

# Direct Numerical Simulation of Three-Dimensional Richtmyer–Meshkov Instability \*

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Direct numerical simulation (DNS) is used to study flow characteristics after interaction of a planar shock with a spherical media interface in each side of which the density is different. This interfacial instability is known as the Richtmyer–Meshkov (R-M) instability. The compressible Navier–Stokes equations are discretized with group velocity control (GVC) modified fourth order accurate compact difference scheme. Three-dimensional numerical simulations are performed for R-M instability installed passing a shock through a spherical interface. Based on numerical results the characteristics of 3D R-M instability are analysed. The evaluation for distortion of the interface, the deformation of the incident shock wave and effects of refraction, reflection and diffraction are presented. The effects of the interfacial instability on produced vorticity and mixing is discussed.

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The study of Richtmyer–Meshkov (R-M) instability is an important subject in many practical problems, such as inertial confinement fusion (ICF) and explosion of supernova problems. Thus, recently people have devoted much more attention to investigation of these kinds of problems.<sup>[1,2]</sup>

In order to correctly capture both the shock and contact discontinuity produced by the different density near the interface, a high order accurate scheme is used to solve the compressible Navier–Stokes (N-S) equations. The basic idea of the numerical method is that the fourth-order accurate central compact difference scheme is used to approximate the viscous terms in the N-S equations, the fourth-order accurate compact difference scheme with group velocity control (GVC)<sup>[3]</sup> is used to approximate the convective terms, and the third-order accurate R-K method is used to approximate the time derivatives. Suppose  $F_j/\Delta x$  is an approximation of the first derivative  $\partial f/\partial x$ . The fourth-order compact scheme with GVC is as follows:

$$\alpha_j^\pm F_{j+1}^\pm + \beta_j^\pm F_j^\pm + \gamma_j^\pm F_{j-1}^\pm = d_j^\pm, \quad (1)$$

$$\begin{aligned} \alpha_j^\pm &= \frac{1}{6} - \sigma_{j+\frac{1}{2}}^\pm, & \gamma_j^\pm &= \frac{1}{6} + \sigma_{j-\frac{1}{2}}^\pm, \\ \beta_j^\pm &= \frac{2}{3} - \sigma_{j+\frac{1}{2}}^\pm + \sigma_{j-\frac{1}{2}}^\pm, \\ d_j^\pm &= \delta_x^0 f_j^\pm - 2[\sigma_{j+\frac{1}{2}}^\pm \delta_x^+ - \sigma_{j-\frac{1}{2}}^\pm \delta_x^-] f_j^\pm, \end{aligned} \quad (2)$$

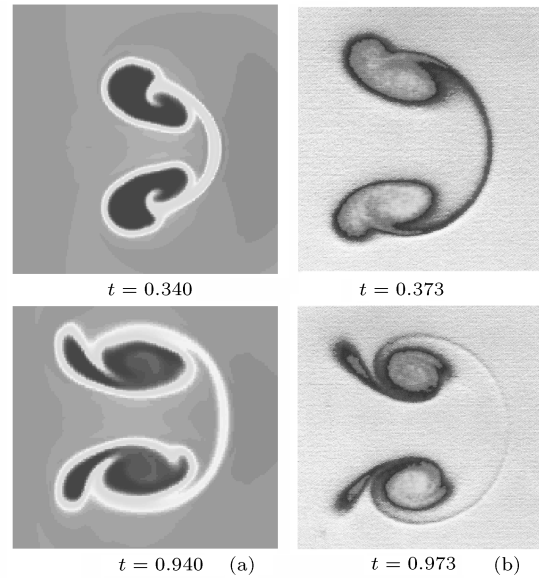
$$\sigma_{j+\frac{1}{2}}^\pm = \pm \sigma_0 [1 \pm \gamma_0 SS_{j+\frac{1}{2}}(P)] \left| \frac{P_{j+1} - P_j}{P_{j+1} + P_j} \right|,$$

$$SS_{j+\frac{1}{2}}(p) = \frac{1}{2} [SS_j(p) + SS_{j+1}(p)], \quad (3)$$

$$SS_j(p) = \text{sign}[\delta_x^0 p_j \cdot \delta_x^2 p_j]. \quad (4)$$

The superscript  $\pm$  corresponds to the positive and

negative flux vectors respectively, and the function  $SS(P)$  is used to control the group velocity.



**Fig. 1.** Density contours for cylindrical interface: (a) numerical results, (b) experimental results.

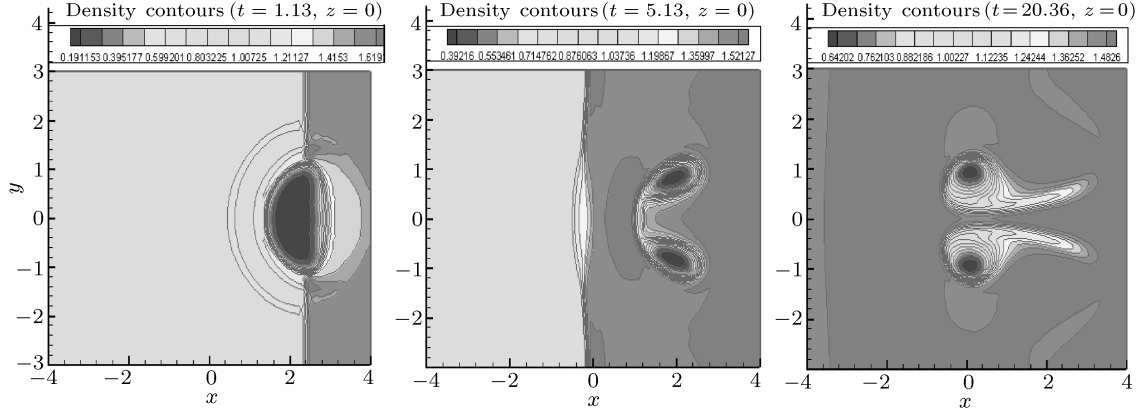
In order to check the accuracy of the numerical method, it is used to solve the 2D compressible N-S equations for simulating interaction of a planar shock with cylindrical interface. The density ratio is  $\rho_1/\rho_2 = 0.138$  with the density  $\rho_1$  inside the cylinder and  $\rho_2$  outside the cylinder. The shock is moving from the right to the left, and the shock Mach number is  $Ms = 1.093$ . The density contours at different time are given in Fig. 1(a). For comparison the corresponding experimental results are given in Fig. 1(b). The time scale in Fig. 1 is defined as the same as Ref. [4].

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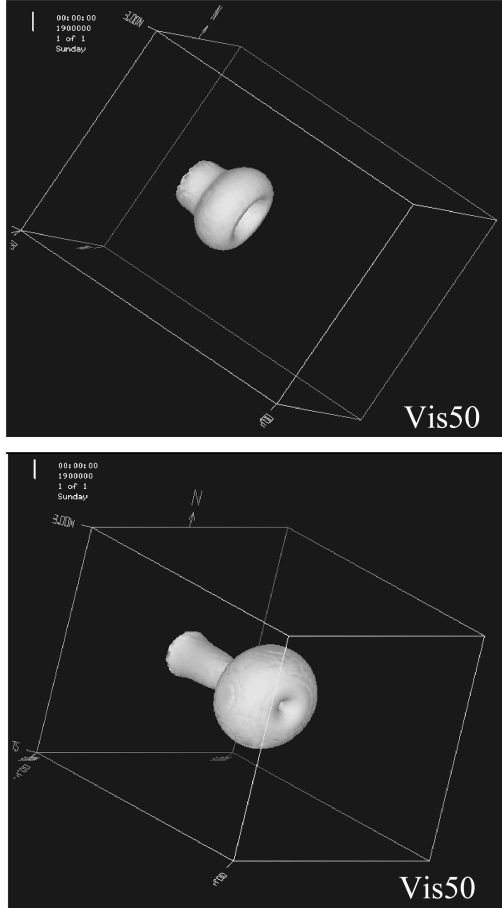
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From these figures it can be seen that numerical re- sults agree well with the experimental data.



**Fig. 2.** Density contours on section  $z = 0$  at different times  $t = 1.13, 5.13, 20.36$ .



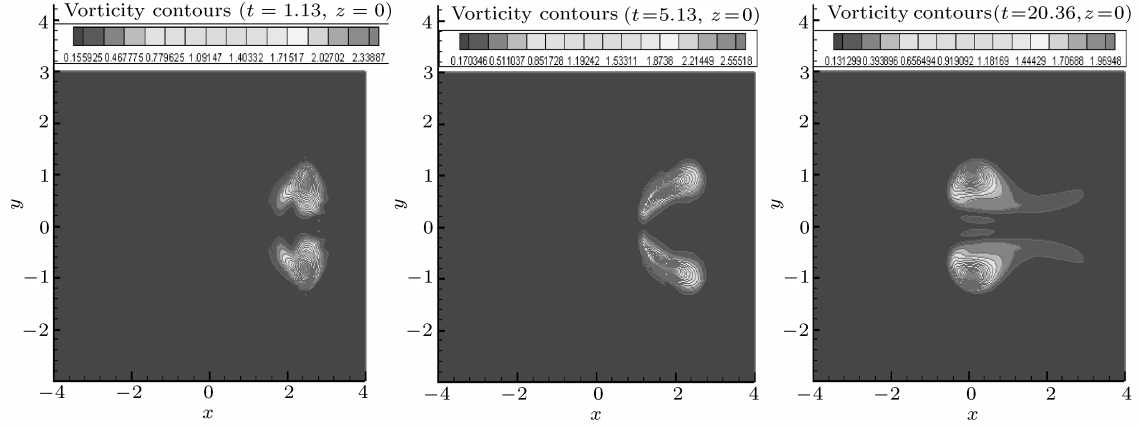
**Fig. 3.** Constant density surface at different times.

The numerical method presented above is used to solve the 3D compressible N-S equations for simulating interaction of a planar shock with a spherical interface. The N-S equations are written in the Cartesian coordinate. The radius of the initial interface  $r_0$  is taken as the characteristic length, the initial shock speed  $u_s$  as the characteristic velocity, and the time

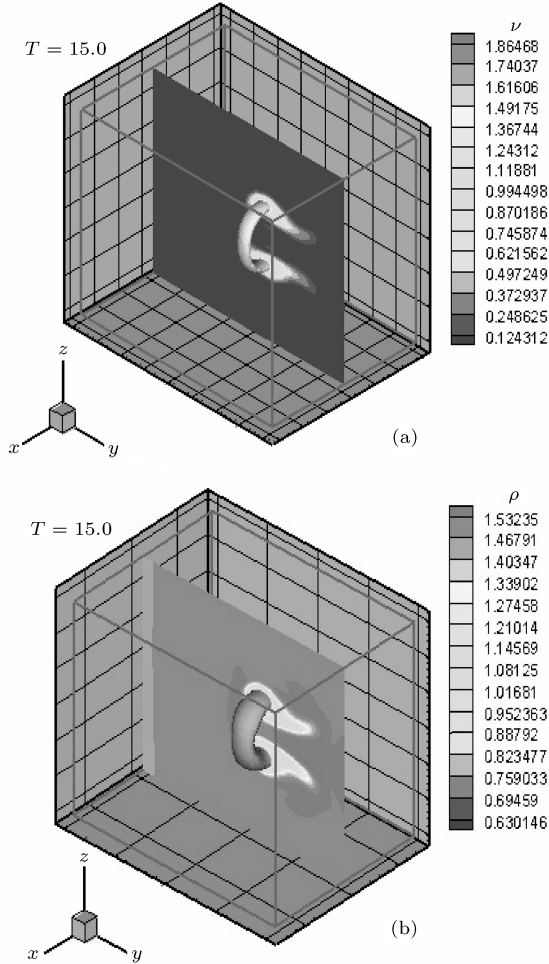
scale is defined as  $r_0/u_s$ . The domain of the computation is  $-L_x < x < L_x$ ,  $-L_y < y < L_y$ ,  $-L_z < z < L_z$ , where  $L_x = L_z = 3.0$  and  $L_y = 4.0$  with grid system  $91 \times 121 \times 91$ . The boundary conditions with free incoming parameters are given on the boundary  $y = -L_y$ ; non-reflecting boundary conditions are used on the downstream boundary  $y = L_y$  and on the side boundaries  $x = \pm L_x$ ,  $z = \pm L_z$ . The shock wave is going from the heavy gas with density  $\rho_2$  outside the interface to the light gas with density  $\rho_1$  inside the interface. The Mach number is  $Ms = 1.25$  and the density ratio is  $\rho_1/\rho_2 = 0.07$ . Figure 2 shows the density contours on section  $z = 0$  at different times ( $t = 1.13, 5.13, 20.36$ ). From Fig. 2(a) it can be seen that as the shock sweeps over the interface the shape of the interface at the upstream face changes due to the significant compression by the shock and Kelvin-Helmholtz instabilities produced due to the shear flow near the spherical interface which becomes unstable. We also can see that the first and second transmitted waves on the left are followed by the interface and the reflect waves are going to the right. At the same time due to the shock going from the heavy gas to the light gas and the shock inside the interface has a higher propagation speed and the effect of the interfacial instability the incident shock is deformed. This phenomenon can also be seen from the pressure contours not presented here. From Fig. 2(b) it can be seen that after interaction due to interface instability the heavy air jet with conical shear layer is formed, then this jet impinges on the downstream interface and pierces it (see Fig. 2(c)). It can be seen clearly from picture of 3D constant density surface at different times (see Fig. 3). During the time of the interaction between the shock and the interface the vorticity is produced due to non-parallelism of the gradients for pressure and density. Because the shock is incident from the right to the left and the light gas is relatively easier to

accelerate, clockwise vorticity is produced at the top and anticlockwise vorticity is generated at the bottom of the interface. Figure 4 shows the vorticity contours on section  $z = 0$  at difference times ( $t = 1.13, 5.13, 20.36$ ). Figure 5 shows the constant vorticity surface

and the vorticity contours on the section  $z=0$ . From these pictures it can be seen that with further development of the flow field the vortex ring structure is formed (see Figs. 4(b) and 5(a)).



**Fig. 4.** Vorticity contours on section  $z = 0$  at difference times  $t = 1.13, 5.13, 20.36$ .



**Fig. 5.** Constant vorticity  $\nu$  (a) and density (b) surfaces, and vorticity and density contours on section  $z = 0$ .

This vortex ring for the sphere-interface is more

distinct than the case for the cylinder interface. For the case of interaction between the planar shock and the cylindrical interface there is the vortex line which is stable, but the 3D vortex ring is unstable. In the further development of the interfacial instability the smaller vortex structures are produced due to breaking up of this unstable vortex ring. It is more effective for mixing of light and heavy gas. From numerical results we can also see that most of the vorticity in the flow is concentrated in the vortex ring and along the conical shear layer at the boundary of the heavy air jet. The obtained numerical results are similar to the basic physical phenomenon occurred in the mode test for collision between the supernova and the ring.<sup>[5,6]</sup> It can be believed that the R-M instability problem of 3D planar shock-spherical interface interaction reflects the basic physical characteristics of this kind of problems.

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