An improved CE/SE scheme for multi-material elastic–plastic flows and its applications

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ABSTRACT

In this paper, a new definition of SE and CE, which is based on the hexahedron mesh and simpler than Chang's original CE/SE method (the space–time Conservation Element and Solution Element method), is proposed and an improved CE/SE scheme is constructed. Furthermore, the improved CE/SE scheme is extended in order to solve the elastic–plastic flow problems. The hybrid particle level set method is used for tracing the interfaces of materials. Proper boundary conditions are presented in interface tracking. Two high-velocity impact problems are simulated numerically and the computational results are carefully compared with the experimental data, as well as the results from other literature and LS-DYNA software. The comparisons show that the computational scheme developed currently is clear in physical concept, easy to be implemented and high accurate and efficient for the problems considered.

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1. Introduction

The material dynamic response and the stress wave propagation under high-velocity impact or explosion load are highly nonlinear phenomena. These problems are hardly described by analytical solutions, and hence we have to seek numerical methods. However, two main challenges are still confronted in the numerical methods: the numerical scheme for the governing equations and the special treatments for the material interfaces.

The governing equations for the elastic–plastic flows can be formulated in two ways known as Lagrangian and Eulerian approach. In the Lagrangian approach, the space grids are fixed within and deformed with the material. Free surfaces and contact surfaces between different materials are easily determined, and remain distinct throughout the calculation. The main limitation of the Lagrangian approach is the results are inaccurate with using the finite difference approximation when the grid cells are distorted significantly. The extreme case is that the computation fails when a grid cell folds over itself. The way to overcome this limitation is to remesh the computational region or to add artificial viscosity, which will reduce the accuracy of the solution. The well-known Lagrangian codes are HEMP [1], TOODY [2], EPIC [3], LS-DYNA [4], etc. In the Eulerian approach, the meshes are fixed in space through which materials flow. On the other hand, although the Eulerian approach can describe the large deformation of materials easily, it is difficult to identify the material interfaces accurately at each time step. Therefore, a

high-accuracy interface tracing algorithm must be adopted for the Eulerian approach. The well-known Eulerian codes are HELP [5], CSQ [6], CTH [7], etc.

The ALE (Arbitrary Lagrangian–Eulerian) method [8], which allows the mesh to move, is quite suitable for coupling problems between fluid and solid structures. However, in the large material deformation region, the adaptive remeshing technologies, as Lagrangian approach, are also needed. Thus, it can be seen that for the large deformation problems, such as high-velocity impact, solid–liquid and gas–liquid interaction, the Eulerian approach with high-resolution solvers for solving the governing equations and capturing interfaces is the best choice.

The traditional codes of Eulerian approach, which have low accuracy and cannot capture strong shock waves very well, mainly use first-order difference schemes and the von Neumann artificial viscosity. Recently, some researchers have applied the high-resolution techniques of the computational fluid dynamics (CFD) to solve the elastic–plastic flow problems. For example, Miller and Colella [9] employed the second-order Godunov method; Howell and Ball [10] developed a free Lagrange scheme with the Godunov method; and Udaykumar et al. [11] adopted the ENO scheme. These efforts not only improve the accuracy of the numerical scheme for the problems of elastic–plastic flows, but also show an important developing area.

The CE/SE method (the space–time Conservation Element and Solution Element method) originally proposed by Chang [12,13] is a novel high-resolution CFD method for hyperbolic conservation laws. It substantially differs from other traditional well-established methods. It has many features, such as a unified treatment of space and time, satisfying both local and global flux conservation in

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space and time by introduction of the Solution Elements (SEs) and Conservation Elements (CEs), and a new shock-capturing strategy. The CE/SE method has been extended to a high-order scheme [14] and has a great success in the field of CFD [13,15,16]. So far, the CE/SE method has been mainly applied in numerical simulations of supersonic flows. Chang et al. [15,17–26] used this method for the aeroacoustic computations and studying the jet of nozzle; Guo et al. [27] extended the CE/SE method to viscous flows; Yu et al. [28–31] simulated chemical detonation waves by using the CE/SE method; and Zhang et al. [32] derived the CE/SE scheme for MHD equations. However, the CE/SE method has not been employed to solve the complex elastic–plastic flows with solid features till now.

The Level Set method [33] is effective for tracing moving interfaces of materials. The key idea of the Level Set method is to introduce an implicit function to describe the advection of a certain interface. It is well suitable to such problems in which the interfaces undergo extremely topological changes. The hybrid particle Level Set method was presented by Enright et al. [34], which combines the merits of the Eulerian Level Set method and the Lagrangian approach based on marker particles. This is a smart technique to solve the issue of mass loss (volume loss for the incompressible case).

In the present work, we propose a new definition of SE and CE and construct an improved CE/SE scheme. The new definition is based on the hexahedron mesh and simpler than the Chang's original one. Furthermore, we apply the improved CE/SE scheme to solve the problems of two-dimensional elastic–plastic flows. As to the knowledge of the authors, it is the first time to use the CE/SE method to solve these problems. The hybrid particle Level Set technique [34] is used for tracing material interfaces. We carefully investigate the free boundary conditions and the contact boundary conditions in detail on the stress, velocity, density, and internal energy. For verifying the accuracy, resolution, and efficiency of the improved CE/SE scheme, we simulate the famous Taylor copper bar impact problem and the penetration of a WHA long rod in to a steel target. The computational results are also carefully compared with the results from experiments, other literature and LS-DYNA software.

2. Governing equations

In two-dimensional Cartesian coordinates, the Eulerian governing equations for the homogenous media without heat conducting, thermal diffusion and external forces can be written in the form of conservation laws as [1,35,36]

$$\frac{\partial \mathbf{Q}}{\partial t} + \frac{\partial \mathbf{E}(\mathbf{Q})}{\partial x} + \frac{\partial \mathbf{F}(\mathbf{Q})}{\partial y} = \mathbf{S}(\mathbf{Q}), \quad (1)$$

where $\mathbf{Q}$ is the vector of conserved variables, $\mathbf{E}$ and $\mathbf{F}$ are the conservation flux vectors in x and y directions respectively, and $\mathbf{S}$ is the source term vector. In Eulerian representation, these vectors are

$$\mathbf{Q} = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ E \\ S_{xx} \\ S_{yy} \\ S_{zz} \\ S_{xy} \\ \varepsilon^p \\ T \end{bmatrix}, \quad \mathbf{E} = \begin{bmatrix} \rho u \\ \rho u^2 - \sigma_{xx} \\ \rho uv - \sigma_{xy} \\ u(E - \sigma_{xx}) - v\sigma_{xy} \\ uS_{xx} \\ uS_{yy} \\ uS_{zz} \\ uS_{xy} \\ u\varepsilon^p \\ uT \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} \rho v \\ \rho uv - \sigma_{xy} \\ \rho v^2 - \sigma_{yy} \\ v(E - \sigma_{yy}) - u\sigma_{xy} \\ vS_{xx} \\ vS_{yy} \\ vS_{zz} \\ vS_{xy} \\ v\varepsilon^p \\ vT \end{bmatrix},$$

$$\mathbf{S} = \begin{bmatrix} \eta \rho u \\ \eta(\rho v^2 - \sigma_{xx} + \sigma_{zz}) \\ \eta(\rho uv - \sigma_{xy}) \\ \eta[v(E - \sigma_{xx}) - v\sigma_{xy}] \\ S_{sxx} \\ S_{syy} \\ S_{szz} \\ S_{sxy} \\ S_{\varepsilon^p} \\ S_T \end{bmatrix}, \quad (2)$$

where $\eta = -i/x$, $i = 0$ for the plane strain problems and for the axisymmetric problems (y is the symmetry axis). In Eq. (2), ρ is the density, u and v are the x and y components of the velocity, respectively, $E = \rho l + 1/2\rho(u^2 + v^2)$ is the total volume energy, l is the volume internal energy, p is the hydrostatic pressure, σ_{xx} , σ_{xy} , σ_{yy} are the components of the stress tensor, S_{sxx} , S_{sxy} , S_{syy} are the components of the deviatoric stress tensor, ε^p is the equivalent plastic strain, and T is the temperature induced by plastic work.

The deviatoric stress tensor s_{ij} reflects the shear deformation and obeys the Hooke's law and the plasticity flow model. The hydrostatic pressure p reflects the volume deformation. If the solid material is under a condition of high strain rate deformation, we can assume that p is determined by the thermodynamic equation of state (EOS) like fluid. In Lagrangian representation, the Hooke's law in the incremental stress form can be written as

$$\dot{s}_{ij} = 2G\left(\dot{D}_{ij} - \frac{1}{3}\dot{D}_{kk}\delta_{ij}\right) - s_{ik}\Omega_{kj} + \Omega_{ki}s_{kj}, \quad (3)$$

where G is the shear modulus, $\dot{s}_{ij}$ is the substantial derivative of s_{ij} , $\dot{D}_{ij}$ is the strain rate tensor, and Ω_{ij} is the spin tensor. The strain rate tensor $\dot{D}_{ij}$ and the spin tensor Ω_{ij} are associated with the velocity derivatives as follows:

$$\dot{D}_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right), \quad \Omega_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} - \frac{\partial v_j}{\partial x_i}\right), \quad (4)$$

where v_i and x_i are the i th velocity and coordinate components, respectively.

With the aid of Eq. (3), in the elastic state, the source terms of S_{sxx} , S_{syy} , S_{szz} , and S_{sxy} in Eq. (2) can be expressed as

$$\begin{aligned} S_{sxx} &= s_{xx}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + 2\Omega_{xy}s_{xy} + 2G\left(\frac{\partial u}{\partial x} - \Phi\right), \\ S_{syy} &= s_{yy}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + 2\Omega_{yx}s_{xy} + 2G\left(\frac{\partial v}{\partial y} - \Phi\right), \\ S_{szz} &= s_{zz}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + 2iG\left(\frac{u}{x} - \Phi\right), \\ S_{sxy} &= s_{xy}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \Omega_{xy}s_{yy} - \Omega_{yx}s_{xx} + G\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \end{aligned}$$

where the spin tensor components $\Omega_{xy} = -\Omega_{yx} = \frac{1}{2}\left[\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right]$ and the volume strain $\Phi = \frac{1}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$.

The relationship between plastic stress and strain is modeled by the von Mises yield criterion condition and the Prandtl Reuss flow rule. The von Mises yield criterion condition can be written in terms of the deviatoric stresses

$$s_{xx}^2 + s_{yy}^2 + s_{zz}^2 + 2s_{xy}^2 \leq \frac{2}{3}\sigma_y^2, \quad (5)$$

where σ_y is the current yield strength of the solid material, i.e., the current von Mises flow stress. In our numerical scheme, if the deviatoric stress components calculated from Eq. (1) do not satisfy the yield criterion condition Eq. (5), they need to be corrected to return to the yield surface. The first correcting scheme was proposed by

Willkins [1,35] in 1964. Recently, Ponthon [37,38] developed another correcting method which can be unified to treat the elasto-plastic and elasto-viscoplastic processes.

The yield stress is determined by Johnson and Cook [39,40] model in the current work. The Johnson–Cook model, including the effects of isotropic strain hardening, strain rate and thermal softening, can be expressed as

$$\sigma_Y = [A + B(\varepsilon^p)^n][1 + C \ln \dot{\varepsilon}^*][1 - T^{*m}].$$

Here, A , B , C , m , and n are the model constants. $\dot{\varepsilon}^* = \frac{\dot{\varepsilon}^p}{\dot{\varepsilon}_0}$ is the relative equivalent plastic strain. If the SI unit system is employed, $\dot{\varepsilon}_0 = 1.0 \text{ s}^{-1}$. $T^* = \frac{T - T_0}{T_m - T_0}$, where T_m and T_r are the melt temperature of the solid material and the room temperature, respectively. Ref. [39] gives the model parameters of 12 kinds of metals commonly used.

The temperature in the Johnson–Cook constitutive model reflects the thermal softening effect of materials. Assume that the temperature increase of materials is mainly from the plastic work

$$\rho C \dot{T} = \beta \dot{W}_p, \quad (6)$$

where T is the temperature, C is the specific heat, β is the Taylor–Quinney parameter [41] which implies the fraction of converting mechanical power into thermal power, and equals 0.9 [42]. $\dot{W}_p$ is the plastic work rate, which is approximated by $\dot{W}_p \approx \dot{\varepsilon}^p \sqrt{\frac{3}{2}} S_{ij} S_{ij}$, where $\dot{\varepsilon}^p$ is the rate of plastic work and defined by

$$\dot{\varepsilon}^p = \sqrt{\frac{2}{3} \dot{D}_{ij}^p \dot{D}_{ij}^p}. \quad (7)$$

Applied Eqs. (6) and (7), the source terms of the equivalent plastic strain and the plastic temperature in Eq. (2) can be expressed as

$$S_{\varepsilon^p} = \dot{\varepsilon}^p \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + \sqrt{\frac{2}{3} \dot{D}_{ij}^p \dot{D}_{ij}^p},$$

$$S_T = T \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + \frac{\beta \dot{W}_p}{\rho C}.$$

In our method, the Mie–Grüneisen equation [4] of state is used to describe the hydrostatic pressure of solid material under high pressure:

$$p = \begin{cases} \frac{\rho_0 c_0^2 \mu [1 + (1 - \frac{\gamma_0}{2}) \mu - \frac{9}{2} \mu^2]}{[1 - (s_1 - 1) \mu - s_2 \frac{\mu^2}{\mu+1} - s_3 \frac{\mu^3}{(\mu+1)^2}]^2} + (\gamma_0 + a \mu) e & \mu \geq 0, \\ \rho_0 c_0^2 \mu + (\gamma_0 + a \mu) e & \mu < 0, \end{cases}$$

where $\mu = \frac{p}{\rho_0} - 1$, s_1 , s_2 , and s_3 are the solid material parameters, and a is the first-order volume correction to the Grüneisen parameter γ_0 .

3. The improved CE/SE scheme

At first, the space–time region is divided into the unified mesh points. For the two-dimensional scheme, let (j, k, n) denote a set of space–time mesh points, where $n = 0, \pm \frac{1}{2}, \pm 1, \pm \frac{3}{2}, \dots$ for time, $j = 0, \pm \frac{1}{2}, \pm 1, \pm \frac{3}{2}, \dots$ for x and $k = 0, \pm \frac{1}{2}, \pm 1, \pm \frac{3}{2}, \dots$ for y . A SE is defined as the vicinity of a mesh point and the whole space–time region is divided into non-overlapping CEs. Assume that the physical variables in every SE are approximated by the Taylor's expansions at the mesh point associated with the SE, and the conservation equation (1) are satisfied in every CE. Different definitions of SE and CE can be obtained different numerical schemes. We design a new type of SE and CE on the general hexahedrons mesh, which is different from that of the Chang's [13] original CE/SE method (see Figs. 1–3). The construction of the new numerical scheme is simpler than Chang's and it is easy to be extended to three-dimensional problems.

Fig. 1. Mesh points projection on x - y plane.

Fig. 2. Solution Element $SE(P)$.

Fig. 3. Conservation Element $CE(P)$.

Let $x_1 = x$, $x_2 = y$, $x_3 = t$ be considered as the coordinates of a Euclidean space E_3 . By using the Gauss' divergence theorem, Eq. (1) can be rewritten in form of

$$\oint_{S(V)} \mathbf{H}_m \cdot \mathbf{ds} = \int_V S_m dV, \quad m = 1, 2, \dots, 10, \quad (8)$$

where $S(V)$ is the boundary of the space–time region V in E_3 , $\mathbf{ds} = \mathbf{n} d\sigma$, in which $d\sigma$ and $\mathbf{n}$ are the area and the unit outward normal vector of a surface element on $S(V)$, and S_m are the components

of the source term vector. Here, $\mathbf{H}_m = (E_m, F_m, \text{ and } Q_m)$ is the space-time flux vector, where E_m, F_m , and Q_m are the components of the vector $\mathbf{E}, \mathbf{F}$, and $\mathbf{Q}$, respectively. Fig. 1 shows the projection of mesh points on the x - y plane, in which the interval between the mesh points $\bullet$ and $\circ$ is $\frac{\Delta t}{2}$ in the time direction or $\frac{1}{2}$ in the mesh number n .

For an arbitrary mesh point $P'(j, k, n)$ on which the variables are solved, define the Solution Element $SE(P')$ constituted by the three vertical planes intersecting at $P'(j, k, n)$ and their neighborhood space as demonstrated in Fig. 2. Suppose that E_m, F_m , and Q_m at point (x, y, t) in $SE(P')$ are approximated by the first-order Taylor's expansions at $P'(j, k, n)$, i.e.,

$$\begin{aligned} Q_m(dx, dy, dt)_{P'} &= (Q_m)_{P'} + (Q_{mx})_{P'} dx + (Q_{my})_{P'} dy + (Q_{mt})_{P'} dt, \\ E_m(dx, dy, dt)_{P'} &= (E_m)_{P'} + (E_{mx})_{P'} dx + (E_{my})_{P'} dy + (E_{mt})_{P'} dt, \\ F_m(dx, dy, dt)_{P'} &= (F_m)_{P'} + (F_{mx})_{P'} dx + (F_{my})_{P'} dy + (F_{mt})_{P'} dt. \end{aligned} \quad (9)$$

Here, $dx = x - x_{P'}$, $dy = y - y_{P'}$ and $dt = t - t_{P'}$, where $x_{P'}, y_{P'}$ and $t_{P'}$ are the position coordinates of point P' . $(X)_{P'}, (X_x)_{P'}, (X_y)_{P'}$ and $(X_t)_{P'}$ are the values of X and its first-order derivatives with respect to x, y , and t at point P' , respectively, where X denotes Q_m, E_m and F_m , respectively.

Substituting Eq. (9) into Eq. (1), we can obtain

$$(Q_{mt})_{P'} = -(E_{mx})_{P'} - (F_{my})_{P'} + (S_m)_{P'}.$$

The above equation implies that the variables required in the computation are merely $(Q_m)_{P'}$, $(Q_{mx})_{P'}$ and $(Q_{my})_{P'}$, because E_m, F_m , and S_m are functions of Q_m .

Define the Conservation Element $CE(P')$ as demonstrated in Fig. 3. From Fig. 3, it can be seen that $CE(P')$ is related to not only $SE(P')$ but also the SEs of $SE(A)$, $SE(C)$, $SE(E)$ and $SE(G)$. Note that the values of physical variables on mesh points A, C, E and G are known. Assume that the integral conservation laws are satisfied in every CE. Integrating Eq. (8) on the surfaces of $CE(P')$ with the aid of Eq. (9), we can obtain

$$(Q_m)_{P'} - \frac{\Delta t}{4} (S_m)_{P'} = \frac{1}{4} \left(\bar{Q} + \frac{\Delta t}{\Delta x} \bar{E} + \frac{\Delta t}{\Delta y} \bar{F} + \frac{\Delta t}{4} \bar{S} \right), \quad (10)$$

where

$$\begin{aligned} \bar{Q} &= Q_m \left(\frac{\Delta x}{4}, \frac{\Delta y}{4}, 0 \right)_A + Q_m \left(-\frac{\Delta x}{4}, \frac{\Delta y}{4}, 0 \right)_C + Q_m \left(-\frac{\Delta x}{4}, -\frac{\Delta y}{4}, 0 \right)_E + Q_m \left(\frac{\Delta x}{4}, -\frac{\Delta y}{4}, 0 \right)_G, \\ \bar{E} &= E_m \left(0, \frac{\Delta y}{4}, \frac{\Delta t}{4} \right)_A - E_m \left(0, \frac{\Delta y}{4}, \frac{\Delta t}{4} \right)_C - E_m \left(0, -\frac{\Delta y}{4}, \frac{\Delta t}{4} \right)_E + E_m \left(0, -\frac{\Delta y}{4}, \frac{\Delta t}{4} \right)_G, \\ \bar{F} &= F_m \left(\frac{\Delta x}{4}, 0, \frac{\Delta t}{4} \right)_A + F_m \left(-\frac{\Delta x}{4}, 0, \frac{\Delta t}{4} \right)_C - F_m \left(-\frac{\Delta x}{4}, 0, \frac{\Delta t}{4} \right)_E - F_m \left(\frac{\Delta x}{4}, 0, \frac{\Delta t}{4} \right)_G, \\ \bar{S} &= (S_m)_A + (S_m)_C + (S_m)_E + (S_m)_G. \end{aligned}$$

Using the continuous conditions at points A', C', E' , and G' , we have the derivatives of Q_m with respect to x and y

$$(Q_{mx})_{P'} = \frac{(Q_{mx})_{P'}^+ + (Q_{mx})_{P'}^-}{2}, \quad (Q_{my})_{P'} = \frac{(Q_{my})_{P'}^+ + (Q_{my})_{P'}^-}{2}, \quad (11)$$

where

$$\begin{aligned} (Q_{mx})_{P'}^- &= -\frac{1}{\Delta x} \left[Q_m \left(0, 0, \frac{\Delta t}{2} \right)_A + Q_m \left(0, 0, \frac{\Delta t}{2} \right)_G - 2(Q_m)_{P'} \right], \\ (Q_{mx})_{P'}^+ &= +\frac{1}{\Delta x} \left[Q_m \left(0, 0, \frac{\Delta t}{2} \right)_C + Q_m \left(0, 0, \frac{\Delta t}{2} \right)_E - 2(Q_m)_{P'} \right], \\ (Q_{my})_{P'}^- &= -\frac{1}{\Delta y} \left[Q_m \left(0, 0, \frac{\Delta t}{2} \right)_A + Q_m \left(0, 0, \frac{\Delta t}{2} \right)_C - 2(Q_m)_{P'} \right], \\ (Q_{my})_{P'}^+ &= +\frac{1}{\Delta y} \left[Q_m \left(0, 0, \frac{\Delta t}{2} \right)_E + Q_m \left(0, 0, \frac{\Delta t}{2} \right)_G - 2(Q_m)_{P'} \right]. \end{aligned}$$

To avoid numerical instability in discontinuous cases, replace Eq. (11) by

$$(Q_{mx})_{P'} = W[(Q_{mx})_{P'}^-, (Q_{mx})_{P'}^+, \alpha], \quad (Q_{my})_{P'} = W[(Q_{my})_{P'}^-, (Q_{my})_{P'}^+, \alpha], \quad (12)$$

where α is an adjustable constant and usually equals $1 \sim 2$, and the weighted equation is $W[x_-, x_+, \alpha] = \frac{|x_-|^{\alpha} x_+ + |x_+|^{\alpha} x_-}{|x_-|^{\alpha} + |x_+|^{\alpha}}$. Obviously, Eq. (12) is same as Eq. (11) when $\alpha = 0$.

It should be noted that $(Q_m)_{P'}$ cannot be obtained explicitly from Eq. (10) due to the source term $(S_m)_{P'}$. As $(S_m)_{P'}$ is a function of $(Q_m)_{P'}$, a local Newton iterative procedure is usually needed to determine $(Q_m)_{P'}$. In the present work, to avoid the iterative procedure and save computation time, we replace $(S_m)_{P'}$ by their linear prediction of current time in Eq. (10), i.e.,

$$\begin{aligned} (Q_m)_{P'} &= \frac{1}{4} \bar{Q} + \frac{\Delta t}{4\Delta x} \bar{E} + \frac{\Delta t}{4\Delta y} \bar{F} + \frac{\Delta t}{2} \tilde{S}, \\ \tilde{S} &= \bar{S} + \frac{\Delta t}{2} \tilde{S}_t, \end{aligned} \quad (13)$$

where $\tilde{S}_t$ is the time derivative of $\bar{S}$. $(Q_m)_{P'}$ can be solved directly without any iteration, since the time derivative of $(S_m)_A, (S_m)_E, (S_m)_C$, and $(S_m)_G$ are all known at current time. In our numerical tests, the results of linear approximation agree with those of Newton iteration, which indicates that the linear approximation of $(S_m)_{P'}$ is reasonable.

4. Interfaces tracing algorithm and boundary conditions

The ghost fluid method proposed by Fedkiw et al. [43] is an effective Eulerian approach to capture interfaces in multi-material flows. Unlike Ref. [43], we adopt different Level Set functions to describe every material interface as illustrated in Fig. 4. The governing equations for each material are solved, respectively, in the region $\phi(\vec{x}, t) < 0$ of each Level Set function, and then the Level Set functions are reevaluated according to the material velocity fields.

The boundary points of a material are defined as that the mesh points are out of the material and needed in the numerical computation. In the two-dimensional case, the CE/SE method needs only one boundary point in x or y direction. All boundary points can be classified into the contact boundary points and the free boundary points. The former describes the interaction between two material blocks, and the latter describes the movement of the free boundary in vacuum. A mesh point is identified as a free boundary point, only if it is a boundary point of one material and does not locate in the other materials. A mesh point is identified as a contact boundary point, only if it is a boundary point of a certain material and locates in the other materials.

4.1 Free boundary conditions

The physical conditions on a free surface are that the normal stress components are zero. For the two-dimensional case, we introduce a local coordinates associated with the interface normal vector. In this local coordinates the stress tensor can be written as

$$\hat{\sigma} = \hat{A}^T \sigma \hat{A}, \quad (14)$$

Fig. 4. Materials, surfaces and level set functions.

where $\hat{\sigma} = \begin{bmatrix} \sigma_{nn} & \sigma_{nt} \\ \sigma_{tn} & \sigma_{tt} \end{bmatrix}$, $A = \begin{bmatrix} n_x & -n_y \\ n_y & n_x \end{bmatrix}$, n_x and n_y are the components of the unit normal vector in x and y directions, and $\sigma = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{bmatrix}$. The normal stress components σ_{nn} and σ_{nt} are zero and the tangential stress component σ_{tt} may not be zero on the free surface.

Suppose point P is a free boundary point. First, calculate the stress components in the local coordinates at the mirror point I of point P of the free surface

$$\sigma_{nn,I} = n_{x,p}^2 s_{xx,I} + n_{y,p}^2 s_{yy,I} + 2n_{x,p}n_{y,p}s_{xy,I} - p_I,$$

$$\sigma_{nt,I} = (n_{x,p}^2 - n_{y,p}^2)s_{xy,I} + n_{x,p}n_{y,p}(s_{yy,I} - s_{xx,I}),$$

$$\sigma_{tt,I} = n_{x,p}^2 s_{yy,I} + n_{y,p}^2 s_{xx,I} - 2n_{x,p}n_{y,p}s_{xy,I} - p_I,$$

where subscript I denotes values at point I , $n_{x,p}$ and $n_{y,p}$ are the components of the unit normal vector in x and y directions at P , which can be computed from the Level Set function. Since the normal stress components are zero on surface and point I is the mirror point of P , it implies

$$\sigma_{nn,P} = -\sigma_{nn,I},$$

$$\sigma_{nt,P} = -\sigma_{nt,I},$$

$$\sigma_{tt,P} = \sigma_{tt,I}.$$

The stress components can be easily converted from the local coordinates to the global Cartesian coordinates by Eq. (14). All other variables such as the density, the velocity and the internal energy are extrapolated linearly from the inside of the material to point P .

4.2. Contact boundary conditions

Here, we consider the sliding interface. The physical conditions imposed on the sliding interface are that the normal velocity and the normal stress components crossing the interface between the two materials are continuous, whereas the tangential velocity and the tangential stress component may remain discontinuous. Suppose point P is a contact boundary point of material 1. So, point P is also in material 2, according to the definition of the contact boundary point.

In the local coordinates the velocity vector is

$$\hat{\mathbf{v}} = A^T \mathbf{v}, \quad (15)$$

$$\text{where } \hat{\mathbf{v}} = \begin{bmatrix} v_n \\ v_t \end{bmatrix} \text{ and } \mathbf{v} = \begin{bmatrix} u \\ v \end{bmatrix}.$$

Given Eq. (15) and the velocity conditions on the sliding interface, the velocity at point P in the local coordinates can be written as

$$v_n = u_{p,2}n_x + v_{p,2}n_y,$$

$$v_t = u_{p,1}n_y - v_{p,1}n_x,$$

where subscript 1 denotes the linearly extrapolated values from material 1, and subscript 2 denotes the actual velocity at point P in material 2. Using Eq. (15), we can obtain the velocity components at point P in the global Cartesian coordinates

$$u_p = v_n n_x + v_t n_y,$$

$$v_p = v_n n_y - v_t n_x.$$

With the stress conditions on the sliding interface and Eq. (14), the components of the stress tensor are calculated from

$$\sigma_{tt} = n_{x,p}^2 s_{yy,1} + n_{y,p}^2 s_{xx,1} - 2n_{x,p}n_{y,p}s_{xy,1} - p_1,$$

$$\sigma_{nn} = n_{x,p}^2 s_{xx,2} + n_{y,p}^2 s_{yy,2} + 2n_{x,p}n_{y,p}s_{xy,2} - p_2,$$

$$\sigma_{nt} = n_{x,p}n_{y,p}(s_{xy,2} - s_{xx,2}) + (n_{x,p}^2 - n_{y,p}^2)s_{xy,2}.$$

It is easy to convert the stress components from the local coordinates to the global Cartesian coordinates by Eq. (14). As the case of the free surface boundary conditions, all other variables such as

the density, the velocity and the internal energy are extrapolated linearly from material 1 to point P .

5. Applications

5.1. Taylor copper bar impact

The famous Taylor bar impact problem is an axisymmetric one where a cylindrical copper bar impacts on a rigid and frictionless wall normally. The length and radius of the copper cylinder is 32.4 and 3.2 mm, respectively. The initial impact velocity is 227 m/s. The mechanical parameters of the copper bar used in the calculation are as follows: the initial density is 8930 kg/m³; the elastic modulus is 117 GPa; the Poisson ratio is 0.35; and the yield strength is 400 MPa. The copper bar is modeled by the linear isotropic hardening constitutive model with tangent hardening modulus $E_t = 100$ MPa. The parameters of Mie–Grüneisen equation of state are listed in Table 1. The rigid wall is treated as the fixed boundary in x direction, the rest boundaries are considered as free surfaces. The computational region is $[0.80 \text{ mm}] \times [0.350 \text{ mm}]$ with 160×700 meshes, which corresponds to $\Delta x = \Delta y = 0.5 \text{ mm}$. The calculation is carried out up to 80 μs , when the normal velocity nearly equals zero.

Fig. 5 shows the deformation and the equivalent plastic strain of the copper bar at four different times 20, 40, 60 and 80 μs . The final maximum radius, the residual bar length, and the maximum equivalent plastic strain calculated by our CE/SE scheme compared with the corresponding results from LS-DYNA and Camacho et al. [42] are listed in Table 2. It can be found that our results are in agreement with the corresponding results from LS-DYNA and Ref. [42]. As shown in Fig. 5, a large gradient of the equivalent plastic strain appears near the rigid wall and the maximum value locates at the axisymmetric axis on the rigid wall. In order to obtain the large gradient of the equivalent plastic strain near the rigid wall, a fine mesh or high-accurate scheme should be used. The agreement of the maximum equivalent plastic strain in Table 2 implies that the efficiency of our CE/SE scheme is quite high.

5.2. Penetration of a WHA long rod into a steel target

In this section, the experiment carried out by Anderson et al. [44] is simulated. A tungsten heavy alloy (WHA) rod projectile penetrates a planar high-strength steel target in the experiment. The WHA rod is 50 mm in length and 2 mm in radius. The steel target is rectangle with 70 mm in length, 40 mm in width and 29 mm in thickness. The incident velocity is 1250 m/s. Camcho et al. [42] have simulated this experiment by their adaptive Lagrangian FEM method. We calculate the same problem under the same conditions using our CE/SE scheme and compare our results with the experimental results and the numerical results from Camcho et al. The target is idealized by the cylinder of the same thickness and 30 mm in radius. Both the tungsten heavy alloy and the high-strength steel are modeled by Johnson–Cook constitutive model and Mie–Grüneisen equation of state, the model parameters are listed in Tables 1 and 3. The computational region is $[0.32 \text{ mm}] \times [0.90 \text{ mm}]$ with 320×900 meshes, which corresponds to $\Delta x = \Delta y = 0.1 \text{ mm}$. The calculation is carried out up to

Table 1

The parameters for Mie–Grüneisen equation of state

Material	c_0 (m/s)	γ_0	s_1	s_2	s_3	A
Copper	3940	1.99	1.489	0	0	0
Tungsten heavy alloy	4030	1.43	1.237	0	0	0
High-strength steel	4570	1.16	1.330	0	0	0.43

Fig. 5. Equivalent plastic strain contours at time 20, 40, 60 and 80 μ s.

Table 2

The comparison of results for the Taylor copper bar impact problem

	Maximum radius	Residual length	Maximum equivalent plastic strain
Our method	7.24	21.50	3.15
LS-DYNA ver970	7.15	21.00	3.30
Ref. [42]	7.21–7.24	21.42–21.44	2.97–3.25

80 μ s, when the velocities of the projectile and the target are nearly equal to zero.

Fig. 6 shows the deformed configuration at four different times 20, 40, 60, and 80 μ s. It can be seen that the long WHA rod becomes shorter and shorter with time developing. The projectile melts and flows back from the bottom of the target crater. Finally the melting projectile protrudes and ejects out of the target crater. The ejecting length of the melting projectile is about 10 mm, which agrees with

Table 3

Parameters of Johnson–Cook Constitutive model

Material	ρ (kg/m ³)	E (GPa)	ν	A (MPa)	B (MPa)	n	C	M	C (J/kg K)	T_m (K)	T_r (K)
Tungsten heavy alloy	17600	323	0.30	1510	177	0.12	0.016	1.00	134	1723	294
High-strength steel	7850	200	0.29	1500	569	0.22	0.003	1.17	477	1777	294

Fig. 6. Deformation at 20, 40, 60 and 80 μ s with impact velocity 1250 m/s; black denotes WHA and gray steel.

Fig. 7. Time histories of projectile nose and tail position for impact velocity 1250 m/s.

Fig. 8. Time histories of projectile nose and tail velocity for impact velocity 1250 m/s.

Comacho et al.'s result [42] well. Being different from the Lagrangian method, the Eulerian method is difficult to calculate the ejecting length accurately. Therefore the computational results indicate that the accuracy of our CE/SE scheme is fairly good.

Fig. 7 shows the projectile nose and tail trajectories as a function of time. As seen, our results agree well with the results from the experiment and Camacho et al. The time histories of the projectile nose and the tail velocity are shown in Fig. 8. As seen in Fig. 8, our results agree with the experiment and our curves are smoother than Camacho et al.'s.

6. Conclusions

In this paper, we develop a new computational scheme for solving the impact problems of multi-material elastic–plastic flows. This scheme is based on our modified CE/SE method, which is simpler and more effective. We describe the solid dynamic response by adopting Eulerian governing equations. The isotropic hardening materials, such as the linear hardening and Johnson–Cook model, are considered. Other constitutive models can also be added to the current frame work conveniently. The interfaces of materials are traced by the high-resolution particle Level Set technique. Suit-

able boundary conditions are also proposed for the current work, which work very well and achieve a high accuracy in the numerical benchmarks.

The main advantages of the current scheme are that it can easily process large boundary deformation and large displacement in space, and it can give the high resolution and accuracy on the physical variables and the positions of boundaries. In the applications of elastic–plastic flow problems, the comparison between our computational results and experiments, as well as results from other numerical methods confirms the high accuracy, resolution, and efficiency of our CE/SE scheme. Our current method should be straightforward extended to three-dimensional problems, since the CE/SE scheme and the hybrid particle level set technique both easy to be extended to three dimensions.

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