. It can be seen from Table 1 that when an AO system can work stably no matter by using our reconstruction matrix or by using the Z-4 matrix, the corresponding instantaneous *STRCC*s with and without dynamic control are comparable, although the results by using the Z-4 matrix are slightly lower. Further, when working instability appears in the AO system, the instantaneous *STRCC*s with dynamic control are obviously lower than the instantaneous *STRCC*s without dynamic control at the same time step.

Table 1 Comparison of phase compensation results *STRCC*s with and without dynamic control (DC) by using different reconstruction matrixes.

Calculation conditions: see Fig. 2.

r_0 (cm)	NN	By using our reconstruction matrix			By using the Z-4 matrix		
		With DC Instanta-neous	With DC Long-exposure	Without DC Instanta-neous	With DC Instanta-neous	With DC Long-exposure	Without DC Instanta-neous
10	1760	0.3269	0.4284	0.3111	0.0038	0.3217	0.2286
	1860	0.6012	0.4287	0.6068	0.0100	0.3070	0.4498
	2010	0.4464	0.4302	0.4556	0.0141	0.2878	0.3302
	2040	0.3650	0.4291	0.3954	0.0132	0.2837	0.3475
20	1760	0.6957	0.7358	0.6884	0.6536	0.6730	0.6381
	1860	0.8596	0.7365	0.8572	0.7934	0.6744	0.7972
	2010	0.7365	0.7361	0.7573	0.7462	0.6742	0.7081
	2040	0.6123	0.7346	0.6905	0.6019	0.6732	0.7114
30	1760	0.8304	0.8365	0.8123	0.8020	0.7998	0.7935
	1860	0.9229	0.8370	0.9159	0.8850	0.8007	0.8785
	2010	0.8429	0.8364	0.8651	0.8590	0.8004	0.8420
	2040	0.7362	0.8356	0.7863	0.7639	0.8000	0.8394

When the working instability appears in an AO system such as when using the Z-4 matrix at NN=1760 (NW=0), the reconstructed wavefront (i.e. the corrected wavefront) is shown in Fig. 3. The corresponding original wavefront i.e. the distorted wavefront before a phase correction is shown in Fig. 4. It can be seen from a comparison of Fig. 3 with Fig. 4 that when the working instability appears the reconstructed wavefront is quite different in comparison to the original one. Especially, some obvious fluctuations in the form of wave appear in the reconstructed wavefront. This great difference of the reconstructed wavefront with the original one destroys the phase correction capability of the AO system. On the other hand, when the AO system works stably the reconstructed wavefront including the dynamic control process should be quite similar to the original wavefront. The reconstructed wavefront obtained from the dynamic iteration at the same time step by using our reconstruction matrix is shown in Fig. 5. Apparently, there is no fluctuation like wave. The great fluctuations in the form of wave in the reconstructed wavefront have very close relationship with the working instability in an AO system.

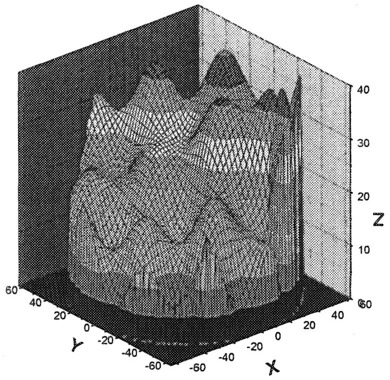

Fig. 3 The reconstructed wavefront with dynamic control by using the Z-4 matrix.

Calculation conditions: see Fig. 2; the time step number NN=1760. The instantaneous dynamic *STRCC*=0.0038.

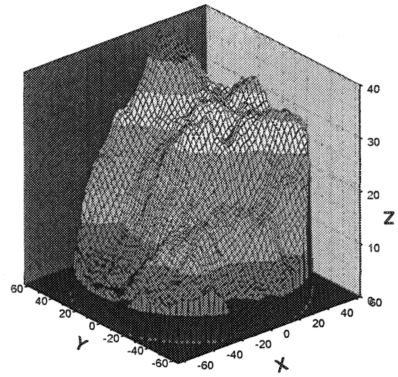

Fig. 4 The original distorted wavefront before phase correction corresponding to that in Fig.3.

Calculation conditions: same as those in Fig. 3.

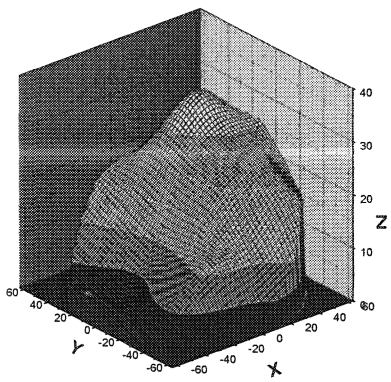

Fig. 5 The reconstructed wavefront with dynamic control by using our reconstruction matrix.

Calculation conditions: same as those in Fig. 3. The instantaneous dynamic *STRCC*=0.3269.

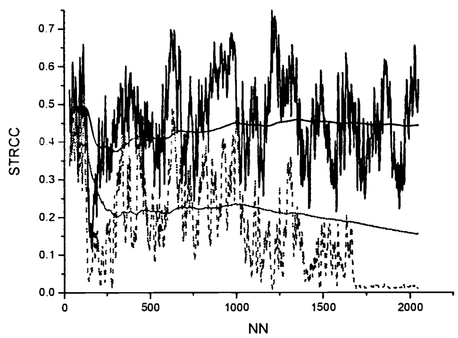

Fig. 6 Comparison of phase compensation results of long time period dynamic iteration by using our reconstruction matrix with those by using the Z-4 matrix. (2)

Calculation conditions: Same as those in Fig. 2 besides the starting position of the dynamic iteration NW=1500.

It can be seen from Fig. 2 that a strong disturbance appears at around NN=1660. In order to study the effect of this strong disturbance on working instability in the AO system in more details, the starting position of the dynamic iteration expressed by time step number NW is changed from NW=0 into NW=1500 and the computational results are shown in Fig. 6. It can be seen from Fig. 6 that indeed there is a strong disturbance at the same position in the turbulence phase screen and its effluences on phase compensation and working stability are very obvious. Again, the AO system by using our reconstruction matrix can work quite well, our reconstruction matrix can "endure" this strong disturbance and the instantaneous dynamic *STRCC*s can recover back to a normal value soon. A change of the starting position NW does not have obvious effects on the AO system by using our reconstruction matrix.

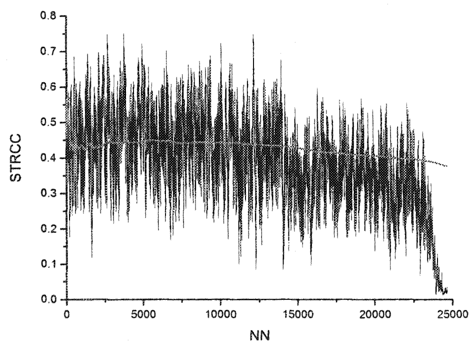

Fig. 7 Instantaneous and long-exposure *STRCC*s of super long time period dynamic iteration by using our reconstruction matrix to show working instability in an AO system. (1)

Calculation conditions: r_0=10 cm; the controller coefficients a_1=1.0, b_1=0.30, others=0; focusing factor in the numerical simulation is 12; others are same as those in Fig. 2.

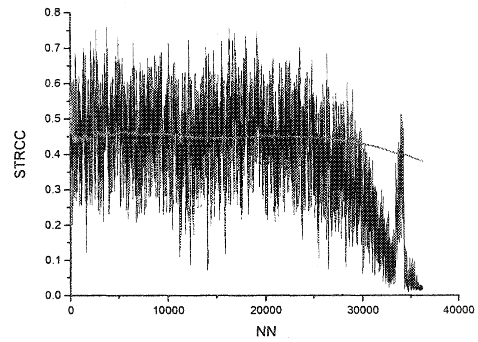

Fig. 8 Instantaneous and long-exposure *STRCC*s of super long time period dynamic iteration by using our reconstruction matrix to show working instability in an AO system. (2)

Calculation conditions: same as those in Fig. 7 besides focusing factor in the numerical simulation is 16.

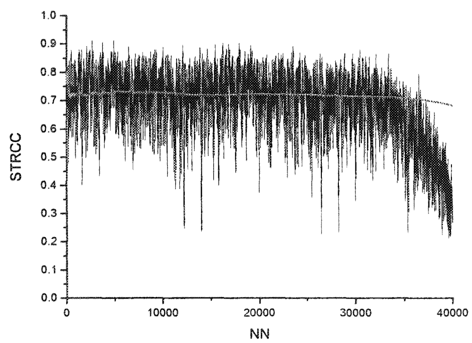

Fig. 9 Instantaneous and long-exposure *STRCC*s of super long time period dynamic iteration by using our reconstruction matrix to show working instability in an AO system. (3)

Calculation conditions: r_0=20 cm; the controller coefficients a_1=1.0, b_1=0.20, others=0; focusing factor in the numerical simulation is 16; others are same as those in Fig. 2.

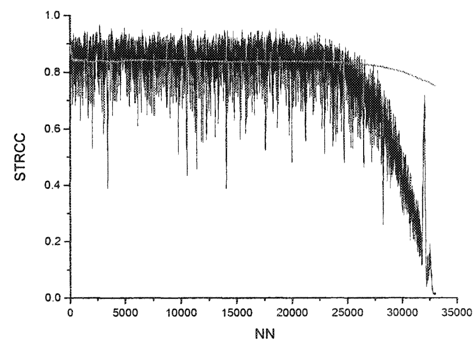

Fig. 10 Instantaneous and long-exposure *STRCC*s of super long time period dynamic iteration by using our reconstruction matrix to show working instability in an AO system. (4)

Calculation conditions: r_0=30 cm; the controller coefficients a_1=1.0, b_1=0.30, others=0; focusing factor in the numerical simulation is 12; others are same as those in Fig. 2.

3.2 Super long time period working stability in an AO system

It is found under some conditions when doing super long time period dynamic iteration a working instability can appear in the AO system even by using our reconstruction matrix. Several typical results obtained by our numerical simulations are shown in Figs. 7-10. It can be seen from these figures that an AO system can work for a quite long time period, but under some conditions it cannot work stably in a longer time period, i.e. in a super long time period. The controller coefficient b_1 and focusing factor have limited effects on the working stability in an AO system (see Figs. 7-9). Furthermore, even in quite weak turbulence it is possible to appear a working instability also (see Figs. 7,9,10) although a working instability seems to appear later in a weaker turbulence. Again, when a working instability appears, some obvious fluctuations in the form of wave also appear in the reconstructed wavefront by using our reconstruction matrix.

4. IMPORTANT EFFILUENCE OF CONTROLLER COEFFICIENT a_1 ON LONG TIME PERIOD WORKING STABILITY

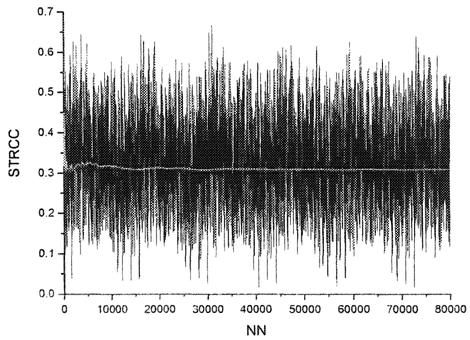

Fig. 11 Instantaneous and long-exposure *STRCCs* of super long time period dynamic iteration by using our reconstruction matrix. (5)

Calculation conditions: r_0=7 cm; the controller coefficients a_1=0.94, b_1=0.50, others=0; focusing factor in the numerical simulation is 20; others are same as those in Fig. 2.

Long-exposure *STRCC* = 0.3081.

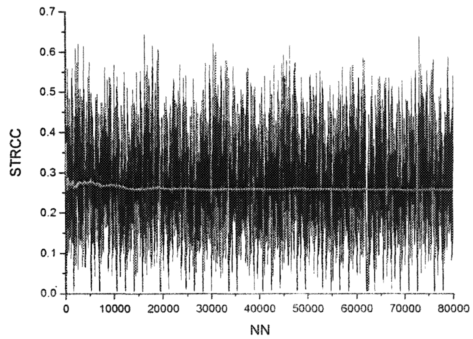

Fig. 12 Instantaneous and long-exposure *STRCCs* of super long time period dynamic iteration by using the Z-4 reconstruction matrix.

Calculation conditions: r_0=7 cm; the controller coefficients a_1=0.94, b_1=0.50, others=0; focusing factor in the numerical simulation is 20; others are same as those in Fig. 2.

Long-exposure *STRCC* = 0.2573.

The bandwidth, phase margin and gain margin in an AO system which are very important and useful parameters can be obtained conveniently from our numerical simulation of the frequency response characteristics.[6] On the base of a numerical simulation of the frequency response characteristics in the present AO system, we expected that when controller coefficient a_1<1.0 the AO system may work more stably because both the bandwidth and the phase margin are obviously larger when a_1<1.0 in comparison to those when a_1=1.0. We did numerical experiments under the condition of a_1<1.0 and found that the controller coefficient a_1 indeed has quite great effect on the long time period working stability in an AO system. The controller coefficient a_1=1.00 only has a marginal stability, in other words, a_1=1.00 is a sub-stable condition for an AO system. When a_1<1, the long time period working stability of an AO system is much improved. Under the condition of a_1<1, an AO system can work stably for a super long time period even by using the Z-4 matrix.

Similarly, the controller coefficients a_1 and b_1, focusing factor and the gain value in the Z-4 matrix can be optimized in our numerical simulation computation. The optimized parameters are chosen to obtain the computational results shown in this paper. Two typical results of super long time period dynamic iterations by using our matrix and by using the Z-4 matrix are shown in Figs. 11 and 12.

It can be seen from a comparison of Figs. 11 and 12 that when a_1<1, the AO system can work stably for a super long time period both by using our reconstruction matrix and by using the Z-4 matrix. However, the result by using the Z-4 matrix shows more portions having very low *STRCC* values and these portions having even lower *STRCC* values in comparison with the result by using our matrix. Therefore, the long-exposure *STRCC* by using the Z-4 matrix is notably lower than that by using our matrix. A comparison of the results by using our matrix with those by using the Z-4 matrix under the condition of different strength turbulence is shown in Table 2. It can be seen from it that under the condition of different strength turbulence, the phase compensation results by using our matrix are notably better than those by using the Z-4 matrix and the difference between them is more obvious when the turbulence is stronger.

Table 2 Comparison of the long-exposure *STRCC*s by using our matrix with those by using the Z-4 matrix under the condition of different strength turbulence.

Calculation conditions: the controller coefficients a_1=0.94, b_1=0.50, others=0; the number of the dynamic iteration NN=80000; others are same as those in Fig. 2.

r_0 (cm)	Focusing factor	Matrix	*STRCC*	Matrix	*STRCC*
7	20	Ours	0.3081	Z-4	0.2573
10	20	Ours	0.4943	Z-4	0.4326
20	16	Ours	0.7639	Z-4	0.7436
30	16	Ours	0.8475	Z-4	0.8403

5. CONCLUSION

Phase compensation effectiveness and long time period working stability are two most important performances of an AO system. The reconstruction matrix has great effects on both phase compensation and working stability in an AO system. A new kind reconstruction matrix is proposed in this paper. In comparison with other existing matrixes in practical application this reconstruction matrix can produce better phase compensation results and much better long time period working stability. A working instability in an AO system is closely related to some obvious fluctuations in the form of wave in the reconstructed wavefront. It is found by super long time period dynamic iterations that the controller coefficient a_1 has even greater effect on the long time period working stability in an AO system. When the controller coefficient a_1<1, the long time period working stability of an AO system is much improved. Even under these conditions the phase compensation results by using our matrix are notably better than those by using conventional matrix especially under the condition of stronger turbulence. It should be noted this paper is characterized in that a pure numerical simulation in a long time period and super long time period is utilized to investigate the phase compensation effectiveness and especially long time period working stability in an AO system. This is a straightforward way although a great amount of computer hours are needed in this way. We hope to develop a more effective and simpler approach to assess the long time period working stability of an AO system in the near future.

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