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DEBONDING AND CRACKING ENERGY RELEASE RATE OF THE FIBER/MATRIX INTERFACE

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Abstract

An investigation of fiber/matrix interfacial fracture energy is presented in this paper. Several existing theoretical expressions for the fracture energy of interfacial debonding are reviewed. For the single-fiber/matrix debonding and pull-out experimental model, a study is carried out on the effect of interfacial residual compressive stress and friction on interface cracking energy release rate. © 1998 Elsevier Science Ltd. All rights reserved

Keywords: C. residual stress, interfacial debonding, fracture work, energy release rate, fibre pull-out

1 INTRODUCTION

The fiber/matrix interfacial fracture energy release rate is one of the most significant micromechanical parameters and has a large effect on the engineering properties of composite materials. Much attention has therefore been paid to experimental methods for characterizing the interfacial fracture behavior and several theoretical models have been established in recent decades. In 1969, Outwater and Murphy[1] proposed the following formula for predicting the fiber/matrix interfacial debonding energy release rate for the first time:

$$G_d = \frac{V_f \sigma_f^2 l_d}{2E_f} \tag{1}$$

where V_f is the fiber volume fraction, σ_f is the fiber fracture stress, E_f is the fiber Young's modulus and l_d is the debonding length. For the single-fiber-reinforced matrix model (the representative unit of a unidirectional fiber composite), this formula can be rewritten as:

$$W_d = \frac{\sigma_f^2 l_d}{2E_f} \pi r_f^2 \tag{2}$$

where W_d is the work of interfacial debonding and r_f is the fiber radius. The last formula can be easily reduced to:

$$G_d = \frac{\sigma_f^2 r_f}{4E_f} \tag{3}$$

Here, G_d is the interfacial debonding energy release rate. The formula for interfacial debonding work, eqn (2), can be found in the two well-known books of Kelly[2] and Hull,[3] where it is given as:

$$W_d = \frac{\pi d^2 \sigma_f^2 l_d}{24E_f} \tag{4}$$

where d is the fibre diameter; the difference between eqns (2) and (4) is only in the two constant coefficients. However, both eqns (2) and (4) are open to question, since eqn (2) actually represents the strain energy stored in the fiber when it is stressed to its fracture strain, and generally, when debonding occurs, the stress in the fiber, σ_d, is much smaller than its fracture stress, σ_f. Therefore the present author prefers the following expression:

$$G_d = \frac{V_f \sigma_d^2 l_d}{2E_f} \tag{5}$$

This formula can be seen in Ref. 4. σ_d is debonding stress in the fiber. Written in terms of the single-fiber debonding release rate, G_i, eqn (5) becomes:

$$G_i = \frac{\sigma_d^2 r_f}{4E_f} \tag{6}$$

G_i has been measured by Chua and Piggott[5] by using the single-glass-fiber pull-out test; however, some errors were present in the formula used and the calculated result. In the analysis of brittle interfacial failure of the single-fiber pull-out test, as shown in Fig. 1, Piggott postulated that when the strain energy stored in the interface reaches or exceeds the interfacial crack propagation work, the interface debonds, and gave the following formula:[6]

$$G_i = \frac{1}{ns\tanh(ns)}\frac{\sigma_d^2 r_f}{4E_f} = f_1\frac{\sigma_d^2 r_f}{4E_f} \tag{7}$$

where

$$n = \left(\frac{E_m}{E_f(1+\nu_m)\ln(r_m/r_f)}\right)^{1/2}$$

$s = L/r_f$, L is the embedded fiber length, ν_m is the matrix Poisson's ratio and r_m is the radius of the matrix (see Fig. 1). It should be pointed out that in this formula G_i is inversely proportional to L.

In view of the discussion above, the conclusion can be drawn that the fiber/matrix interfacial fracture energy is one of the most interesting topics. Proposition of a variety of formulae has deepened our insight into this problem. However, the problem still requires further investigation. In this paper, besides reviewing the effect of the matrix elastic modulus and of the Poisson's ratios of fiber and matrix on interfacial debonding energy, the contribution of the interfacial residual compressive stress and friction to G_i will be analyzed.

2 INTERFACIAL FRACTURE ENERGY RELEASE RATE WITHOUT RESIDUAL STRESS

In an analysis of the cylindrical single-fiber/matrix pull-out test model, in which there is partial interface debonding (Fig. 2), Charalambides and Evans[7] derived the following formula by using the energy balance method:

$$G_{II} = \left(\frac{\lambda}{\lambda + \frac{V_f}{1-V_f}}\right)\frac{\sigma_d^2 r_f}{4E_f} = f_2\frac{\sigma_d^2 r_f}{4E_f} \tag{8}$$

where $\lambda = E_m/E_f$ and G_{II} is the mode II fracture energy release rate of the interface; for the experimental model shown in Fig. 2, $G_{II} \cong G_i$. This formula indicates that G_{II} is independent of L and the crack length, a, and is also independent of the Poisson's ratios of the two component materials.

The present author[8] has examined the derivation process of eqn (8) and noticed that the assumption of uniform strain distribution has been made in deriving eqn (8); i.e. it is postulated that

- both in the matrix and in the fiber, strain is uniformly distributed; and
- the strain values in the matrix and in the fiber are equal.

Computations of the strain and stress distributions of this model were carried out by means of the finite-element method (FEM), the results indicating that the strain distribution is widely different from this assumption. Nevertheless, for a wide range of variations of λ and V_f, this formula gives a good prediction of G_{II}. When E_m is too small compared with E_f, eqn (8) yields results with greater inaccuracy. For instance, if $\lambda = 6{\cdot}57\times10^{-3}$, the relative deviation of G_{II} of eqn (8) from the FEM result is greater than 10%. For this case, a modification factor α was recommended and a revised formula was given as:

$$G_{II} = \left(\frac{\lambda^\alpha}{\lambda^\alpha + \frac{V_f}{1-V_f}}\right)\frac{\sigma_d^2 r_f}{4E_f} \tag{9}$$

where α is a number approximating to 1; e.g. for the above case, if $\alpha = 0{\cdot}97$, the deviation reduced to less than 3·5%.

In Ref. 9 the following formula can be found, which is similar to eqn (8) but takes the Poisson's ratios into account:

$$G_{II} = \frac{(1-2k\nu_f)^2}{\left[(1-2k\nu_f) + \frac{\gamma}{\lambda}(1-2k\nu_m)\right]}\frac{\sigma_d^2 r_f}{4E_f} = f_3\frac{\sigma_d^2 r_f}{4E_f} \tag{10}$$

where

$$k = \frac{\lambda\nu_f + \gamma\nu_m}{\lambda(1-\nu_f) + 1 + \nu_m + 2\gamma} \tag{11}$$

ν_f and ν_m are the Poisson's ratios of fiber and matrix, respectively, and

$$\gamma = \frac{r_f^2}{r_m^2 - r_f^2} = \frac{V_f}{1-V_f}$$

V_f being the fiber volume fraction.

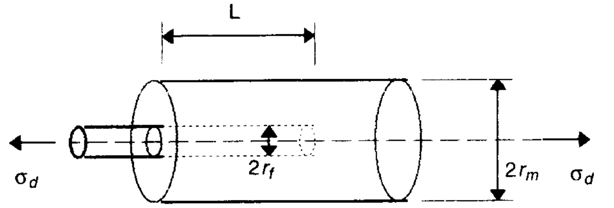

Fig. 1. Schematic drawing of the fiber pull-out experimental model.

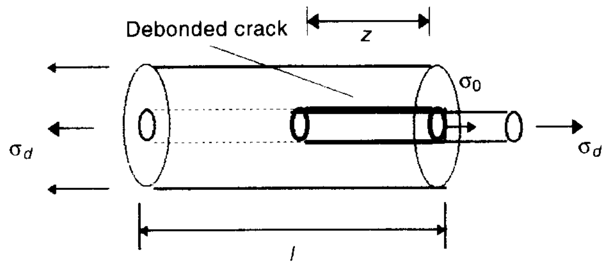

Fig. 2. Cylindrical fiber/matrix pull-out test model with partially debonded interface.

It is easily seen that all the expressions of eqns (7), (8) and (10) have a common term, $\sigma_d^2 r_f/4E_f$, with different non-dimensional coefficients. To compare these formulae, calculations were conducted by using the material constants of a commonly used carbon-fiber-reinforced epoxy composite, for which $E_f = 258{\cdot}6$ GPa, $E_m = 3{\cdot}4$ GPa, $\nu_f = 0{\cdot}25$ and $\nu_m = 0{\cdot}35$. As for the geometry of the test model, as shown in Fig. 2, different diameters may be assigned by different investigators; however, for a case study, $r_f = 4{\cdot}5$ mm and $r_m = 16{\cdot}2$ mm are adopted here. The results are listed in Table 1.

The data in Table 1 illustrate that $f_2 \cong f_3$, indicating that Poisson's ratio has little influence on G_{II}. However, f_1 is very different from f_2 and f_3, and the smaller is L, the larger will be f_1; this may be ill grounded.

3 INFLUENCE OF INTERFACIAL RESIDUAL STRESS AND FRICTION ON G_{II}

For most polymer-based composites, the thermal expansion coefficient of the fiber, α_f, is far smaller than that of the matrix, α_m. Because the service temperature, T_s, of composite structures is much lower than the cure temperature, T_c, there exists considerable residual compressive stress at the fiber/matrix interface in the service condition. By using the axisymmetric thermal elasticity method, the following equation predicting the residual stress, q_0, was derived. A brief derivation is given in Appendix A, where the sign of q_0 is assumed to be opposite to that of Kim and Mai[9]

$$q_0 = \frac{-(\alpha_m - \alpha_f)\Delta T}{\left(\frac{\nu_m + \bar{f}}{E_m} - \frac{1-\nu_f}{E_f}\right)} \tag{12}$$

where $\Delta T = T_s - T_c$, $\bar{f} = (1 + V_f)/(1 - V_f)$ and $V_f = r_f^2/r_m^2$ is the fiber volume fraction. Since ΔT is negative, q_0 is positive. Taking carbon/epoxy as an example, $\alpha_m \cong 22\times10^{-6}\,K^{-1}$, $\alpha_f \cong 3{\cdot}1\times10^{-6}\,K^{-1}$ and $\Delta T \cong -160°C$; the values of E_m, E_f, ν_m, ν_f, r_m and r_f are the same as those for the calculations of Table 1. Substituting these data into eqn (12) results in $q_0 = 6{\cdot}732$ GPa. It should be pointed out that the absolute value of q_0 stands for compression.

For the cylindrical single-fiber/matrix pull-out test model, Kim and Mai[9] present the following equation describing the dependence of the fiber pull-out stress, σ_d^p, on the debonding length, z (see Fig. 2):

Table 1. Results of the comparison calculation of different formulas

	Formula		
	eqn (7)	eqn (8)	eqn (10)
Coefficient	$f_1{}^a$	f_2	f_3
Result	0·4022	0·1349	0·1343

$^a f_1$ depends on embedded length, L; here $L = 130$ mm.

$$\sigma_d^p = \sigma_0 + (\bar{\sigma} - \sigma_0)\left[\frac{\exp(\bar{\lambda}z) - 1}{\exp(\bar{\lambda}z) - 1 + \theta}\right] \tag{13}$$

where $\bar{\sigma}$ is the term considering the influence of residual stress, its expression being given as

$$\bar{\sigma} = \left(\frac{q_0}{k}\right)\left[1 + \left(\frac{\gamma}{\lambda}\right)\left(\frac{\nu_m}{\nu_f}\right)\right]$$

(note here that the sign is opposite to that in Kim and Mai[9]);

$$\theta = \frac{1 + \left(\frac{\gamma}{\lambda}\right)\left(\frac{\nu_m}{\nu_f}\right)}{1 + \left(\frac{\gamma}{\lambda}\right)\left(\frac{1-2k\nu_m}{1-2k\nu_f}\right)}$$

$\bar{\lambda} = 2\mu k/r_f$; k is as given as in eqn (11); μ is the friction coefficient which can be very different depending on the fiber surface treatment;[10] σ_0 is the fiber pull-out stress when debonding length $z = 0$ or friction is zero ($\mu = 0$, $\bar{\lambda} = 0$). It can be seen from eqn (12) that as the debonding crack grows the fiber pull-out stress becomes larger.

By using eqn (13) the debonding energy release rate, G_{II}, with consideration of the interfacial residual stress and friction can be estimated:

$$G_{II} = \lim_{\Delta z \to 0} \frac{\left[\left(\sigma_d^p\right)^2_{|z+\Delta z}(z + \Delta z) - \left(\sigma_d^p\right)^2_z z\right]}{4E_f} \frac{r_f}{\Delta z} \tag{14}$$

Using a Taylor series expansion method and taking the limit of eqn (14), we have

$$G_{II} = \frac{\left(\sigma_d^p\right)\left(\sigma_d^p\right)'_z z r_f}{2E_f} = \frac{\left(\sigma_d^p\right) r_f}{4E_f} \frac{2z\left(\sigma_d^p\right)'_z}{\sigma_d^p} \tag{15}$$

where $\left(\sigma_d^p\right)'_z$ represents the derivative of the pull-out debonding stress with respect to z. Numerical calculations have to be used to obtain the approximations to σ_d^p and G_{II}. Actually, the curve of σ_d^p versus z is nearly a straight line, (Figs 3 and 4), so

$$G_{II} \cong \frac{r_f\left(\sigma_d^p\right)^2}{2E_f}$$

In order to illustrate the influence of residual stress, q_0, and interfacial friction, μ or $\bar{\lambda}$, on G_{II}, calculations of the variations of σ_d^p and G_{II} with cracking length, z, and friction factor, μ, were conducted. The values of the parameters used are: $\sigma_0 = 1$ GPa and $\mu = 0{\cdot}25$ (the other parameters are the same as those given in the previous section). Thus, we obtained $k = 0{\cdot}0213$, $\theta = 1{\cdot}351$,

$\bar{\lambda}=0{\cdot}002367\,\mathrm{mm}^{-1}$, $\bar{\sigma}=3{\cdot}1316\,\mathrm{GPa}$. Substituting these parameters into eqns (13) and (15), the variations of σ_d^p and G_{II} with z and μ were obtained. The resulting curves are shown in Figs 3 and 4. Figure 3 shows the curves of σ_d^p and G_{II} versus z, and Fig. 4 shows σ_d^p and G_{II} versus μ. The right ordinate represents normalized fiber pull-out stress,

$$\overline{\sigma_d^p}=\frac{\sigma_d^p}{\sigma_0}$$

and the left ordinate is the normalized energy release rate,

$$\bar{G}_{II}=\frac{G_{II}}{\left(\frac{\sigma_0^2 r_f}{4E_f}\right)}$$

It can be seen that residual stress and friction exert considerable effects on the pull-out stress and debonding energy release rate. As the debonded crack grows, the pull-out stress, σ_d^p, and debonding energy release rate, G_{II}, are enhanced, and when the interfacial friction is larger, σ_d^p and G_{II} are larger.

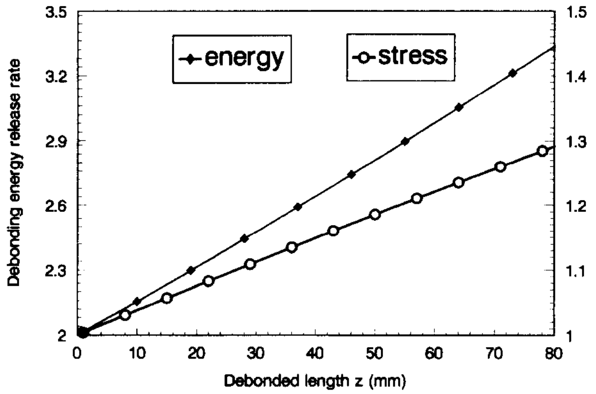

Fig. 3. Curves of normalized fiber pull-out stress and interfacial debonding energy release rate versus debonded length, z.

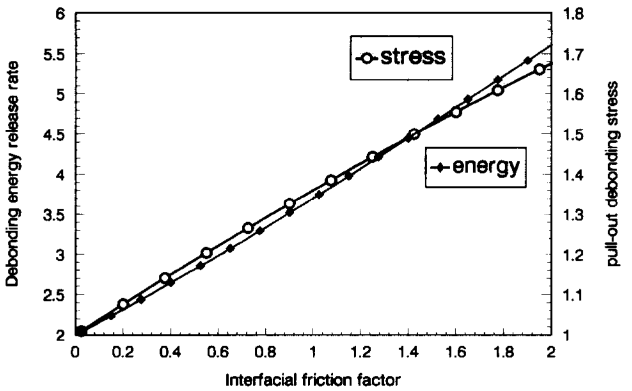

Fig. 4. Curves of normalized fiber pull-out stress and interfacial debonding energy release rate versus interfacial friction, μ.

4 CONCLUDING REMARKS

Although investigations of the problem of interfacial fracture have been carried out for more than 20 years, the problem is not yet completely solved. Theoretically, obtaining the strictly exact solution of the bimaterial model is not easy; both shear-lag analysis and the simplified elasticity solution are approximate results. Experimentally, the theorem for pull-out of a fiber from a matrix using the cylindrical bimaterial model is simple, but in reality, many factors, such as the measurement of fiber embedded and free lengths and rigidity of the test fixture, exert large effects on the results. If the viscoelasticity of the materials is taken into consideration the problem is further complicated.

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APPENDIX

The residual thermal stress acting on the interface between fiber and matrix, q_0, can be analyzed by using classical elastic mechanics. The model of Fig. 2 is a concentric cylinder model consisting of a solid cylinder (the fiber) and a thick-walled cylindrical shell (the matrix). Here, the problem is typically a two-dimensional plane strain problem in polar coordinates.

- For the fibre, under the action of the compressive residual stress, q_0, the stress in the fibre is uniform and $\sigma_r=\sigma_\theta=-q_0$ (the sign of q_0 is opposite to that in Ref. 9).

$$\begin{Bmatrix} \varepsilon_r \\ \varepsilon_\theta \end{Bmatrix} - \Delta T \begin{Bmatrix} \alpha_r \\ \alpha_\theta \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_f} & \frac{-\nu_f}{E_f} \\ \frac{-\nu_f}{E_f} & \frac{1}{E_f} \end{bmatrix} \begin{Bmatrix} \sigma_r \\ \sigma_\theta \end{Bmatrix} \tag{A1}$$

$$\frac{u_r|_{r=r_f}}{r_f} = \varepsilon_\theta = -q_0\left(\frac{1-\nu_f}{E_f}\right) + \Delta T \alpha_f \tag{A2}$$

- For the matrix, under action of q_0 the stresses in the cylindrical shell are

$$\sigma_r = -q_0 \frac{r_m^2 r_f^2}{r^2(r_m^2 - r_f^2)} + q_0 \frac{r_f^2}{r_m^2 - r_f^2} \tag{A3}$$

$$\sigma_\theta = q_0 \frac{r_m^2 r_f^2}{r^2(r_m^2 - r_f^2)} + q_0 \frac{r_f^2}{r_m^2 - r_f^2} \tag{A4}$$

At the position of $r = r_f$

$$\sigma_r|_{r=r_f} = -q_0 \tag{A5}$$

$$\sigma_\theta|_{r=r_f} = q_0 \frac{r_f^2 + r_m^2}{r_m^2 - r_f^2} \tag{A6}$$

According to the stress/strain relationship of the matrix with consideration of thermal expansion, we have

$$\begin{Bmatrix} \varepsilon|_{r=r_f} \\ \varepsilon_\theta|_{r=r_f} \end{Bmatrix} - \Delta T \begin{Bmatrix} \alpha_m \\ \alpha_m \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_m} & \frac{-\nu_m}{E_m} \\ \frac{-\nu_m}{E_m} & \frac{1}{E_m} \end{bmatrix} \begin{Bmatrix} -q_0 \\ q_0\left(\frac{1+V_f}{1-V_f}\right) \end{Bmatrix} \tag{A7}$$

Since

$$\varepsilon_\theta|_{r=r_f} = \frac{u_r}{r_f}$$

we have

$$\frac{u_r}{r_f} = \Delta T \alpha_m + \frac{q_0}{E_m}\left(\nu_m + \frac{1+V_f}{1-V_f}\right) \tag{A8}$$

Equating eqn (A2) with eqn (A8) yields

$$q_0 = -\frac{\Delta T(\alpha_m - \alpha_f)}{\frac{1}{E_m}\left(\nu_m + \frac{1+V_f}{1-V_f}\right) + \frac{1-\nu_f}{E_f}} \tag{A9}$$