

Cracks in saturated sand

X.B. Lu & S.Y. Wang

Institute of Mechanics, Chinese Academy of Sciences, Beijing, China

Peng Cui

Institute of Mountain Hazard and Environment, Chinese Academy of Sciences, Chengdu, China

ABSTRACT: The formation mechanism of water film (or crack) in saturated sand is analyzed numerically. It is shown that there will be no stable “water film” in the saturated sand even if the strength of the skeleton is zero and no positions are choked. The stable water films initiate and grow if the choking state keeps unchangeable once the fluid velocities of one position decreases to zero in a liquefied sand column. A simplified method for evaluating the thickness of water film is presented according to a solidification wave theory. The theoretical results obtained by the simplified method are compared with the numerical results and the experimental results of Kokusho.

1 INTRODUCTION

It is often occurred on the ground slope that sand deposit translates to lateral spreading or even landslide or debris flow not only during, but also after earthquakes. If the sand deposit on a slope are composed of many sublayers, there will be a water film forms once it liquefied (Kokush et al. 1998) which may serves as a sliding surface for postliquefaction failure. As a result, landslide or debris flow may happen on a slope with very gentle slope-angle. Seed (1987) was the first to suggest that the existence of water film (crack) in sand bed is the reason of slope failures in earthquakes. Some researchers (Fiegel, G.L. & Kutter, B.L. 1994, Kokusho, T. 1999, Zhang Junfeng 1998) performed some experiments to investigate the formation of crack in layered sand or in a sand containing a seam of non-plastic silt. Nevertheless, the mechanism of the formation of cracks or “water film” in a sand layer with the porosity distributed continuously is not very clear.

In the viewpoints above, Firstly, we present a pseudo-three-phase model describing the moving of liquefied sand and numerically simulates. Secondly, we present a simplified method to analyze the evolution of the water film.

2 FORMULATION OF THE PROBLEM

It is considered here a horizontal sand layer, which is water saturated and the porosity changes only vertically. The fine grains may be eroded from the skeleton and the eroding relation is assumed to be proportional

to the velocity difference of grains and pore water (Cheng, C.M. et al. 2000). The x axis is upward.

$$\frac{1}{\rho_s} \left(\frac{\partial Q}{\partial t} + u_s \frac{\partial Q}{\partial x} \right) = \frac{\lambda}{T} \left(\frac{u - u_s}{u^*} - q \right)$$

$$\text{if } -\varepsilon(x, 0) \leq \frac{Q}{\rho_s} \leq \frac{Q_c(x)}{\rho_s} \quad (1)$$

$$\frac{1}{\rho_s} \left(\frac{\partial Q}{\partial t} + u_s \frac{\partial Q}{\partial x} \right) \leq 0 \quad \text{otherwise} \quad (2)$$

in which Q is the sand mass eroded per unit volume of the sand/water mixture, ρ_s is the grain density, u and u_s are the velocities of pore fluid and sand grains, q is the volume fraction of sand carried in percolating fluid, T and u^* are physical parameters, λ is a small dimensionless parameter, $\varepsilon(x, t)$ is the porosity, $Q_c(x)$ is the maximum Q that can be eroded at x .

3 MODEL OF THE PROBLEM

The mass conservation equations are as follows:

$$\begin{cases} \frac{\partial(\varepsilon - q)\rho}{\partial t} + \frac{\partial(\varepsilon - q)\rho u}{\partial x} = 0 \\ \frac{\partial q \rho_s}{\partial t} + \frac{\partial q \rho_s u}{\partial x} = \frac{\partial Q}{\partial t} + u_s \frac{\partial Q}{\partial x} \\ \frac{\partial(1 - \varepsilon)\rho_s}{\partial t} + \frac{\partial(1 - \varepsilon)\rho_s u_s}{\partial x} = -\frac{\partial Q}{\partial t} - u_s \end{cases} \quad (3)$$

in which ρ is the density of water. A general equation may be obtained by eq. (3)

$$\varepsilon u + (1 - \varepsilon)u_s = U(t) \quad (4)$$

in which $U(t)$ is the total mass of fluid and grains at a transect. The momentum equations are:

$$\left\{ \begin{aligned} & [(\varepsilon - q)\rho + q\rho_s] \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} \right) \\ & = -\varepsilon \frac{\partial p}{\partial x} - \frac{\varepsilon^2(u - u_s)}{k(\varepsilon, q)} - [(\varepsilon - q)\rho + q\rho_s]g \\ & \quad \times [(\varepsilon - q)\rho + q\rho_s] \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} \right) \\ & \quad + (1 - \varepsilon)\rho_s \left(\frac{\partial u_s}{\partial t} + u_s \frac{\partial u_s}{\partial x} \right) \\ & = -\frac{\partial p}{\partial x} - \frac{\partial \sigma_e}{\partial x} - [(\varepsilon - q)\rho + q\rho_s]g \\ & \quad - (1 - \varepsilon)\rho_s g - \left(\frac{\partial Q}{\partial t} + u_s \frac{\partial Q}{\partial x} \right) (u - u_s) \end{aligned} \right. \quad (5)$$

in which p is the pore pressure.

Here k is assumed as following

$$k(\varepsilon, q) = k_0 f(q, \varepsilon) = k_0(-\alpha q + \beta \varepsilon) \quad (6)$$

in which α, β are parameters and $1 < \beta \ll \alpha$, which indicates that changes in q overweighs that of ε .

The mass conservation equation (4) yield assuming both u and u_s are zero at $x = 0$.

$$\varepsilon u + (1 - \varepsilon)u_s = U(t) = 0 \quad (7)$$

Taking T as characteristic time. u_t the characteristic velocity and L the characteristic length of the problem. We make eq. (1) non-dimensional. Letting

$$\bar{u} = \frac{u}{u_t}, \quad \tau = \frac{t}{T}, \quad \xi = \frac{x}{Lu_t} \quad (8)$$

Instituting equ. (1), (2), (4), (7) (8) into eqs. (3), we may obtain

$$\begin{aligned} \frac{\partial \varepsilon}{\partial \tau} + \frac{\partial \varepsilon \bar{u}}{\partial \xi} &= \bar{u} \frac{u_t}{u^*(1 - \varepsilon)} - q \\ \frac{\partial q}{\partial \tau} + \frac{\partial q \bar{u}}{\partial \xi} &= \bar{u} \frac{u_t}{u^*(1 - \varepsilon)} - q \end{aligned} \quad (9)$$

For $Tg/u_t \gg 1$, the inertia terms are negligible, the last equation of eq. (5) becomes

$$\begin{aligned} \bar{u} &= \left(\frac{1 - \varepsilon}{\varepsilon} \right)^2 (\varepsilon - q) f(q, \varepsilon) \frac{k_0 \rho_s g (1 - \rho/\rho_s)}{u_t} \\ &= \left(\frac{1 - \varepsilon}{\varepsilon} \right)^2 (\varepsilon - q) f(q, \varepsilon) \end{aligned} \quad (10)$$

$$u_t = k_0 \rho_s g (1 - \rho/\rho_s) \quad (11)$$

The initial conditions are:

$$\varepsilon(\xi, 0) = \varepsilon_0(\xi), \quad q(\xi, 0) = 0. \quad (12)$$

4 NUMERICAL RESULTS

Numerical simulation is carried out based on eq. (8). The parameters adopted in simulation are as follows: $\Delta t = 9 \times 10^{-4}$, $\Delta \zeta = 5 \times 10^{-3}$, $\beta = 46 \sim 56$, $\kappa = 50.0$, $a = 0.08$, $\rho_s = 2400 \text{ kg/m}^3$, $\rho_w = 1000 \text{ kg/m}^3$, $u^* = 0.04$, $k_0 = 4 \times 10^{-6} \text{ m/s}$, $\alpha = 1$.

The boundary conditions:

1. The initial porosity distribution is $\varepsilon_0(x) = \bar{\varepsilon}_0(1 - a \tanh((x - 0.5L)/2) \cdot \kappa)$, in which $\bar{\varepsilon}_0 = 0.4$, $L = 1$, $0 \leq x \leq 1$, L is the length of sand column. There is an assumption that u keeps zero once it drops to zero.
2. The distribution of initial porosity is the same as that in condition 1, there is no assumption.

Figure 1 shows that if we assume that once the sand column at some point is jammed, they keep this state forever, then the sand above the jammed position will be prevented to drop cross the jammed point and so the porosity becomes smaller and smaller, while the sand below the point will settle gradually and makes

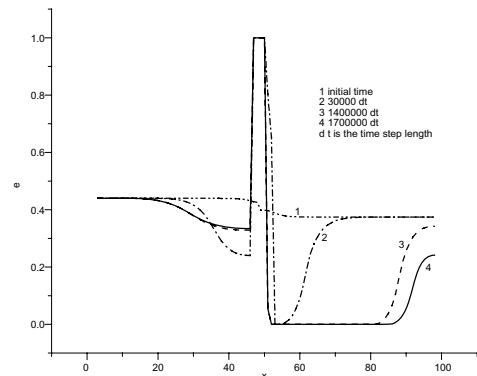


Figure 1. The evolution of cracks in the condition 1.

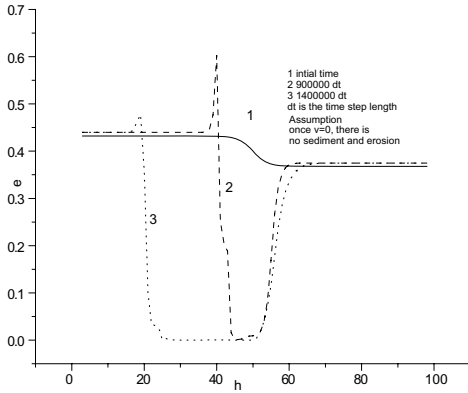


Figure 2. The evolution of cracks in condition 2.

the crack extends gradually. But if we do not adopt the assumption as in Figure 1, the crack will form first and then disappear gradually (Figure 2). The results show that the forming conditions of stable water film are: (1) the porosity of the upper part of the sand column must be smaller than that of the lower. (2) The keeping of the jamming state to prevent the free dropping of the grain or the skin friction.

5 A SIMPLIFIED EVALUATION METHOD

A simplified method is presented here for analyze the evolution of cracks fast and practically. Florin and Ivanov (1961) pointed out that when the settling particles reach solid material, such as the unliquefied underlying sand, or the container base in a experiment, they accumulate to form a solidified zone which increases in thickness with time. A solidification front therefore moves upward until it reaches the surface, or overlying unliquefied material. Scott et al. (1986) had analyzed the development of the solidification.

Assuming that the whole mass reaches its terminal velocity, k , which is the permeability, instantaneously at the end of liquefaction, Florin have given an expression for the constant velocity \dot{z} , of the solidification front:

$$\dot{z} = \frac{\rho}{\rho_w} \frac{1 - n_1}{n_0 - n_1} k \quad (13)$$

in which $\rho' = \rho_s - \rho_w$ is the buoyant unit weight of the liquefied sand n_0 is the porosity of the liquefied sand, n_1 is the porosity of the solidified sand.

From eq. (13), we can obtained the duration of liquefaction and subsequent excess pore pressure decline for any point in the sand column.

$$t = \frac{\rho_w}{\rho} \frac{n_0 - n_1}{1 - n_1} \frac{h}{k} \quad (14)$$

in which h is the height of any point in the sand column.

The final settlement of the top surface of the sand layer is

$$\Delta L = \frac{n_0 - n_1}{1 - n_0} H \quad (15)$$

in which H is the maximum height of sand layer.

The rate of settlement is

$$\dot{s}_a = \frac{\gamma' k}{\gamma_w} \quad (16)$$

The settlement at any time is

$$s_a = \frac{\gamma' k}{\gamma_w} t \quad (17)$$

The settlement velocity v_e of the elements above the water film is determined by the combined permeability k_{es} of the middle layer and the upper layer as (Kokusho, 2002)

$$k_{es} = \sum_{i=1}^m L_i / \sum_{i=1}^m L_i / k_i \quad (18)$$

The upward seepage flow and the settlement of grains have the following Velocities:

$$v = k_{es} i_e; \quad u = -\frac{mv}{1 - n} \quad (19)$$

i_e is the average hydraulic gradient, n is the porosity.

The deform of the skeleton by the geostatic stress in the solidification zone after the solidification may be expressed as^[5]

$$s_2 = \frac{1}{2} \frac{\rho' g}{m_s} z^2(t) \quad (20)$$

in which m_s is the compressible modulus. The total deformation is:

$$\Delta s = \Delta L + \Delta s_2 \quad (21)$$

Instituting eqs. (15) and (20) into eq. (21), considering that the initial height is equal to maximum height of the liquefied zone and the solidified zone and the water layer above the sand surface:

$$\Delta s = \frac{n_0 - n_2}{1 - n_2} (\Delta z + \Delta s) + \frac{\rho' g}{m_s} z \Delta z \quad (22)$$

because $h = h_0 + L + s$, which yields $\Delta h = (\Delta L + \Delta s)$.

Eq. (16) may be written as

$$\frac{\Delta s}{\Delta t} = k \frac{\rho'}{\rho_w} \quad (23)$$

According to eqs. (22) and (23), the increase velocity of the solidification thickness:

$$\frac{\Delta z}{\Delta t} = \frac{k \rho'}{\rho_w} \left/ \left(\frac{n_0 - n_1}{1 - n_1} + \frac{1 - n_1}{1 - n_0} \frac{\rho' g_z}{m_s} \right) \right. \quad (24)$$

Then we can obtain the duration of any location that the solidification front arrives at as follows:

$$t = \frac{\rho_w}{k \rho'} \left(\frac{n_0 - n_1}{1 - n_1} z + \frac{1}{2} \frac{1 - n_1}{1 - n_0} \frac{\rho' g_z}{m_s} z^2 \right) \quad (25)$$

Side friction may be expressed if it should be considered

$$\sigma_s = \mu K_0 \sigma_z \quad (26)$$

The effect of the changes of porosity on the permeability k is considered as a linear relation:

$$k = k_0 [1 - \alpha(n_0 - n)] \quad (27)$$

in which k_0 is the initial porosity, α is a parameter, n_0 is the initial porosity.

By considering eqs. (26) and (27) in the pore pressure gradient and considering the consolidation of the solidification zone, we can compute the development of cracks at these conditions.

6 COMPARISON WITH EXPERIMENTAL RESULTS

It is shown that the results computed by numerical method and the simplified method are close to the experimental results (Figure 3, parameters used in computing is the same given in literature 7). The simplified method presented in this paper may be used to compute the evolution of the water film.

7 CONCLUSIONS

Numerical simulations show that there are stable water films only in the conditions that: (1) the porosity of the upper part of the sand column must be smaller than that of the lower. (2) The keeping of the jamming state. A simplified method for evaluating the thickness of water film is presented. It is shown that the simplified method agree with the experimental results.

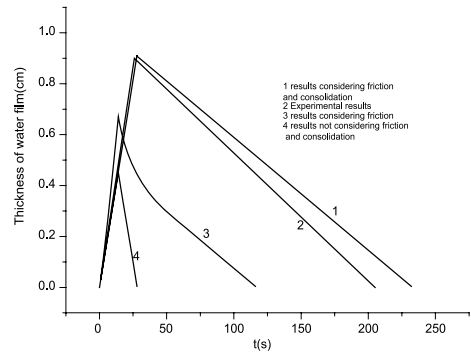


Figure 3. The comparison of the results computed by simplified method with the experiment of Kokusho.

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