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Parametric Studies of Tension Leg Platform with Large Amplitude Motions

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ABSTRACT

Tension leg platform (TLP) is an important kind of working station for deep water exploration and development in ocean, whose dynamic responses deserve a serious thought. It is shown that for severe sea state, the effects of nonlinearities induced by large displacements of TLP may be noteworthy, and then employment of small displacements model should be restrained. In such situation, large amplitude motion model may be an appropriate alternative. The numerical experiments are performed to study the differences of dynamic responses between the two models. It is shown that for most cases, differences between results of the two models are significant. The variances of the differences vs. the wave period are the most remarkable, and that of the differences vs. wave heading angle are also apparent.

KEY WORDS: Tension leg platform (TLP); large amplitude motions; nonlinear dynamic response; wave loads; numerical simulation.

INTRODUCTION

As well known, ocean is an attractive region for mankind to exploit oil and natural gas resources, and the most promising areas include lots of deep water blocks. Tension leg platform (TLP) is a typical kind of compliant offshore working station, which has become a competitive alternative for deep water exploration and development all over the world. Similarly, it is selected by petroleum industries in China as a candidate for offshore oil exploitation to meet the rapidly growing domestic demand.

TLP can move with 6 degrees of freedom, which are surge, sway and yaw in the horizontal plane and heave, roll and pitch in the vertical plane. The dynamic response of TLP is an important problem of offshore mechanics, and there are many researches on it. Williams and Rangappa (1994) developed an approximate semi-analytical technique to calculate hydrodynamic loads, added mass and damping coefficients for idealized TLP consisting of arrays of circular cylinder. Ahmad (1996) conducted stochastic response analysis considering viscous hydrodynamic force, variable added mass and large excursion. Ahmad, Islam and Ali (1997) investigated TLP's sensitivity to dynamic effects of the wind. Yilmaz, Incecik and Barltrop (2001) calculated free surface elevations for an array of four cylinders. Chandrasekaran and Jain (2002a, b) developed a method to analyze the dynamic behavior of triangular and square TLP; and they performed numerical studies to compare the dynamic responses of a triangular TLP with that of a square TLP.

To the best of the author's knowledge, the existing investigations on TLP mostly adopt the small displacements assumption explicitly or implicitly, which treat the translational displacements and angular displacements as small magnitude. In this sense, we call such method as linear model. Therefore, the nonlinear effects induced by large displacements are omitted. In fact, in severe sea state, or extreme adverse state, the displacements of TLP may be distinctly large and should not be taken for small quantities. There are very few investigations ostensibly claim to have considered arbitrary displacements. However, it may not be the fact. Zeng et al (2006) have explained the reason.

Zeng, Liu, Shen and Wu (2006) developed a theoretical model for analyzing the nonlinear behavior of a TLP with large amplitude motions, in which multifold nonlinearities are taken into account, such as nonlinear restoring forces, coupling of the six degrees of freedom, instantaneous position, instantaneous wet surface, free surface effects and viscous drag force. The nonlinear dynamic analysis of ISSC TLP in regular waves was performed in the time domain. It was found that nonlinear responses of TLP considering effects induced by large amplitude motions differ from that of the linear model significantly. By contrary to the small displacements linear model, we call this large displacements method as nonlinear model.

This paper is a continuation of the paper by Zeng et al (2006). Herein we investigate the variance trend of dynamic responses differences between the linear and nonlinear model as the wave conditions vary, by using the method developed in that paper (Zeng et al (2006)). The motivation of this paper is to study quantificationally whether the linear model may be appropriate to a certain extent even severe sea state is considered. The wave height, wave period and wave heading angle are the main variable parameters concerned. The numerical calculations are performed for a typical TLP (ISSC TLP) consisting of four columns and four pontoons.

The major assumptions we adopt are listed as followings

i) The amplitude of motions of TLP may be large, not confined to small quantities.
ii) The component cylinders of hull are assumed sufficient slender, and then the wave diffraction effects have been neglected. (For

extreme or swell sea state we considered in this paper, the low frequency components are dominant, and then the wavelengths are large enough). In addition, the hydrodynamic interactions between cylinders are omitted.

iii) Wave forces are evaluated by Morison's type equation, with the instantaneous displaced position and the instantaneous wet surface considered.

iv) The free surface effects are taken into account following the stretching method Chakrabarti (1987) proposed.

PRIMARY FORMULAS

A typical TLP consisting of four columns and pontoons is shown in Figs. 1(a) (b) (c). Three right-hand Cartesian coordinate systems oxyz, OXYZ, $G\xi\eta\zeta$ are used. The oxyz is space fixed coordinate system, plane oxy coincides with the undisturbed calm water surface, and the positive z-axis is pointing upwards. This coordinate system is used to define wave. The OXYZ is also space fixed coordinate system, which has its origin located at the center of gravity (C.G.) of the undisturbed TLP. Three axes of coordinate system OXYZ are in parallel with those of oxyz. The $G\xi\eta\zeta$ is body fixed coordinate system, which coincides with the OXYZ when the TLP is stationary. The motions of TLP are denoted by the displacements X_1, X_2, X_3, X_4, X_5 and X_6 of $G\xi\eta\zeta$ with respect to OXYZ. X_1, X_2, X_3 are the coordinates of G in OXYZ, which denote the translation (surge, sway and heave) of TLP. X_4, X_5, X_6 represent the three angular motions, which are the Eulerian angles of $G\xi\eta\zeta$ with reference to OXYZ.

For nonlinear model, X_1, X_2, X_3, X_4, X_5 and X_6 may be large quantities, while for linear model they are small. Then the transformation of coordinates can be written as follows

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} + \begin{pmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{pmatrix} \begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix} \tag{1a}$$

Where t_{ij} (i,j= 1, 2, 3) are functions of X_4, X_5 and X_6, i.e.

$$t_{ij} = t_{ij}\left(X_4, X_5, X_6\right)$$

For nonlinear model, t_{ij} are nonlinear functions of X_4, X_5 and X_6, whereas for linear model they are linear functions of angular motions if X_4, X_5 and X_6 are small quantities. The detailed formulas for t_{ij} are given in Zeng et al (2006).

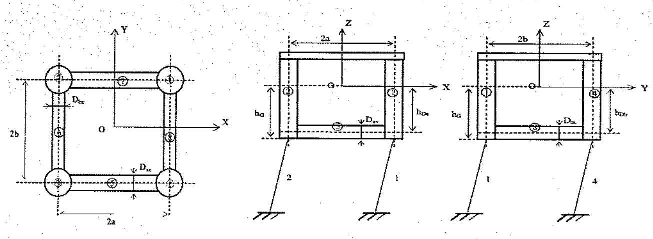
The motion equations of six components X_i of TLP are

$$\begin{pmatrix} M & 0 & 0 & 0 & 0 & 0 \\ 0 & M & 0 & 0 & 0 & 0 \\ 0 & 0 & M & 0 & 0 & 0 \\ 0 & 0 & 0 & I_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & I_3 \end{pmatrix} \begin{pmatrix} \ddot{X}_1 \\ \ddot{X}_2 \\ \ddot{X}_3 \\ \dot{\omega}_1 \\ \dot{\omega}_2 \\ \dot{\omega}_3 \end{pmatrix} = \begin{pmatrix} F_1(X_i, \dot{X}_i, \ddot{X}_i) \\ F_2(X_i, \dot{X}_i, \ddot{X}_i) \\ F_3(X_i, \dot{X}_i, \ddot{X}_i) \\ F_4(X_i, \dot{X}_i, \ddot{X}_i) - (I_3 - I_2)\omega_2\omega_3 \\ F_5(X_i, \dot{X}_i, \ddot{X}_i) - (I_1 - I_3)\omega_3\omega_1 \\ F_6(X_i, \dot{X}_i, \ddot{X}_i) - (I_2 - I_1)\omega_1\omega_2 \end{pmatrix} \tag{2}$$

in which M is the body mass of TLP in air, I_i (i=1, 2, 3) are the moments of inertia with respect to the principal axes through C.G.; F_i are the components of external force (i=1, 2, 3) and moment (i=4, 5, 6) vectors, respectively; ω_i (i=1, 2, 3) are the components of angular velocity, dot over variable means time derivative. ω_i are nonlinear functions of X_4, X_5 and X_6 for large rotations (nonlinear model) and linear functions for small rotations (linear model).

TLP endures tension of tethers, hydrodynamic and hydrostatic forces acting on columns and pontoons and the self-gravities. After doing vector sums of those forces, we can obtain the principal vector $\vec{F}$ of external forces acting on TLP

$$\vec{F} = \vec{F}_w + \vec{F}_B + \vec{F}_t - Mg\vec{k} = F_1\vec{i} + F_2\vec{j} + F_3\vec{k} \tag{3}$$



(a) (b) (c)

Fig. 1 Sketch of TLP and the coordinate systems

where $\vec{i}$, $\vec{j}$, $\vec{k}$ are base vectors of system OXYZ; g is the acceleration due to gravity. Similarly, we can get the resultant moment $\vec{M}$ by summing external moment vectors together:

$$\vec{M} = \vec{M}_{Gw} + \vec{M}_{GB} + \vec{M}_{Gt} = F_4\vec{e}_1 + F_5\vec{e}_2 + F_6\vec{e}_3 \tag{4}$$

where $\vec{e}_1$, $\vec{e}_2$, $\vec{e}_3$ are base vectors of system $G\xi\eta\zeta$. For large amplitude motions, F_i (i=1, 2,...,6) are nonlinear functions of the responses of TLP as shown in Eq. 2. The detailed formulas for calculating the components of external forces and moments in Eq. 3 and Eq. 4 are given by Zeng et al (2006). As long as the formulas for calculating F_i are obtained, we can perform calculations of Eq. 2. We solve Eq. 2 by using a fourth-order Runge-Kutta numerical time integration procedure.

NUMERICAL STUDIES

It is an important problem that to what extent the differences between the results of linear and nonlinear model (i.e. small displacements model and large displacements model) can be ignored. If we can find the bound in which the results given by linear model are adequate, the employment of the linear model can simplify the analysis process. If it is on the contrary, although employing nonlinear model makes the problem very complex, it is worthwhile.

We perform numerical studies on the dynamic responses of a typical TLP (ISSC TLP), by employing both the linear and nonlinear model. The primary properties of ISSC TLP are shown in Table 1.

Table 1. Primary properties of ISSC TLP (Eatock Taylor and Jefferys, 1986)

Description	Value
Spacing between column centres (m)	86.25
Column radius (m)	8.44
Pontoon width (m)	7.5
Pontoon height (m)	10.5
Draft (m)	35.0
Displacement (kg)	54.5×10^6
Mass (kg)	40.5×10^6
Length of tendons (m)	415.0
Roll moment of inertia (kg m^2)	82.37×10^9
Pitch moment of inertia (kg m^2)	82.37×10^9
Yaw moment of inertia (kg m^2)	98.07×10^9
Vertical position of C.G. above keel (m)	38.0

The object we compare between the linear and nonlinear model in this paper is the response variation range, which is defined here as the distance between the crest and the trough of certain steady state responses. The responses concerned are the 6 degrees of freedom and the stresses of the four tethers of TLP. In the following sections, the differences between the respective solutions of response variation range by linear and nonlinear model are shown. Hereinafter, with the exception of special declaration, the data in text, tables and figures describe the differences in response variation ranges between the nonlinear and linear models, i.e. the data shown represent

$$difference = \frac{nonlinear\ solutions - linear\ solutions}{linear\ solutions} \times 100\%$$

In this paper, we focus our attention on the regular wave case, which can give a hint and lay the foundation for the irregular wave case. The solutions of the steady state responses of ISSC TLP in regular waves by both linear and nonlinear models are shown in Figs. 2~11. The wave height is 8 meters, the wave period is 14 seconds and the wave heading angle is 22.5 degrees respectively. It can be found that except surge and sway, nonlinearities exert a distinct influence on the steady state responses of TLP. The differences in surge and sway between the two models, which are -2.3% and -2.4% respectively, are not unacceptable. Nevertheless the differences in heave, roll, pitch and stresses of the four tethers are -69.9%, -65.2%, -24.1%, -62.3%, -57.1%, -58.8%, -59.4% respectively, which are fairly obvious. In addition, we can easily find high order components (low or high frequency components) differences in heave, surge, sway and yaw. We can see that the heave results of nonlinear model reveal high-frequency components, whereas the linear counterparts do not. Surge, sway and yaw show differences in drift components. Moreover, for the wave condition we show here, the phase of nonlinear solutions of heave, roll, pitch and stresses of the four tethers are shifted from the linear counterparts by about 180 degrees (heave, roll and pitch) and 150 degrees (stresses of tethers) respectively. Investigating other solutions further, we find that such phase shifts can change over from 0 to 180 degrees as the wave periods vary. The phase shifts and high order components can be attributed to the quadratic, cubic and higher order terms in the motion equations introduced by both the complicated nonlinear coupling among six degrees of freedom and the loads-responses interactions induced by finite displacements. Quantificational and detailed investigations on variation trend of high order components differences and phase shifts are beyond this paper, which will be dealt in future.

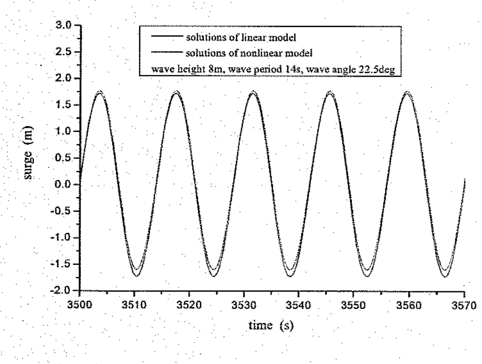


Fig. 2 Steady state response of surge

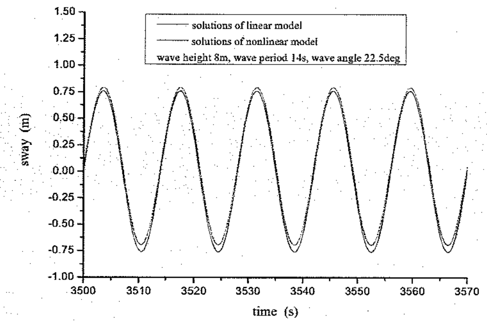


Fig. 3 Steady state response of sway

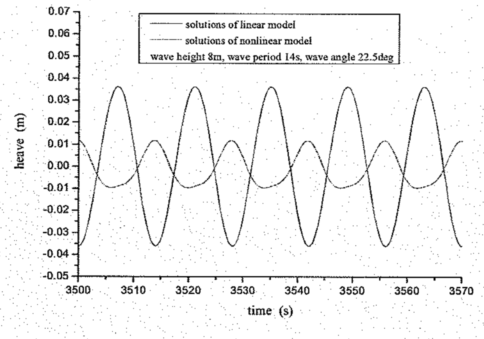


Fig. 4 Steady state response of heave

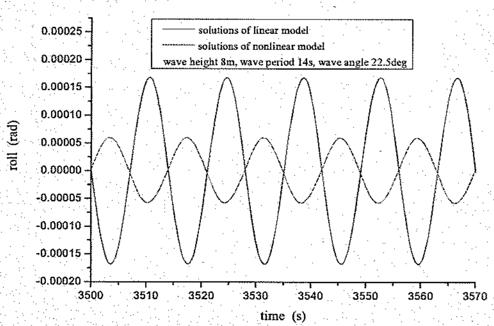


Fig. 5 Steady state response of roll

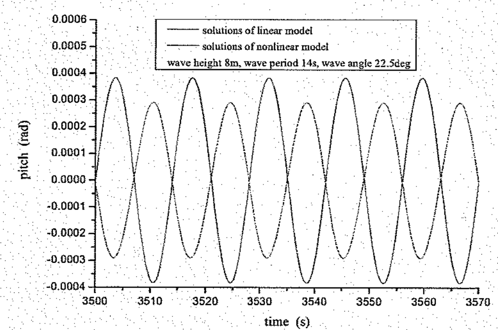


Fig. 6 Steady state response of pitch

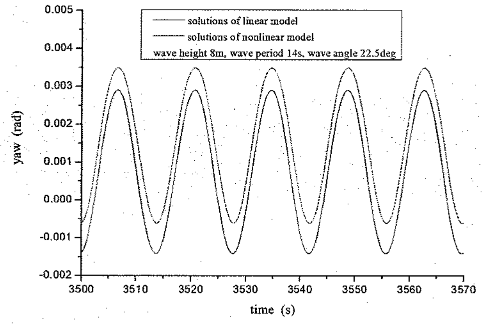


Fig. 7 Steady state response of yaw

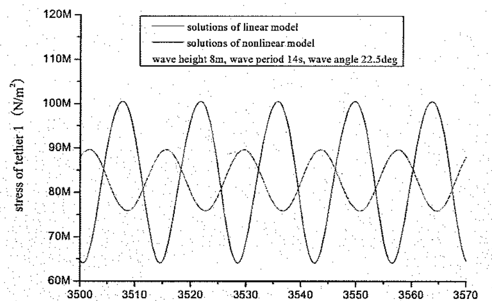


Fig. 8 Steady state response of stress of tether 1

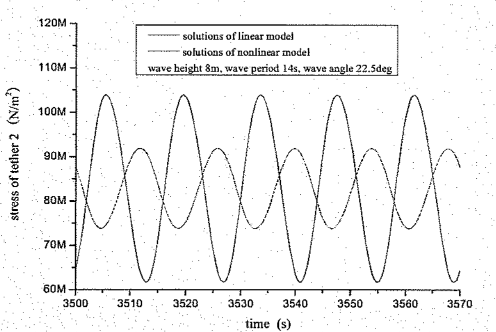


Fig. 9 Steady state response of stress of tether 2

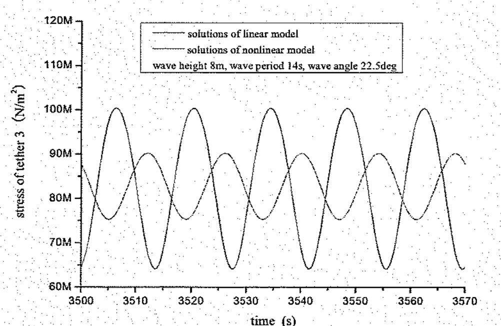


Fig. 10 Steady state response of stress of tether 3

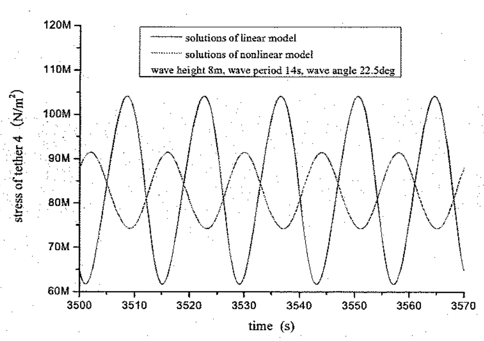


Fig. 11 Steady state response of stress of tether 4

For the TLP we study, the main parameters related to sea state are wave height, wave period and wave heading angle. Then we shall investigate the relationship of differences of response variation range vs. wave height, wave period and wave heading angle respectively.

Differences of Response Variation Range vs. Wave Height

For the sake of examining the differences of response variation range between the linear model and nonlinear model vs. wave height, we perform numerical calculations employing both models for several wave conditions. The wave heading angle is 22.5 degrees, the wave periods are 11 and 13 seconds respectively and the wave heights are 6, 8, 10 and 12 meters respectively. The results of 6 degrees of freedom and stresses of four tethers given by the two models are compared. Then the differences are shown in Table 2 and 3. It can be found that for different wave height, the variations of differences between the two models are indistinctive. That is to say, the differences are insensitive to wave height. Therefore, we can select a certain wave height (combining with different wave periods and heading angles to constitute some wave conditions) as representative of others to carry out subsequent analysis.

Table 2. Differences between linear and nonlinear model (Wave period is 11 seconds; the wave heading angle is 22.5 degrees.)

Wave height (m)	6	8	10	12
Surge (%)	-3.4	-3.7	-4.3	-4.6
Sway (%)	-2.1	-2.5	-2.8	-3.1
Heave (%)	-84.1	-84.4	-84.3	-84.4
Roll (%)	-62.9	-63.8	-65.2	-66.4
Pitch (%)	+35.5	+34.5	+34.6	+34.5
Yaw (%)	-4.5	-4.6	-3.8	-2.3
Stress of tether 1 (%)	+9.2	+8.6	+8.0	+7.6
Stress of tether 2 (%)	-38.6	-38.2	-37.7	-37.3
Stress of tether 3 (%)	+12.0	+11.9	+12.5	+12.6
Stress of tether 4 (%)	-41.5	-42.1	-42.5	-43.2

Table 3. Differences between linear and nonlinear model (Wave period is 13 seconds; the wave heading angle is 22.5 degrees.)

Wave height (m)	6	8	10	12
Surge (%)	-2.6	-2.2	-2.4	-2.4
Sway (%)	-2.1	-2.3	-2.5	-2.6
Heave (%)	-80.3	-79.8	-79.3	-77.5
Roll (%)	-76.1	-73.7	-70.8	-67.5
Pitch (%)	-13.0	-13.3	-13.8	-14.2
Yaw (%)	-4.9	-4.4	-3.9	-3.2
Stress of tether 1 (%)	-59.7	-60.1	-60.5	-61.2
Stress of tether 2 (%)	-56.8	-56.7	-56.2	-56.2
Stress of tether 3 (%)	-56.9	-55.8	-55.3	-54.7
Stress of tether 4 (%)	-58.9	-59.3	-59.8	-60.4

Differences of Response Variation Range vs. Wave Period

Comparing Table 2 and 3 correspondingly, we can see that differences are sensitive to wave period. For instance, the differences of pitch and differences of stress of tether 1 and 3 change over from positive to negative number as the wave period varies from 11 to 13 seconds. Thus the variance relationships of differences vs. wave period are investigated in this section.

Following the same procedure as the last section, we perform numerical calculations for both nonlinear and linear models. The wave height is 8 meters, the wave heading angels are 0, 22.5, 45 degrees respectively, and the wave period are 11, 12, 13, 14, 15, 16, 17, 18, 20, 22, 24, 26, 28 seconds respectively. The differences vs. period curves are shown in Figs. 12~21.

It is perceived from these figures that on the whole, the differences of 6 degrees of freedom and stresses of tethers change obviously as the period varies, i.e. differences are sensitive to wave period. The exceptions occur in sway, roll and yaw when wave heading angle is 0 degree. For 0 degree heading angle, steady state responses of sway, roll and yaw given by both nonlinear and linear model are all 0, which can be understood easily. The steady state responses of yaw given by both models are also 0 when the heading angle is 45 degrees, because the plan view of ISSC TLP is square (i.e. a=b in Fig. 1(a)).

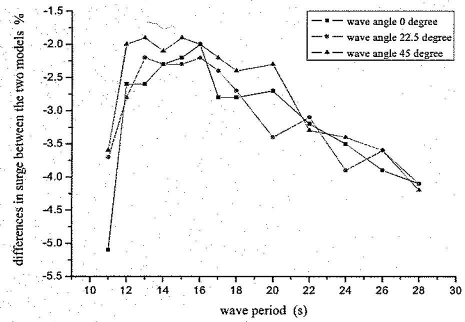


Fig. 12 Surge differences vs. period

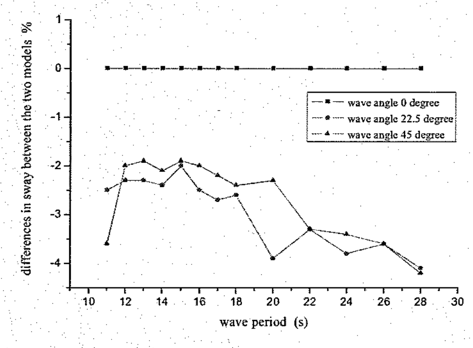


Fig. 13 Sway differences vs. period

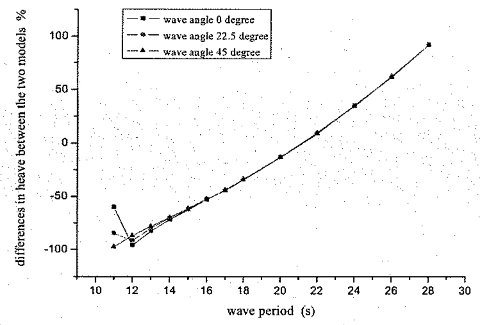


Fig. 14 Heave differences vs. period

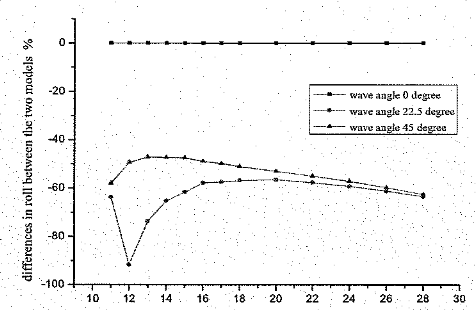


Fig. 15 Roll differences vs. period

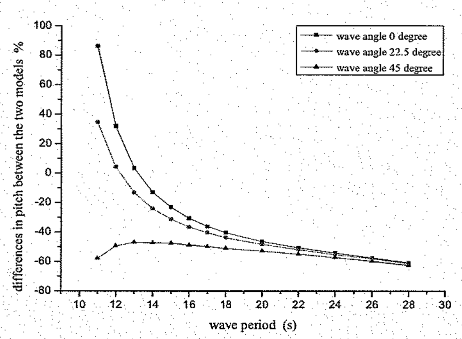


Fig. 16 Pitch differences vs. period

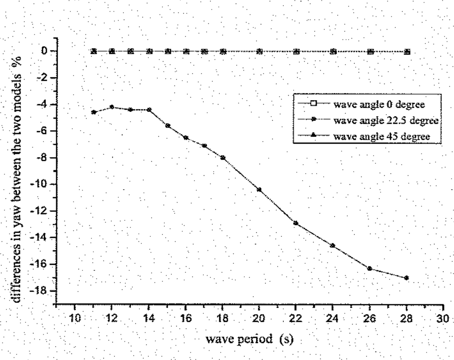


Fig. 17 Yaw differences vs. period

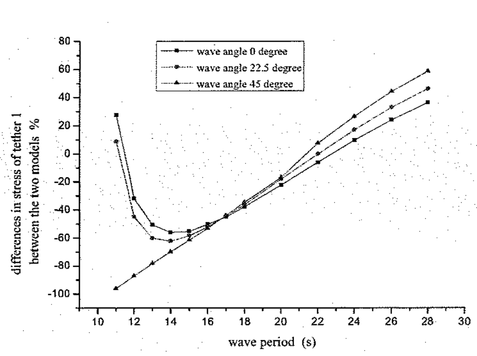


Fig. 18 Stress of the 1st tether differences vs. period

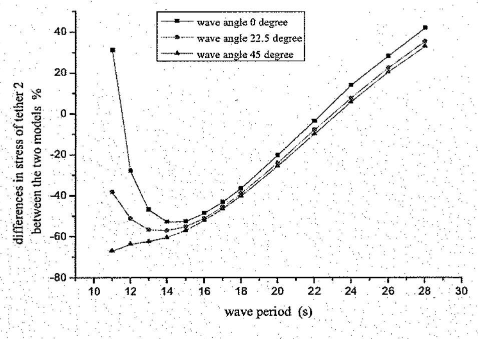


Fig. 19 Stress of the 2nd tether differences vs. period

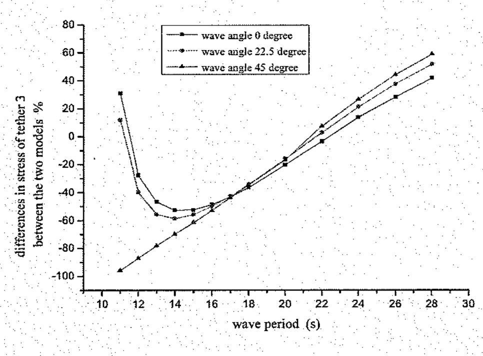


Fig. 20 Stress of the 3rd tether differences vs. period

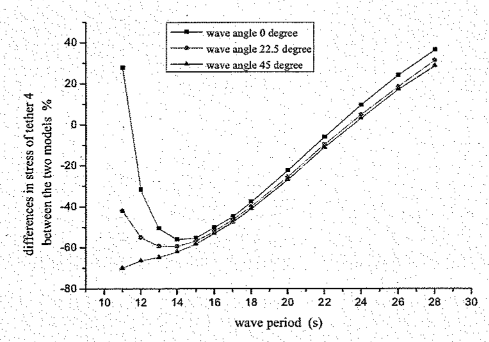


Fig. 21 Stress of the 4th tether differences vs. period

Moreover, it is discovered from Figs. 12~21 that the quantities of differences in heave, roll, pitch and stresses of the four tethers are comparatively large. Differences of surge, sway and yaw are relative small. It seems that differences of stresses of tethers may be mainly attributed to differences of heave, roll and pitch, although the numerical value of heave, roll and pitch are small relative to surge, sway and yaw (Figs. 2~7).

Differences of Response Variation Range vs. Wave Heading Angle

From Figs. 12~21, it can be found that differences also change as the wave heading angle varies. Then in this section we study the variance trend of differences vs. heading angle. The results of preceding section are rearranged, which are shown in Figs. 22~31.

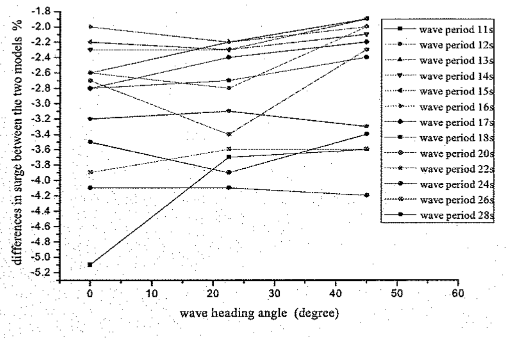


Fig. 22 Surge differences vs. wave heading angle

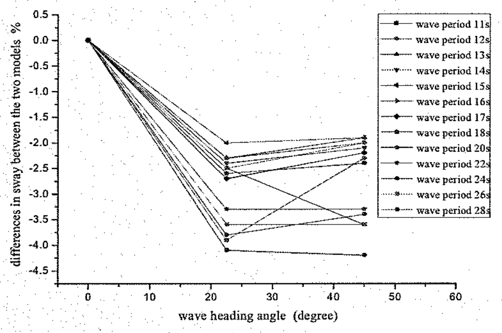


Fig. 23 Sway differences vs. wave heading angle

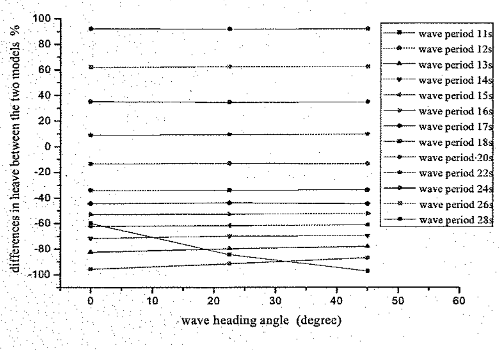


Fig. 24 Heave differences vs. wave heading angle

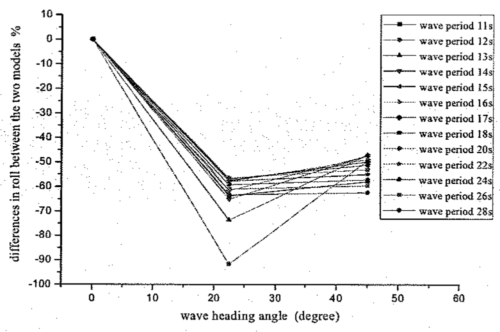


Fig. 25 Roll differences vs. wave heading angle

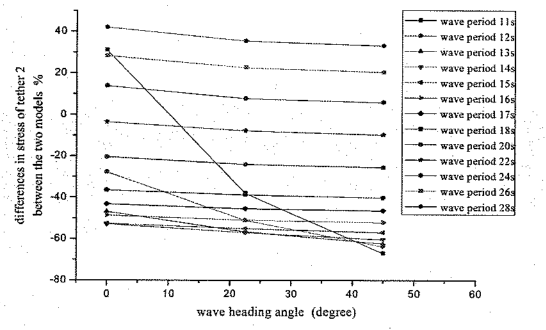


Fig. 29 Stress of the 2nd tether differences vs. wave heading angle

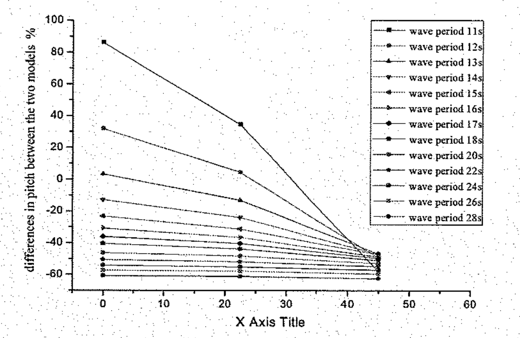


Fig. 26 Pitch differences vs. wave heading angle

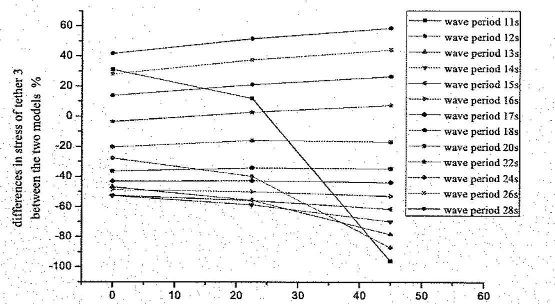


Fig. 30 Stress of the 3rd tether differences vs. wave heading angle

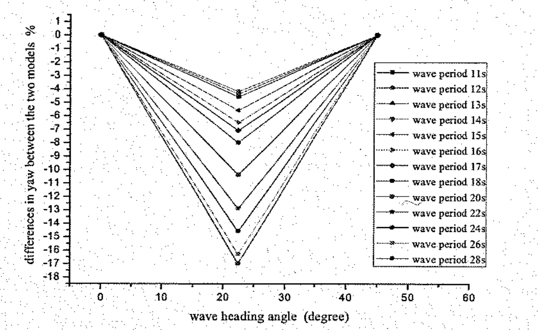


Fig. 27 Yaw differences vs. wave heading angle

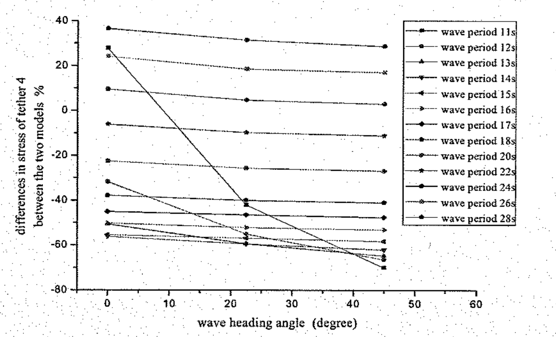


Fig. 31 Stress of the 4th tether differences vs. wave heading angle

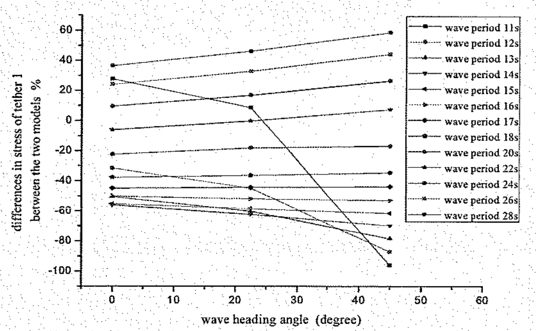


Fig. 28 Stress of the 1st tether differences vs. wave heading angle

We can see from Figs. 22~31 that in principle, the variation of wave heading angle also influence the differences of responses, with the exception of differences of heave. For differences of heave, differences do not change clearly as the wave heading angle varies, though the value of heave differences are large.

CONCLUSIONS

The numerical studies of dynamic responses of TLP with large amplitude motions (nonlinear model) and with small displacements (linear model) are performed, and the results of both models are compared. The differences of 6 degrees of freedom and stresses of four tethers between the two models are investigated.

The differences in steady state responses of heave, roll, pitch and stresses of four tethers between nonlinear and linear model are remarkable. The differences between the two models are sensitive to wave period and wave heading angle; and insensitive to wave height for the case we studied. Because the majority of differences of responses are significant, it seems that employment of linear model may induce unallowable errors. Then the nonlinear model may be a better alternative. For the circumstances when the maximum stresses of tethers are the main concerned parameters (the accuracy of evaluation will directly influence the design of tethers), nonlinear model is intensively recommended.

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