Non-slipping JKR model for transversely isotropic materials

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Abstract

A generalized plane strain JKR model is established for non-slipping adhesive contact between an elastic transversely isotropic cylinder and a dissimilar elastic transversely isotropic half plane, in which a pulling force acts on the cylinder with the pulling direction at an angle inclined to the contact interface. Full-coupled solutions are obtained through the Griffith energy balance between elastic and surface energies. The analysis shows that, for a special case, i.e., the direction of pulling normal to the contact interface, the full-coupled solution can be approximated by a non-oscillatory one, in which the critical pull-off force, pull-off contact half-width and adhesion strength can be expressed explicitly. For the other cases, i.e., the direction of pulling inclined to the contact interface, tangential tractions have significant effects on the pull-off process, it should be described by an exact full-coupled solution. The elastic anisotropy leads to an orientation-dependent pull-off force and adhesion strength. This study could not only supply an exact solution to the generalized JKR model of transversely isotropic materials, but also suggest a reversible adhesion sensor designed by transversely isotropic materials, such as PZT or fiber-reinforced materials with parallel fibers.

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1. Introduction

The mechanics of contact between solid hemispheres has been used extensively in studying the surface energy of materials during the past decade (for examples, Barquins, 1988; Carpick et al., 1996; Baney and Hui, 1997; Greenwood, 1997; Johnson and Greenwood, 1997; Barthel, 1998; Greenwood and Johnson, 1998; Kim et al., 1998; Robbe-Valloire and Barquins, 1998; Morrow et al., 2003; Schwarz, 2003). Four main theories have been developed to describe the history of this contact problem: those of Hertz (1882), Johnson et al. (1971), Derjaguin et al. (1975), and Maugis (1992). While the Hertz theory (Hertz, 1882) assumes that adhesion between the spheres cannot be sustained, the JKR, DMT and MD theories do allow for adhesion by taking into account the surface energies of the bodies. In the JKR (Johnson et al., 1971) model, an equilibrium contact area is established via Griffith energy balance between elastic energy and surface energy, which results
in compressive stress in the central region of contact and crack-like singular tensile stress near the edge of contact; the contact area remains finite until a critical pull-off force is reached. In the DMT model (Derjaguin et al., 1975), molecular forces outside the Hertz contact area are considered, but these forces are assumed not to change the contact profile of the Hertz solution and the tensile stress is finite in the cohesive zone outside the contact area but zero inside it. This paradox was solved by Maugis (1992), who developed a unified model linking the JKR and DMT models by extending the Dugdale model (Dugdale, 1960) of a plastic crack to the case of adhesive contact between two elastic spheres.

Most of the existing models on contact mechanics are related to isotropic materials and the tangential tractions in the contact region are neglected or regarded as frictional one to uncouple the normal and tangential stresses (Johnson, 1985). For a nanometer-sized contact, most of the phenomena will be different from those predicted by JKR theory which is not the only description of bodies in adhesive contact but is rather the limiting case of a continuous regime of contact mechanics. For example, Carpick et al. (1996) did nanometer-sized contact experiment and found that the tip-sample adhesion and the measured frictional forces unexpectedly decreased by more than one order of magnitude when scanning the tip in contact with the mica sample. They attribute the friction and adhesion decreases to changes of the interface, either structure or chemical, as opposed to changes in bulk structure or properties. Although several features unique to the nanoscale were observed in the experiment study of adhesion and friction for nano-asperities (Enachescu et al., 1999; Carpick et al., 2004), they found that the friction is proportional to the true contact area, the interfacial shear strength and the work of adhesion can be determined in the framework of fracture mechanics.

With the developing of bio-mechanics and bionics, contact mechanics has been used to understand biological adhesion mechanism (Arzt et al., 2003; Geim et al., 2003; Persson, 2003; Glassmaker et al., 2004; Gao and Yao, 2004; Hui et al., 2004; Chu et al., 2005; Gao et al., 2005). In some cases, perfect bonding should be considered because the effects of tangential traction on the pull-off process cannot be neglected (Chen and Gao, 2006a,b,c, 2007). Recent studies on elastic bodies in non-slipping adhesive contact with a laterally stretched substrate (Chen and Gao, 2006a,b) indicate that the substrate strain can have significant effect on the contact area. The pull-off process of two elastic spheres in non-slipping adhesive contact under a pair of pulling forces and a mismatch strain will be influenced by the mismatch strain significantly (Chen and Gao, 2006c). Furthermore, isotropy is not enough sometimes to describe the material characters (Gao et al., 2005; Yao and Gao, 2006; Chen and Gao, 2007), such as the attachment pad of grasshopper and cicada (Slifer, 1950; Roth and Willis, 1952; Arnold, 1974), the tissue of gecko’s feet (Autumn et al., 2000).

As an extension of the adhesive contact model for isotropic materials (Chen and Wang, 2006), in the present paper, a generalized adhesive contact model between two dissimilar transversely isotropic solids is investigated. The contact region is also assumed to be perfect bonding, then, an external loading is added obliquely on the cylinder. Exact solutions to the generalized JKR model for transversely isotropic materials will be given and analyzed. The results suggest that an orientation-dependent adhesion sensor could be designed by transversely isotropic materials, such as PZT-4 and fiber-reinforced composites, where all fibers are in parallel. Furthermore, explicit solutions can be deduced from the generalized model for some special and useful cases, such as adhesive contact between a rigid cylinder and a transversely isotropic half plane or between an isotropic solid and a transversely isotropic substrate, only if the direction of pulling force is normal to the contact interface.

2. Elastic constants and Barnett–Lothe tensors for transversely isotropic material

A special class of orthotropic materials are those that have the same properties in one plane (e.g., the $x - z$ plane) and different properties in the direction normal to this plane (e.g., the $y$-axis). Such materials are called transversely isotropic, and they are described by five independent elastic constants, instead of nine for fully orthotropic.
Examples of transversely isotropic materials include some piezoelectric materials (e.g., PZT-4, barium titanate) and fiber-reinforced composites where all fibers are in parallel. By convention, the five elastic constants in transverse isotropic constitutive equations are the Young’s modulus and Poisson ratio in the $x - z$ symmetry plane, $E_1$ and $v_1$, the Young’s modulus and Poisson ratio in the $y$-direction, $E_2$ and $v_2$, and the shear modulus in the $y$-direction $G_{12}$.

For orthotropic materials, Dongye and Ting (1989) and Hwu (1993) have shown the explicit expressions for the Barnett–Lothe tensors $S$ and $L$, which are real matrices composed of the elasticity constants. For plane strain transversely isotropic materials, the corresponding Barnett–Lothe tensors $S$ and $L$ can be reduced as

$$S = \begin{bmatrix} 0 & s_{12} \\ s_{21} & 0 \end{bmatrix}, \quad L = \begin{bmatrix} L_{11} & 0 \\ 0 & L_{22} \end{bmatrix},$$  \tag{1}

where

$$s_{12} = -a_2 \gamma_1 z_1, \quad s_{21} = a_4 \gamma_2 z_1,$$

$$L_{11} = a_1 \gamma_1 E_1, \quad L_{22} = a_2 \gamma_2 E_2,$$

$$\gamma_1 = \left( E_1 / G_{12} + 2 \epsilon_1 \sqrt{E_1 / E_2} \right)^{-1/2}, \quad \gamma_2 = \left( E_2 / G_{12} + 2 \epsilon_1 \sqrt{E_2 / E_1} \right)^{-1/2},$$

$$a_1 = (1 - v_1^2)^{-1/2}, \quad a_2 = \left( 1 - \frac{v_1^2}{E_2} \right)^{-1/2},$$

$$z_1 = \sqrt{(1 - v_1^2) \left( 1 - \frac{v_1^2}{E_2} \right) - (1 + v_1) v_2 \sqrt{E_1 / E_2}},$$  \tag{2}

$s$ and $L$ tensors are often used to express the influence of material properties. In the present paper, they will be used as coefficients in the following Green equations (7).

3. Adhesive contact model for two dissimilar transversely isotropic solids

In this paper, a plane strain model as shown in Fig. 1 is investigated, in which an elastic cylinder is in adhesive contact with an elastic half plane. Both solids are transversely isotropic materials. The upper material

![Fig. 1. The adhesive contact model between an elastic transversely isotropic cylinder of radius $R$ and an elastic transversely isotropic half plane with contact width $2a$. Two kinds of coordinate systems $(x,y_1)$ and $(x,y_2)$ are fixed on the cylinder and the half plane, respectively, with the origins at the center of the contact interface.](image-url)
denotes as material “1” and the lower one is material “2”. \( R \) is the radius of the upper cylinder and \( 2a \) is the contact width. After adhesive contact, an external loading \( F \) in \( x - y \) plane acts on the cylinder with an angle \( \phi \) inclined to the normal of the contact interface as shown in Fig. 1.

Two Cartesian reference coordinate systems \((x_1, y_1)\) and \((x_2, y_2)\) lie at the center of the contact region with \( y_1 \) and \( y_2 \) pointing into the upper cylinder and the lower half-space, respectively. The contact region is assumed to be perfect bonding, so that the edges of the contact region resemble two opposing interfacial cracks under plane strain deformation.

As in almost all contact mechanics theories (Johnson, 1985), the contact width is assumed to be small compared to the radius of the cylinder such that the deformation of the cylinder can be approximated by that of an elastic half-space. The pulling force is assumed to be properly added such that no net bending moment on the contact region is produced.

Under the above assumptions, the continuity condition of displacements across the contact interface can be expressed as

\[
\begin{aligned}
\tilde{u}_{x1} - \tilde{u}_{x2} &= 0, \\
\tilde{u}_{y1} + \tilde{u}_{y2} &= \delta - \frac{x}{R}, \quad |x| \leq a,
\end{aligned}
\]

where \( \tilde{u}_{ij} \) \((\tilde{u}_{ij})\) denotes the displacement in the \( x \) \((y_i, i = 1, 2)\) direction of each material \( j \) \((j = 1, 2)\) along the interface, \( \delta \) is a constant and \( R \) the radius of the cylinder (Johnson, 1985).

The displacement gradients with respect to \( x \) yields

\[
\begin{aligned}
\frac{\partial \tilde{u}_{x1}}{\partial x} - \frac{\partial \tilde{u}_{x2}}{\partial x} &= 0, \\
\frac{\partial \tilde{u}_{x1}}{\partial x} + \frac{\partial \tilde{u}_{x2}}{\partial x} &= -\frac{x}{R}.
\end{aligned}
\]

Using the Green’s functions of an elastic transversely isotropic half-space subjected to surface tractions (Ting, 1996) yields

\[
\begin{aligned}
\frac{D_{11}}{\pi} \int_{-a}^{a} \frac{\theta_{j}^2}{s^2} ds + W_{12} P(x) &= 0, \\
\frac{D_{22}}{\pi} \int_{-a}^{a} \frac{P(s)}{s^2} ds + W_{21} Q(x) &= -\frac{x}{R},
\end{aligned}
\]

where \( P(x) \) and \( Q(x) \) denote the normal and tangential tractions along the contact interface of the cylinder, respectively. The coefficients are expressed as

\[
D_{11} = \left( \frac{1}{L_{11}} \right)_1 + \left( \frac{1}{L_{11}} \right)_2, \quad D_{22} = \left( \frac{1}{L_{22}} \right)_1 + \left( \frac{1}{L_{22}} \right)_2
\]

and

\[
W_{12} = \left( \frac{s_{12}}{L_{22}} \right)_1 - \left( \frac{s_{12}}{L_{22}} \right)_2, \quad W_{21} = \left( \frac{s_{21}}{L_{11}} \right)_1 - \left( \frac{s_{21}}{L_{11}} \right)_2,
\]

\[
W_{21} = -W_{12},
\]

\( ()_j \) denotes the term of \( j \)th \((j = 1, 2)\) material.

Eq. (7) can be rewritten in a matrix form as

\[
\frac{1}{\pi} \int_{-a}^{a} A \cdot f(s) ds + B f(x) = C,
\]

where

\[
A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & -\frac{w_{21}}{D_{11}} \\ \frac{w_{21}}{D_{22}} & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 \\ \frac{w_{21}}{D_{22}} \end{bmatrix}, \quad f(x) = \begin{bmatrix} Q(x) \\ P(x) \end{bmatrix}.
\]

The calculations are quite lengthy but the methodology of solving such integral Eq. (11) is standard, which can be found in Chen and Gao (2006a), so that we skip the details and only give a brief introduction as follows.
Introducing the following transformation

\[
F_k(z) = \frac{1}{2\pi i} \int_{-a}^{a} \frac{f_k(s)}{s - z} \, ds \quad k = 1, 2,
\]

where \( z = x + iy \) and here \( i = \sqrt{-1} \), Eq. (11) can be decoupled into two inhomogeneous Hilbert equations. Following the standard procedure (Carrier et al., 1983; Chen and Gao, 2006a) to solve Hilbert equation, we can obtain the interfacial tractions \( Q(x) \) and \( P(x) \),

\[
\begin{align*}
Q(x) &= 2\text{Re}\{I(x)\} + \frac{\pi i}{R\sqrt{D_1 D_22 (1 - \eta^2)}} \int_{-a}^{a} \frac{t(a + t)^{\eta}(a - t)^{-\eta}}{t - x} \, dt, \\
P(x) &= \left[ -2\text{Im}\{I(x)\} + \frac{2\pi i (a + x)^{-\eta}(a - x)^{-\eta}}{2i(1 - \eta^2)} \right] \sqrt{\frac{D_1}{D_2}}.
\end{align*}
\]

where

\[
I(x) = \frac{(a + x)^{-\eta}(a - x)^{-\eta}}{2\pi i \sqrt{D_1 D_22 (1 - \eta^2)}} \int_{-a}^{a} \frac{t(a + t)^{\eta}(a - t)^{-\eta}}{t - x} \, dt. \tag{15}
\]

The stress singularity is

\[
r = \frac{1}{2} + i\varepsilon, \quad \varepsilon = \frac{1}{2\pi} \ln \left( \frac{1 + \eta}{1 - \eta} \right) 
\]

and

\[
\eta = |W_{21} (D_1 D_22)^{-1/2}|
\]

denotes a bimaterial constant, which is analogous to Dundurs’ parameters \( \beta \) for isotropic materials. Using the following equilibrium conditions

\[
\int_{-a}^{a} P(x) \, dx = -F \cos \phi, \quad \int_{-a}^{a} Q(x) \, dx = F \sin \phi \tag{18}
\]

yields

\[
K_1 = \frac{-\sqrt{1 - \eta^2} \left( F \sin \phi + iF \cos \phi \sqrt{D_{22}/D_{11}} \right)}{4i \int_{-a}^{a} (a + x)^{-\eta}(a - x)^{-\eta} \, dx},
\]

\[
K_2 = \frac{-\sqrt{1 - \eta^2} \left( F \sin \phi - iF \cos \phi \sqrt{D_{22}/D_{11}} \right)}{4i \int_{-a}^{a} (a + x)^{-\eta}(a - x)^{-\eta} \, dx}.
\]

Substituting Eqs. (19) and (20) into Eqs. (14), the interfacial tractions can be rewritten as

\[
\begin{align*}
Q(x) &= 2\text{Re}\{I(x)\} + \frac{\pi i}{R\sqrt{D_1 D_22 (1 - \eta^2)}} \int_{-a}^{a} \frac{t(a + t)^{\eta}(a - t)^{-\eta}}{t - x} \, dt, \\
\end{align*}
\]

\[
P(x) = \left[ -2\text{Im}\{I(x)\} + \frac{4\text{Re}\{I(x)\}}{\sqrt{1 - \eta^2}} \right] \sqrt{\frac{D_1}{D_2}}.
\]

According to Wu (1990), Hwu (1993) and Ting (1996), the stress intensity factors for an interface crack can be given by

\[
K = \lim_{x \to d} \sqrt{2\pi(a - x)} \mathcal{A} \left( \left( \frac{a - x}{l} \right)^{\nu_i} \right) \mathcal{A}^{-1} \phi = \left\{ K_{Il}, -K_{I} \right\}, \tag{22}
\]

where

\[
\nu_1 = \nu, \quad \nu_2 = -\nu
\]

\[
\nu = 1
\]
and

\[
A = \begin{bmatrix}
\sqrt{2D_{11}} & \sqrt{2D_{12}} \\
\sqrt{2D_{21}} & \sqrt{2D_{22}}
\end{bmatrix}, \quad \varphi = \begin{Bmatrix}
Q(x) \\
P(x)
\end{Bmatrix}.
\] (24)

Substituting Eqs. (23), (24) into (22) yields

\[
\begin{bmatrix}
K_{H} \\
-K_{I}
\end{bmatrix} = \lim_{x \to a} \sqrt{2\pi} \begin{bmatrix}
\text{Re}\{H(x)\} \\
-\text{Im}\{H(x)\}
\end{bmatrix} \begin{bmatrix}
\sqrt{\frac{D_{22}}{D_{11}}} \text{Im}\{H(x)\} \\
\text{Re}\{H(x)\}
\end{bmatrix} \begin{Bmatrix}
Q(x) \\
P(x)
\end{Bmatrix},
\] (25)

where

\[
H(x) = (a - x)^{\frac{1}{2} + i\phi} I^{-i\phi}.
\] (26)

Using Eq. (21), the stress intensity factors can be explicitly expressed as

\[
K_{I} = \frac{-a^{3/2}}{\sqrt{\pi RD_{22}}(1 - \eta^{2})} \text{Re}\left\{ 2^{i\phi}(\frac{a}{\eta})^{i}\int_{-1}^{1} \frac{\xi(1 + \xi)(1 - \xi)^{\nu}}{\xi - 1} d\xi \right\} + \frac{\sqrt{\pi}}{\sqrt{a}}
\]

\[
\times \sqrt{\frac{D_{22}}{D_{11}}} \text{Im}\left\{ \int_{-1}^{1} \frac{2^{i\phi}(\frac{a}{\eta})^{i}}{1 + \xi^{2}} (\frac{a}{\eta})^{i} \left( F \sin \phi + iF \cos \phi \sqrt{\frac{D_{22}}{D_{11}}} \right) \xi d\xi \right\}
\] (27)

and

\[
K_{H} = \frac{a^{3/2}}{\sqrt{\pi R \sqrt{D_{11}D_{22}}(1 - \eta^{2})}} \text{Im}\left\{ 2^{i\phi}(\frac{a}{\eta})^{i}\int_{-1}^{1} \frac{\xi(1 + \xi)(1 - \xi)^{\nu}}{\xi - 1} d\xi \right\}
\]

\[
+ \frac{\sqrt{\pi}}{\sqrt{a}} \text{Re}\left\{ \int_{-1}^{1} \frac{2^{i\phi}}{1 + \xi^{2}} (\frac{a}{\eta})^{i} \left( F \sin \phi + iF \cos \phi \sqrt{\frac{D_{22}}{D_{11}}} \right) \xi d\xi \right\}
\] (28)

Substituting the stress intensity factors in Eqs. (27) and (28) into the energy release rate

\[
G = \frac{1}{4 \cosh^{2} \pi \phi} (D_{22}K_{I}^{2} + D_{11}K_{H}^{2}),
\] (29)

then, using the Griffith energy balance criterion at the contact edges

\[
G = \Delta_{G},
\] (30)

where \( \Delta_{G} \) is the work of adhesion, leads to the following governing equation

\[
\frac{F^{2} \pi D_{11}}{a} \left( \frac{\sin^{2} \phi + \frac{D_{22}}{D_{11}} \cos^{2} \phi}{\sqrt{R(1 - \eta^{2})}} \right)^{2} - 2aF \left( \int_{-1}^{1} (1 + \xi^{2})^{-\nu} (1 - \xi)^{-\nu} d\xi \right)^{2}
\]

\[
\times \sqrt{\frac{D_{22}}{D_{11}}} \text{Im}\left\{ \int_{-1}^{1} \frac{\xi^{2}(1 + \xi)^{\nu}(1 - \xi)^{\nu}}{\xi - 1} d\xi \left( \sin \phi + i\sqrt{\frac{D_{22}}{D_{11}}} \cos \phi \right) \right\}
\]

\[
+ \frac{a^{3}}{\pi R^{2} D_{22}(1 - \eta^{2})} \left( \int_{-1}^{1} \frac{(1 + \xi)(1 - \xi)^{\nu}}{\xi - 1} d\xi \right)^{2} - 4\Delta_{G} \cosh^{2} \pi \phi = 0
\] (31)

which describes the contact size \( a \) as a function of the pulling force \( F \) and the pulling angle \( \phi \).

For a special case, i.e., \( \phi = 0 \), which corresponds to the direction of the pulling force normal to the contact interface, the relation between the contact size \( a \) and the pulling force \( F \) can be simplified as
\[
\frac{F^2\pi D_{22}}{a} \left| \int_{-1}^{1} (1 + \zeta)^{-\gamma}(1 - \zeta)^{-\gamma} d\zeta \right|^2 - \frac{2aF}{R(1 - \eta^2)} \Re \left\{ \int_{-1}^{1} \frac{\zeta (1 + \zeta)^{\gamma} (1 - \zeta)^{\gamma}}{\zeta - 1} d\zeta \right\} \right\}
\]
\[
+ \frac{a^3}{\pi R^2 D_{22}(1 - \eta^2)^2} \left( \int_{-1}^{1} \frac{\zeta (1 + \zeta)^{\gamma} (1 - \zeta)^{\gamma}}{\zeta - 1} d\zeta \right)^2 - 4\gamma \cosh^2 \pi \eta = 0. \tag{32}
\]

4. Analysis and discussion

4.1. The effects of mismatch parameter \(\eta\)

From Eq. (31), one can find that the relation between the pulling force \(F\) and the contact width \(2a\) is affected by the elastic mismatch parameter \(\eta\), which leads to an oscillatory singularity and a very complex expression. Motivated by recent studies (Chen and Gao, 2006a,b,c; Chen and Gao, 2007; Chen and Wang, 2006), where the Dundurs’ parameter \(\beta\) has a minor effect on the pull-off process when the pulling force is normal to the contact interface.

It can be shown in the present model that the normalized pulling force \(F/\Delta\gamma\) depends on the normalized contact half-width \(a/R\) via four parameters, \(D_{11}/D_{22}\), \(\Delta\gamma D_{22}/R\), \(\eta\) and \(\phi\). For the case with a fixed pulling angle \(\phi \neq 0\), numerical calculation find that \(\eta\) has a significant effect on the pull-off process and the difference between the non-oscillatory \((\eta = 0)\) and oscillatory solutions \((\eta \neq 0)\) increases with an increasing value of \(\eta\).

For a special case, \(\phi = 0\), i.e., the pulling force normal to the contact interface, the relation between the pulling force and the contact half-width is expressed by Eq. (32), where we find that the normalized pulling force \(F/\Delta\gamma\) depends on the normalized contact half-width \(a/R\) via three parameters, \(\Delta\gamma D_{22}/R\), \(\eta\) and \(\phi\), except for the non-dimensional parameter \(D_{11}/D_{22}\). For a fixed value of \(\Delta\gamma D_{22}/R\), the difference between the non-oscillatory and oscillatory solutions with different values of \(\eta\) is very small when \(\eta \leq 0.25\). Especially, the critical pull-off force and pull-off contact half-width are hardly influenced by a wide range of \(\eta\), so that the non-oscillatory solution can be a very good approximation to the oscillatory cases. Fig. 2 is shown as an example.

Furthermore, one can easily infer from the conclusions of Fig. 2 that the non-oscillatory solution is also adaptive to the model of a rigid or an elastic isotropic cylinder in contact with an isotropic or a transversely isotropic substrate only if the direction of pulling is normal to the contact interface. This inference is consistent well with the former studies about adhesive contacts (Chen and Gao, 2006a,b,c; Chen and Gao, 2007; Chen

Fig. 2. Normalized pulling force \(F/\Delta\gamma\) as a function of normalized contact half-width \(a/R\) for \(\Delta\gamma D_{22}/R = 0.1\) and different values of \(\eta = 0.0, 0.1, 0.25, 0.5\).
An interesting issue is to study the critical pull-off force, pull-off contact width as well as the adhesion strength, which can be obtained from numerical calculation if $\phi \neq 0$. Examples are shown in Figs. 3–5. In each figure, we take $\eta = 0.1$, four different values of $D_{11}/D_{22}$ and two values of $\Delta \gamma D_{22}/R$. The results for isotropic case, i.e., $D_{11}/D_{22} = 1.0$, are also shown in the figures. Fig. 3 shows the normalized critical pull-off force $F^p/F_0^p$ as a function of the pulling angle $\phi$, where the subscript “0” denotes $\phi = 0$ and the superscript “p” denotes pull-off. From Fig. 3, one can see that the normalized critical pull-off force does not depend on the non-dimensional parameter $\Delta \gamma D_{22}/R$ but depends strongly on the non-dimensional value of $D_{11}/D_{22}$ and pulling angle $\phi$. As the value of $D_{11}/D_{22}$ increases, the normalized pull-off force varies stronger and stronger with the direction of pulling. The normalized pull-off force attains the maximum value always at $\phi = 0$ when the elastic anisotropy becomes stronger. Especially, for the case of $D_{11}/D_{22} \geq 1000$, the maximum pull-off force is almost an order of magnitude larger than the minimum one.
Fig. 4 shows the normalized pull-off contact width as a function of pulling angle $\phi$ with the same parameters used in Fig. 3. From Fig. 4, one can see that the normalized pull-off contact width also does not depend on the non-dimensional parameter $D_{22}/R$ but senses the value of $D_{11}/D_{22}$ strongly. The maximum pull-off contact width always emerges at $\phi = \pi/2$. An interesting phenomenon can be found from Figs. 3 and 4. The pull-off force and pull-off contact half-width for $D_{11}/D_{22} = 1.0$ in Figs. 3 and 4, respectively, increase when the pulling angle $\phi$ increases, which means that the friction force is proportional to the contact area in the isotropic case and is consistent with the experiment results (Carpick et al., 1996, 2004; Enachescu et al., 1999). However, in the transversely isotropic case, i.e., $D_{11}/D_{22} < 1.0$, the pull-off force decreases while the pull-off contact half-width increases for a determined value of $D_{11}/D_{22}$, which should be verified by the corresponding experiment in future.

The adhesion strength $\sigma^p$ is defined as the ratio of the pull-off force to pull-off contact width. Normalized adhesion strength $\sigma^p/\sigma^p_0$ as a function of the pulling angle $\phi$ is shown in Fig. 5 with the same parameters used in Figs. 3 and 4. It shows that $D_{11}/D_{22}$, i.e., the elastic anisotropy, has significant effects on the normalized adhesion strength $\sigma^p/\sigma^p_0$. As the value of $D_{11}/D_{22}$ increases, the normalized adhesion strength $\sigma^p/\sigma^p_0$ varies strongly with the direction of the pulling force. Furthermore, when the value of $D_{11}/D_{22}$ is larger than 1000, the maximum adhesion strength, which emerges at $\phi = 0$, can be an order of magnitude larger than the minimum one attained at $\phi = \pi/2$ and the value of adhesion strength decreases quickly when the angle deviates $\phi = 0$.

From above, one can see that a switch between attachment and detachment can thus be accomplished just by shifting the pulling angle. For practical application, it is helpful for designing synthetic adhesion systems in engineering using piezoelectric materials (e.g., PZT-4, barium titanate) or fiber-reinforced composites where all fibers are in parallel.

4.2. The non-oscillatory solution for the case of $\phi = 0$

For the case of $\phi = 0$, we have found from above that the non-oscillatory solution can be a good approximation to oscillatory one for the bimaterial adhesive contact model. Explicit expressions of the pull-off force, pull-off contact width and the adhesion strength in the non-oscillatory solution will be given, which is very convenient to be used for engineering.

The non-oscillatory condition corresponds that

$$\eta = 0$$

(33)
which yields

\[ e = 0, \quad r = \frac{1}{2}. \] (34)

Substituting the condition of \( \phi = 0 \) and Eqs. (33) and (34) into Eq. (21) yields

\[ Q(x) = 0 \] (35)

and

\[ P(x) = \frac{a^2 - x^2}{2RD_2} \frac{F}{\pi\sqrt{a^2 - x^2}}. \] (36)

Then the stress intensity factor can be obtained

\[ K_I = \frac{\sqrt{\pi a}^{3/2}}{2RD_2} + \frac{F}{\sqrt{\pi a}}. \] (37)

Substituting the stress intensity factor in Eq. (37) into the following Griffith energy balance criterion,

\[ G = \frac{D_2 K_I^2}{4} = \gamma \] (38)

yields the external loading \( F \) as a function of the contact half-width \( a \)

\[ F = -\frac{\pi a^2}{2RD_2} + 2\sqrt{\frac{\pi a \gamma}{D_2}}. \] (39)

Using the following equation

\[ \frac{\partial F}{\partial a} = 0 \] (40)

yields the pull-off contact half-width as

\[ a_0^p = \left( \frac{R^2D_2 \gamma}{\pi} \right)^{\frac{1}{3}} \] (41)

and the pull-off force

\[ F_0^p = \frac{3}{2} \left( \frac{\pi R \gamma}{D_2} \right)^{\frac{1}{3}}, \] (42)

where the subscript “0” denotes the case of \( \phi = 0 \).

The adhesion strength \( \sigma_p \) can be obtained from Eqs. (31) and (42) as,

\[ \sigma_p^0 = F_0^p = \frac{3}{2} \left( \frac{\pi^2 \gamma}{D_2 R} \right)^{\frac{1}{3}} \] (43)

Eqs. (39)–(43) can be a good approximation to the adhesive contact model of two transversely isotropic materials with the direction of the pulling force normal to the contact interface.

4.3. Special case – isotropic bimaterial case

If the transversely isotropic materials degenerate to isotropic materials, i.e.,

\[ D_{11} = D_{22} = \frac{2(1 - v_1^2)}{E_1} + \frac{2(1 - v_2^2)}{E_2} = \frac{2}{E}, \] (44)

\[ W_{21} = -W_{12} = \frac{(1 - 2v_1)(1 + v_1)}{E_1} - \frac{(1 - 2v_2)(1 + v_2)}{E_2}, \] (45)
where $E_1$ and $\nu_1$ denote the Young’s modulus and Poisson ratio of the upper cylinder, $E_2$ and $\nu_2$ are those of the lower substrate.

Then, the oscillatory solution will be identical to that in Chen and Gao (2006a) and the non-oscillatory solution will be the same as the classical two-dimensional JKR theory (Barquins, 1988; Chaudhury et al., 1996; Chen and Gao, 2006a).

For the cases of adhesive contact between a rigid cylinder and a transversely isotropic material or an isotropic elastic cylinder and a transversely isotropic material, the solutions can be obtained very easily from above.

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References


