The plane wave solutions (3) will be stable if and only if the inequality (11) be satisfied for all k. When k < 1, $k^s \sim 0$ ($s \ge 4$), then after much simplification, the necessary condition for $\text{Re}\gamma(k) < 0$ is contained in the hypothesis.

Remark The time-decay estimates of the perturbation can be found in [5]. More details will be given out in part (II) of this paper.

References

- [1] Kapitula, T., On the nonlinear stability of plane waves for the Ginzburg-Landau equation, Commun. Pure Appl. Math., 1994, 47:831-841.
- [2] Matkowsky, B. J. and Volpert, V., Stability of plane wave solutions of complex Ginzburg-Landau equations, Quar. Appl. Math., 1993, 2:265-281.
- [3] Xin, J. X., Multidimensional stability of traveling waves in a bistable reaction-diffusion equations, Commun. Part. Diff. Equ., 1992, 17:1889-1899.
- [4] Kopell, N. and Howard, L. N., Plane wave solutions to reaction-diffusion equations, Stud. Appl. Math., 1973, 52:291-328.
- [5] Guo, B. L. and Jiang, M. R., Nonlinear stability of plane waves for the Ginzburg-Landau-BBM equations, to appear.

Stream-vorticity NS Equation ADI Scheme to Investigate Properties of Impulsively Started Translatory Flows Around the Rotational Circular Cylinder without Wall Vorticity Conditions ⁷

Caimao LUO and Xuequan E

(Institute of Mechanics, Chinese Academy of Sciences, Beijing 100080, China) E-mail: lcm@imech.ac.cn exq@imech.ac.cn

Abstract: The properties of flow around a circular cylinder impulsively started into translatory and rotatory motion with rotational parameter α less than or equal to 8.0 and Reynolds number Re=100 and 200 are investigated in the present paper. The vorticity and stream function N-S equations are adopted here, with a 2nd-order spatial and temporal accuracy ADI (alternating direction implicit) scheme. Moreover the wall vorticity obtain through the principle of conservation of the total computational domain vorticity is determined by domain vorticity and stream function, therefore, through the wall vorticity iteration, the wall vorticity condition is not fixed during the time step. And the present model results indicate: (1) when $\alpha > 4.0$, vortex street suppression is obvious for the computational period (t < 60) for all the Re numbers here studied; (2) the higher the α number for the same Reynolds number, the slower the upper main vortex proceeds; (3) the maximum instantaneous transverse coefficient exceeds the limitation 4π .

Keywords: wake, vortex shedding mode, rotation and translation, ADI, wall vorticity condition.

⁷The paper was received on Nov. 30, 1998

Introduction

Flow around a spinning circular cylinder, known as the Magnus effect, is a classical fluid dynamics problem. Also this problem is prototypical in the study of unsteady flow separation, and particularly, has applications in the boundary layer control on airfoils.

The structure of the flow around an impulsively started rotating circular cylinder depends mainly on two parameters. One is the rotational speed ratio α , defined by $\alpha = \Omega a/U_{\infty}$, where a is the radius of the cylinder, and U_{∞} the approaching flow velocity and another is the Reynolds number Re, defined by $Re = 2U_{\infty}a/\nu$, in which ν is the kinematic viscosity.

Many scholars have been attracted to this problem since the well-known Magnus experiment in 1853, such as Prantl^[1], Batchelor^[2], Coutanceau^[3,4], Chang^[5], Chen et al.^[6], Chew^[7].

Prandtl^[1] carried out a flow visualization experiment on this problem. And Coutanceau & Menard^[3,4] investigated the early phase of the establishment of the flow past a circular cylinder started impulsively into rotation and translation through visualization with solid tracers. Their experimental Re and α ranges are 200–1000 and 0–3.25 respectively. A lot of detailed photographs were presented for later comparison with calculations.

For computational studies of this problem, the computational time, far field radius, wall vorticity condition (adopting integral form or one-sided differential formula) are key problems. From the earlier documents, Only Chew et al.^[7] computation period is long enough, nondimensional time reaching 100, but their wall vorticity condition, adopting the 2nd order one-sided differential formula, is in doubt for large α case and long-time computation, according to Wu^[8], because it violates the vorticity conservation law; though Chen et al.^[6] computation period is 54, their maximum computation radius is only 24; they present only the force coefficient for $\tau \leq 24$ and the results are questionable for $\tau > 24$. And the high α case, with long computational period, has never been investigated up to now.

Above all, it's necessary to investigate the long period characteristics of the flow around the circular cylinder started impulsively into rotary and rotational motion, especially for the high α case. Through the conservation law of total computational-domain vorticity, the integral form of the wall vorticity condition is adopted here, identical to the formula proposed by Badr^[9,10].

The present model combines a second-order-accurate, alternating-direction, implicit stream-function/vorticity form of the Navier-Stokes equations (temporal accuracy strictly reaching the 2nd order by some prediction formula) with an integral wall vorticity formula, achieving satisfactory results compared to the earlier documents. And the present model has the following properties: (1) the initial and final computation domains are the same and large enough for the present research; (2) the initial circular cylinder surface vorticity distribution matches well with the analytical solutions proposed by Badr et al.^[10]; (3) the nondimensional computational periods reach 60; (4) the U_x velocity distributions along the x-axis are presented to examine the existence of the Von Karman vortex street.

2. Fundamental Equations

2.1 Physical Model

A non-rotating coordinate system translating with the cylinder is used during the present study. And the fluid is supposed to be incompressible, two dimensional. The fluid at infinity has a uniform velocity of magnitude of U_{∞} in the x direction, and the cylinder rotates in the counterclockwise direction with angular velocity Ω , as shown in Fig.1.



Fig.1 Definition sketch

A stream function/vorticity formulation is adopted here. In two dimensions the equations and boundary conditions are:

 \sim

$$\frac{\partial \widetilde{\omega}}{\partial \widetilde{t}} - \frac{1}{\widetilde{r}} \left[\frac{\partial}{\partial \widetilde{r}} (\widetilde{\omega} \, \frac{\partial \psi}{\partial \theta}) - \frac{\partial}{\partial \theta} (\widetilde{\omega} \, \frac{\partial \widetilde{r}}{\partial \theta}) \right] = \nu \nabla^2 \widetilde{\omega}$$
(1)

$$\widetilde{\omega} = \nabla^2 \, \widetilde{\psi} \tag{2}$$

in which,

$$\nabla^{2} = \frac{\partial^{2}}{\partial r^{2}} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{\tilde{r}^{2}} \frac{\partial^{2}}{\partial \theta^{2}}$$
$$\widetilde{u} = -\frac{1}{\tilde{r}} \frac{\partial}{\partial \theta}, \quad \widetilde{v} = \frac{\partial}{\partial \tilde{\psi}}, \quad \widetilde{\omega} = -\frac{1}{\tilde{r}} [\frac{\partial}{\partial r} (\widetilde{v} \, \widetilde{r}) - \frac{\partial}{\partial \theta}]$$
$$\widetilde{r} = a e^{\pi\xi}, \quad \theta = \pi\eta, \quad t = U_{\infty} \, \widetilde{t} \, / a$$

in which $\widetilde{\omega}$, $\widetilde{\psi}$, \widetilde{r} , θ , t stand for dimensional vorticity, stream function, radial and angular coordinates and time respectively.

2.2 Mathematical Model

Assuming that the cylinder radius a is the characteristic length for r, a/U_{∞} the time scale, and U_{∞} the velocity scale, we obtain the following non-dimensional equations:

$$\frac{R}{2}[g(\xi,\eta)\frac{\partial\omega}{\partial t} + \frac{\partial}{\partial\eta}(\frac{\partial\psi}{\partial\xi}\omega) - \frac{\partial}{\partial\eta}(\frac{\partial\psi}{\partial\eta}\omega)] = \nabla^2\omega$$
(3)

$$abla^2 \psi = g(\xi,\eta) \omega$$
 (4)

$$abla^2 = rac{\partial^2}{\partial \xi^2} + rac{\partial^2}{\partial \eta^2}, \quad g(\xi,\eta) = \pi^2 \mathrm{e}^{2\pi\xi}$$

in which, $R = \frac{2U_{\infty}a}{\partial\xi}$, $\psi = \frac{\psi}{U_{\infty}a}$, $\omega = \frac{\omega a}{U_{\infty}}$, $\alpha = \frac{\Omega a}{U_{\infty}}$, subject to the non-slip condition on the cylinder surface:

$$t \ge 0, \quad \psi = 0, \quad \frac{\partial \psi}{\partial \xi} = \pi \alpha, \quad \xi = 0$$
 (5)

there's no restriction on ω for surface $\xi = 0$; remote condition: $\psi = 2 \sin h \pi \sin \pi \eta$, for $\xi \to \infty$;

$$u_r = -\frac{1}{\pi e^{\pi\xi}} \frac{\partial \psi}{\partial \eta}, \quad v_\theta = \frac{1}{\pi e^{\pi\xi}} \frac{\partial \psi}{\partial \xi}$$
(6)

$$\omega = \frac{1}{\pi e^{\pi\xi}} \left[\frac{1}{e\pi\xi} \frac{\partial}{\partial\psi} (v e^{\pi\xi}) - \frac{\partial u}{\partial\xi} \right]$$
(7)

in which u, v are defined as: $u = \frac{\partial \psi}{\partial \xi}, v = -\frac{\partial \psi}{\partial \eta}$. Therefore, we can get:

 $\xi = 0, \quad {\partial^2 \psi \over \partial \xi^2} = g(\xi,\eta) \omega$ (8)

2.3 Discretization (ADI)

The ADI (alternating direction implicit) scheme is adopted for the discretization of equation (3), with 2nd-order accuracy O $(\frac{1}{4}\Delta\tau^2 + \Delta\xi^2 + \Delta\eta^2)$. • Along the direction:

$$g(\xi,\eta)\frac{\omega_{ij}^{n+\frac{1}{2}} - \omega_{ij}^{n}}{\frac{1}{2}\Delta\tau} - \frac{\omega_{ij+1}^{n+\frac{1}{2}} - 2\omega_{ij}^{n+\frac{1}{2}} + \omega_{ij-1}^{n+\frac{1}{2}}}{Re\Delta\eta^{2}} + \frac{u_{ij}^{n+\frac{1}{4}}(\omega_{ij+1}^{n+\frac{1}{2}} - \omega_{ij-1}^{n+\frac{1}{2}})}{4\Delta\eta} = \frac{\omega_{ij+1}^{n} - 2\omega_{ij}^{n} + \omega_{ij-1}^{n}}{Re\Delta\eta^{2}} - \frac{u^{n+\frac{1}{4}}(\omega_{ij+1}^{n} - \omega_{ij-1}^{n})}{4\Delta\eta} + 2\frac{\omega_{i=1}^{n+\frac{1}{4}} - 2\omega_{ij}^{n+\frac{1}{4}} + \omega_{i-1}^{n+\frac{1}{4}}}{Re\Delta\xi^{2}}$$
(9)
$$+ \frac{v_{ij}^{n+\frac{1}{4}}(\omega_{i+1}^{n+\frac{1}{4}} - \omega_{i-1}^{n+\frac{1}{4}})}{2\Delta\xi}$$

in which

 $\omega^{n+\frac{1}{4}} = \frac{1}{4} (5\omega_{ij}^n - \omega_{ij}^{n-1}) = \frac{1}{2} (3\omega_{ij}^n - \omega_{ij}^{n-\frac{1}{2}}), u_{ij}^{n+\frac{1}{4}} = \frac{1}{4} (5u_{ij}^n - u_{ij}^{n-1}), \nu_{ij}^{n+\frac{1}{4}} = \frac{1}{4} (5\nu_{ij}^n - \nu_{ij}^{n-1}).$ • Along the direction:

$$g(\xi,\eta) \frac{\omega_{ij}^{n+1} - \omega^{n+\frac{1}{2}}}{\frac{1}{2}\Delta\tau} - \frac{\omega_{i+1j}^{n+1} - 2\omega_{ij}^{n+1} + \omega_{i-1j}^{n+1}}{Re\Delta\xi^2} - \frac{v_{ij}^{n+\frac{2}{4}}(\omega_{i+1j}^{n+1} - \omega_{i-1j}^{n+1})}{4\Delta\xi} = \frac{\omega_{i+1j}^{n+\frac{1}{2}} - 2\omega_{ij}^{n+\frac{1}{2}} + \omega_{i-1j}^{n+\frac{1}{2}}}{Re\Delta\xi^2}$$
(10)

in which

$$\omega_{ij}^{n+\frac{3}{4}} = \frac{1}{2}(3\omega_{ij}^{n+\frac{1}{2}} - \omega_{ij}^{n}), u_{ij}^{n+\frac{3}{4}} = \frac{1}{4}(7u_{ij}^{n} - 3u_{ij}^{n-1}), \nu_{ij}^{n+\frac{3}{4}} = \frac{1}{4}(7\nu_{ij}^{n} - 3\nu_{ij}^{n-1}).$$

2.4 Vorticity Boundary Conditions

According to Gresho^[11], the total non dimensional vorticity Q in the computational fluid domain is given by

$$\int_{\Gamma} -\frac{\partial \psi}{\partial n} = \int_{\Gamma} u_{\tau} = \int_{\Omega} \omega \tag{11}$$

Therefore.

$$\iint_C \zeta \mathrm{d}A = \int_0^{2\pi} \int_0^R r \zeta \mathrm{d}r \mathrm{d}\theta = \int_0^2 \int_0^{\xi_\infty} 2\pi^2 \mathrm{e}^{2m\xi} \zeta \xi \mathrm{d}\xi \mathrm{d}\eta = -2\pi\alpha \tag{12}$$

And referring to the method by Badr^[10], the Fourier expansions of the ψ , ξ are as follows

$$\psi(\xi,\eta,\tau) = \frac{1}{2}F_0(\xi,\tau) + \sum_{n=1}^{\infty} \{F_n\xi,\tau) \cos n\pi\eta + f_n(\xi,\tau) \sin n\pi\eta\}$$
(13)

$$\chi(\xi,\eta,\tau) = \frac{1}{2}G_0(\xi,\tau) + \sum_{n=1}^{\infty} \{G_n\xi,\tau\} \cos n\pi\eta + g_n(\xi,\tau)\sin n\pi\eta\}$$
(14)

Substituting (15), (16) into (4), we can obtain

$$\frac{\partial^2 F_n}{\partial \xi^2} - n^2 F_n = e^{2\xi} G_n \qquad n = 0, 1, 2 \cdots$$
(15)

$$\frac{\partial^2 f_n}{\partial \xi^2} - n^2 f_n = e^{2\xi} g_n \qquad n = 1, 2 \cdots$$
(16)

Given χ , we have

$$G_n = \frac{1}{\pi} \int_0^{\pi} \chi \cos n\theta d\theta \quad n = 0, 1, 2 \cdots \text{ for } \xi \neq 0$$
(17)

$$g_n = \frac{1}{\pi} \int_0^{\pi} \chi \sin n\theta d\theta \quad n = 0, 1, 2 \cdots \text{ for } \xi \neq 0$$
(18)

$$\int_{0}^{\chi \infty} \pi^{2} \mathrm{e}^{(2-n)\pi\chi} G_{n} \mathrm{d}\chi = \begin{cases} -2\pi\alpha, & n=0\\ 0 & n\neq 0 \end{cases}$$
(19)

$$\int_0^{\chi \infty} \pi^2 e^{(2-n)\pi\chi} g_n d\chi = \begin{cases} -2\pi, & n=1\\ 0 & n \neq 1 \end{cases}$$
(20)

If (14) is substituted into (12), the total non-dimensional vorticity can be derived and matches with Gresho's^[11] result. Therefore, the nature of the integral formula is the conservation of the total computational domain vorticity.

3. Results and Discussion

The present calculation is carried out for $100 \le Re \le 200$ and $0.5 \le \alpha \le 8.0$ on a Pentium PC with main frequency 120MHz. The computation domain is $0 \le \xi \le 15/8$, $0 \le \eta 2$, or $a \le r \le 361.58a$, $0 \le \theta \le 2\pi$ (the corresponding computation parameters are: $\Delta \xi = 1/120, \Delta \eta = 1/80$, grid 151×161 for ξ , η direction respectively). For calculation details and verification of the present model, refer to Caimao Luo & Xuequan E^[12].

3.1 The First Upper Vortex Development for Re=100 and 200, $3.25 \le \alpha \le 8$

Fig.2 gives the first upper vortex development for Re=200, $\alpha=4$, showing that only the upper vortex is generated at $\tau=1$, becoming stronger and stronger at the subsequent instants $\tau=2$, 3, 4, 5, 6. From $\tau=7$ it begins to shed from the cylinder. And the upper vortex is brought downstream by the oncoming flow since $\tau=7$. For the case $\alpha >3$, the first upper vortex evolving process is similar. But the first vortex locations for various α and Reynolds number cases at $\tau=36$ are different, as can be seen in Figs.2, 3. It can be observed from Figs.2, 3 that the main vortex moves downstream more slowly as α increases, which can also be inferred from Fig.7.





Fig.2 Instantaneous streamline patterns for Re=200, $\alpha=4$, $\Delta\xi=1/80$, $\Delta\eta=1/80$ (151×161)





Fig.3 Instantaneous streamline patterns for $Re=100, \tau=36, \Delta\xi=1/80, \Delta\eta=1/80$ (151×161)

(d) $\alpha = 8$

3.2 Vortex Shedding Suppression at Various α 's

It can be shown from Figs.3, 4 that the gap between the zero streamline (the darkest area in the streamline pattern) and the circular surface becomes wider as α increases, without regular vortex shedding at $\tau=36$. Moreover, the vortex shedding suppression can be confirmed by Fig.5, which gives the long time situation of the cylinder wake, indicating that no further vortex is shed except for the first upper main vortex. Figs.6, 7 shows that the main vortex moves faster as α decreases, with the minimum of the U_x corresponding to the first shed vortex.

Above all, at least at $\tau=60$, for Re=100, 200, $\alpha=4$, 5, 6, 7, 8 there is no Von Karman vortex street appearing, the vortex shedding being suppressed completely.

3.3 Force Coefficients for Re=100 and Re=200, $\alpha \leq 8$

Figs.8 and 9 give the in-line and transverse force coefficient temporal variation for $Re=100, \alpha=0.5, 1.0, 2.07, 4.0, 5.0, 6.0, 7.0, 8.0$ respectively, showing that the first peak increases with α , and the periodicity deteriorates with α growth. After the in-line force coefficient reaches its first peak, it will decrease with time to a stable state, with time the in-line force curve reaching steady state, multiplying with α . Figs.10 and 11 present the in-line and transverse force coefficient temporal variation for $Re=200, \alpha=0.5, 1.0, 2.07, 3.25, 4.0, 5.0, 6.0, 7.0, 8.0$ respectively, indicating that the periodicity of the force coefficient co-incides with Figs.10 and 12. Other features of the curve variation are similar to those of Re=100 case.

From Fig.9 and Fig.11, it can be observed that the instantaneous maximum value of the transverse force coefficient exceeds 4π , limitation proposed by B.K.Batchelor, for $\alpha = 7, 8$.

4. Conclusions

The present article combines successfully the strictly 2nd-order accurate spatial and temporal ADI scheme with the integral wall vorticity condition derived from the total computation domain vorticity reservation law. Based on the model, the long time flow development for Re=200, $\alpha=4.0$, 5.0, 6.0, 7.0, 8.0 and Re=100, $\alpha=4.0$, 5.0, 6.0, 7.0, 8.0 has been investigated. The final conclusions are: (1) for Re=100, $200 \alpha=4$, 5, 6, 7, 8, no vortex street appears for $\alpha=60$, with the vortex shedding being completely suppressed; (2) the first peak of high α number of the in-line force coefficient case can last for comparatively longer time than low α number case, and the peak value increases with α ; (3) the first upper main vortex moves more slowly as the α number becomes larger.

Acknowledgment

This study is supported by The State Key Laboratory of Ocean Engineering of Shanghai Jiaotong University.



Fig.4 Instantaneous streamline patterns for $Re=200, \tau=36, \Delta\xi=1/80, \Delta\eta=1/80$ (151×161)



Fig.5 Instantaneous streamline patterns for $Re=200, \tau=60, \Delta\xi=1/80, \Delta\eta=1/80$ (151×161)



Fig.6 U_x distribution along the x-axis for Re=100, $\tau=58$, $\Delta\xi=1/80$, $\Delta\eta=1/80$ (151×161) (a) $\alpha=4$; (b) $\alpha=5$; (c) $\alpha=6$; (d) $\alpha=7$; (e) $\alpha=8$



Fig.7 U_x distribution along the x-axis for Re=200, $\tau=58$, $\Delta\xi=1/80$, $\Delta\eta=1/80$ (151×161) (a) $\alpha=4$; (b) $\alpha=5$; (c) $\alpha=6$; (d) $\alpha=7$; (e) $\alpha=8$



Fig.8 In-line force coefficient temporal variation for Re=100, $\alpha=0.5$, 1.0, 2.07, 4.0, 5.0, 6.0, 7.0, 8.0



Fig.9 Transverse force coefficient temporal variation for Re=100, $\alpha=0.5$, 1.0, 2.07, 4.0, 5.0, 6.0, 7.0, 8.0



Fig.10 In-line force coefficient temporal variation for Re=200, $\alpha=0.5$, 1.0, 2.07, 3.25, 4.0, 5.0, 6.0, 7.0, 8.0



Fig.11 Transverse force coefficient temporal variation for Re=100, $\alpha=0.5$, 1.0, 2.07, 3.25, 4.0, 5.0, 6.0, 7.0, 8.0

References

- [1] Prandtl, L. and Tietjens, O. G., Applied Hydro- and Aeromechanics, Dover, 1934.
- [2] Batchelor, G. K., An Introduction to Fluid Dynamics, Cambridge University Press, 1967.
- [3] Coutanceau, M. and Menard, C., Influence of rotation on the near-wake development behind an impulsively started circular cylinder, J. Fluid Mech., 1985, 158:399-446.
- [4] Coutanceau, M. and Menard, C., Visualization of the eddy formation behind a rotating and translating circular cylinder, In: Proc. Int. Symp. on Physical & Numerical Flow Visualization ASME-FED, 1985, b 22:9-19.
- [5] Chang, C. C. and Chern, R. L., Vortex shedding from an impulsively started rotating and translating circular cylinder, J. Fluid Mech. 1991, 233:265-298.
- [6] Chen, Y. M., Ou, Y. R. and Arne, J. Pearlstein, Development of the wake behind a circular cylinder impulsively started into rotary and rectilinear motion, J. Fluid Mech., 1993, 253:449-484.
- [7] Chew, Y. T., Cheng, M. and Luo, S. C., A numerical study of flow past a rotating circular cylinder using a hybrid vortex scheme, J. Fluid Mech. 1995, 299:35-71.
- [8] Wu, J. C., Numerical boundary conditions for viscous flow problems, AIAA Journal, 1976, 14(8):1042.
- [9] Badr, H. M., Coutanceau, M., Dennis, S. C. R. and Menard, C., Unsteady flow past a rotating circular cylinder at Reynolds number 103 and 104, J. Fluid Mech., 1990, 220:459-484.
- [10] BADR, H. M. and Dennis, S. C. R., Time-dependent viscous flow past an impulsively started rotating and translating circular cylinder, J. Fluid Mech., 1985, 158:447-488.
- [11] Gresho, P. M., Incompressible fluid dynamics: some fundamental formulation issues, Annu. Rev. Fluid Mech., 1991, 23:53-413.
- [12] Caimao Luo and Xuequan E, Integral wall vorticity condition being directly combined with ω - ψ ADI scheme, submitted to Computational Fluid Dynamics Journal, 1998.